Alan Teorisi

Ders 10 – Dirac delta fonksiyonu ve yük dağılımları

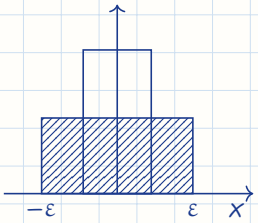
Orijinal not: Ders10.pdf

UYARI Bu belge, el yazısı ders notlarından Claude (Anthropic) yardımıyla dijitale aktarılmıştır. Yardımcı kaynak olarak hazırlanmıştır; hata veya eksik içerebilir. Lütfen orijinal notları ve ders kaynaklarını esas alın.

NOTICE *This document was transcribed from handwritten lecture notes with the help of Claude (Anthropic). It is meant as a supplementary resource and may contain errors or omissions. Always check against the original notes and official course material.*

orj. s. 1

$$\delta(x-a) = \begin{cases} x \neq a, & 0 \\ x = a, & \infty \end{cases}$$

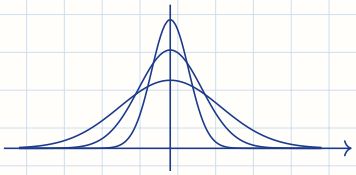


$$f_\varepsilon(x) = \begin{cases} 0; & |x| > \varepsilon \\ 1/2\varepsilon & |x| < \varepsilon \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(x-a)\,dx = 1$$

$$S_T = \frac{1}{2\varepsilon} \cdot 2\varepsilon = 1 \Rightarrow \lim_{\varepsilon\to 0} S_T = \lim_{\varepsilon\to 0} \frac{1}{2\varepsilon} 2\varepsilon = \lim_{\varepsilon\to 0} 1 = 1$$

$$\delta(x) = \lim_{z\to\infty} \frac{z}{\sqrt{\pi}} e^{-z^2x^2} = \begin{cases} 0, & x \neq 0 \\ \infty, & x = 0 \end{cases}$$

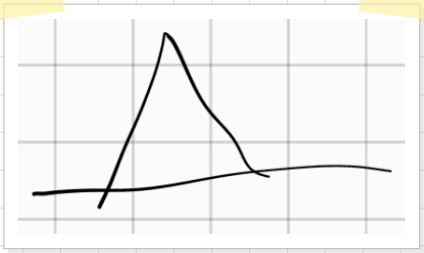


$$\int_{-\infty}^{\infty} \delta(x)\,dx = 1 = \int_{-\infty}^{\infty} \lim_{z\to\infty} \frac{z}{\sqrt{\pi}} e^{-z^2x^2}\,dx$$
$$= \frac{1}{\sqrt{\pi}} \lim_{z\to\infty} z \int_{-\infty}^{\infty} e^{-z^2x^2}\,dx$$

$$zx = x' \Rightarrow dx = \frac{dx'}{z}$$

$$\int_{-\infty}^{\infty} \delta_z(x)\,dx = \int_{-\infty}^{\infty} \lim \frac{z}{\sqrt{\pi}} e^{-z^2x^2}\,dx$$
$$= \frac{1}{\sqrt{\pi}} \lim_{z\to\infty} z \int_{x=-\infty}^{\infty} e^{-x'^2} \frac{dx'}{z}$$
$$= \frac{1}{\sqrt{\pi}} \lim_{z\to\infty} z \cdot \frac{1}{z} \int_{-\infty}^{\infty} e^{-x'^2}$$
$$= \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-x'}\,dx' = \frac{\sqrt{\pi}}{\sqrt{\pi}} = 1$$ [1]

[1] Claude notu: üs muhtemelen $-x'^2$ olmalı.



orijinal çizim, s. 1

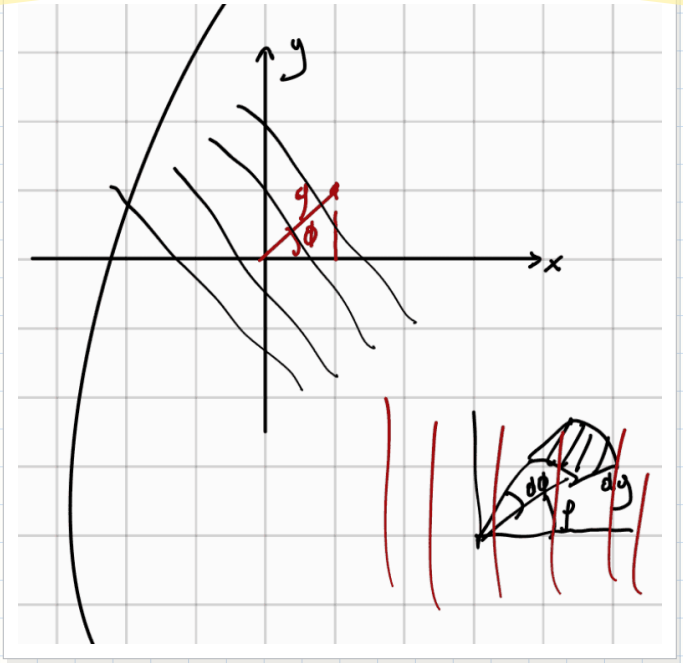
orj. s. 2

$$I = \int_{-\infty}^{\infty} e^{-x^2}\,dx = \int_{-\infty}^{\infty} e^{-y^2}\,dy > 0$$

$$\int_{-\infty}^{\infty} e^{-x^2}\,dx \int_{-\infty}^{\infty} e^{-y}\,dy = I^2 \quad [2]$$

$$I^2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-x^2-y^2}\,dx\,dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \underbrace{e^{-(x^2+y^2)}}_{f(x,y)}\,dx\,dy$$



orijinal çizim, s. 2

$$e^{-(x^2+y^2)} = e^{-\rho^2}$$

$$ds = dx\,dy$$

$$ds = \rho\,d\phi\,d\rho$$

$$0 < \rho < \infty$$

$$0 < \phi < 2\pi$$

[2] Claude notu: üs muhtemelen $-y^2$ olmalı.

$$\int_{\rho=0}^{\infty}\int_{\phi=0}^{2\pi} e^{-\rho^2}\,\rho\,d\rho\,d\phi = \frac{2\pi}{-2}\int_{\rho=0}^{\infty} -2\rho\,e^{-\rho^2}\,d\rho$$

$$I^2 = -\pi\int_{\rho=0}^{\infty} -2\rho\,e^{-\rho^2}\,d\rho = -\pi\,e^{-\rho^2}\Big|_{\rho=0}^{\infty} = -\pi\,(e^{-\infty} - e^{0}) = \pi$$

$$I = \int_{-\infty}^{\infty} e^{-x^2}\,dx = 2\int_{0}^{\infty} e^{-x^2}\,dx = \sqrt{\pi} \;\Rightarrow\; \int_{0}^{\infty} e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2}$$

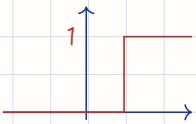
orj. s. 3

$$\star\quad f(x)\,\delta(x-a) = f(a)\,\delta(x-a)$$

$$\star\quad \int_{-\infty}^{\infty} f(x)\,\delta(x-a)\,dx = \int_{-\infty}^{\infty} f(a)\,\delta(x-a)\,dx$$
$$= f(a)\int_{-\infty}^{\infty}\delta(x-a)\,dx = f(a)$$

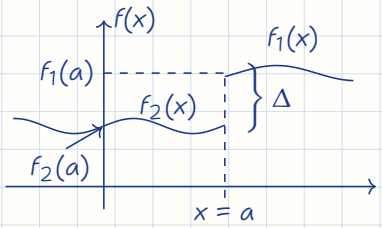
$$\int_{-\infty}^{\infty}\delta(x-a)\,dx = 1 = \int_{\xi_1}^{\xi_2}\delta(x-a)\,dx = \begin{cases} 1, & \xi_1 < a < \xi_2 \\ 0, & \text{aksi halde} \end{cases}$$

$$f(x) = \int_{-\infty}^{x}\delta(z-a)\,dz = \begin{cases} 1; & a < x \\ 0; & a > x \end{cases} = \upsilon(x)$$



$$\upsilon(x-a) \overset{\Delta}{=} \int_{-\infty}^{x}\delta(z-a)\,dz \;\Rightarrow\; \frac{d\upsilon(x-a)}{d(x-a)} = \upsilon'(x-a) = \delta(x-a)$$

$$\upsilon'(x) \overset{\Delta}{=} \delta(x)$$



$$f(x) = \begin{cases} f_1(x); & x > a \\ f_2(x); & x < a \end{cases} = f_1(x)\,\upsilon(x-a) + f_2(x)\,[1-\upsilon(x-a)]$$

$$f_1(a) - f_2(a) = \Delta$$

$$f'(x) = \frac{df(x)}{dx} = \frac{d}{dx}[f_1(x)\,\upsilon(x-a)] + \frac{d}{dx}[f_2(x)\,[1-\upsilon(x-a)]]$$
$$= f_1'(x)\,\upsilon(x-a) + f_1(x)\,\upsilon'(x-a)^{3} + f_2'(x)\,[1-\upsilon(x-a)] + f_2(x)\,[1-\upsilon(x-a)]'$$
$$= f_1'(x)\,\upsilon(x-a) + f_2'(x)\,[1-\upsilon(x-a)] + f_1(x)\,\delta(x-a) - f_2(x)\,\delta(x-a)$$
$$= \begin{cases} f_1'(x); & x > a \\ f_2'(x); & x < a \end{cases} + [f_1(x) - f_2(x)]\,\delta(x-a)$$
$$= \begin{cases} f_1'(x); & x > a \\ f_2'(x); & x < a \end{cases} + \underbrace{[f_1(a) - f_2(a)]\,\delta(x-a)}_{\text{tekil kısmı distribüsyondur}}$$

[3] Claude notu: ikinci terim muhtemelen $f_1(x)\,\upsilon'(x-a)$ olmalı (çarpım türevi).

orj. s. 4

$$V = \begin{cases} V_1(x,y,z); & x > a \\ V_2(x,y,z) & x < a \end{cases}$$

$$\Rightarrow \operatorname{grad} V = \{\operatorname{grad} V\} + \vec{e}_x \underbrace{[V_1 - V_2]|_{x=a}}_{\Delta V|_{x=a}} \delta(x-a)$$

$$= \begin{cases} \operatorname{grad} V_1; & x > a \\ \operatorname{grad} V_2; & x < a \end{cases}$$

$$= \vec{e}_x (V_1 - V_2)|_{x=a}$$

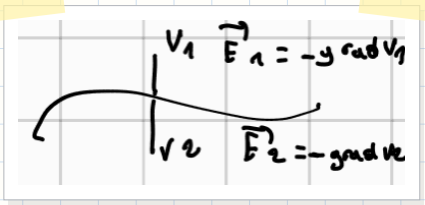
$$= -\vec{e}_x (V_2 - V_1)|_{x=a}$$

$$\vec{D} = \begin{cases} \vec{D}_1, & x > a \\ \vec{D}_2, & x < a \end{cases}$$

$$\operatorname{div} \vec{D} = \begin{cases} \operatorname{div} \vec{D}_1; & x > a \\ \operatorname{div} \vec{D}_2; & x < a \end{cases} + \vec{e}_x (\vec{D}_1 - \vec{D}_2)|_{x=a} \delta(x-a)$$

$$\vec{n} \Delta f|_S \delta(S)$$

$$\vec{E} = \begin{cases} \vec{E}_1, & x > a \\ \vec{E}_2, & x < a \end{cases} \Rightarrow \operatorname{rot} \vec{E} = \left\{ \begin{array}{ll} \operatorname{rot} \vec{E}_1; & x > a \\ \operatorname{rot} \vec{E}_2; & x < a \end{array} \right\}$$



orijinal çizim, s. 4

$$\vec{E} = -\operatorname{grad} V = \begin{cases} \vec{E}_1 \\ \vec{E}_2 \end{cases} + \vec{n}[V_1 - V_2]|_S \delta(S)$$

$$\downarrow$$

$$\vec{n}\,[V_1|_S - V_2|_S]\,\delta(S)$$

$$\vec{D}_1 = \varepsilon_1 \vec{E}_1$$

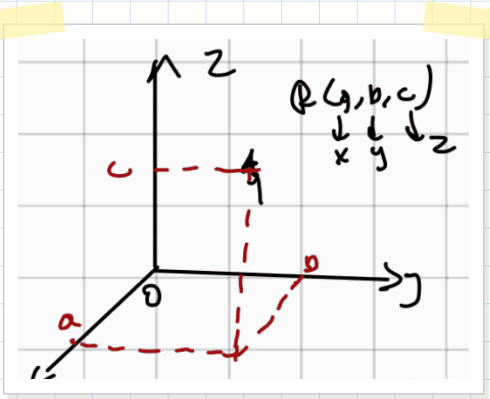
$$\vec{D}_2 = \varepsilon_2 \vec{E}_2$$

$$\vec{D} = \begin{cases} \vec{D}_1; & \text{üst}(S) \\ \vec{D}_2; & \text{alt}(S) \end{cases}$$

$$\operatorname{div} \vec{D} = \left\{ \begin{array}{ll} \operatorname{div} \vec{D}_1 & \text{üst}(S) \\ \operatorname{div} \vec{D}_2 & \text{alt}(S) \end{array} \right\} + \underbrace{\vec{n}\,(\vec{D}_1 - \vec{D}_2)|_S}_{\rho_s} \delta(S)$$

orj. s. 5

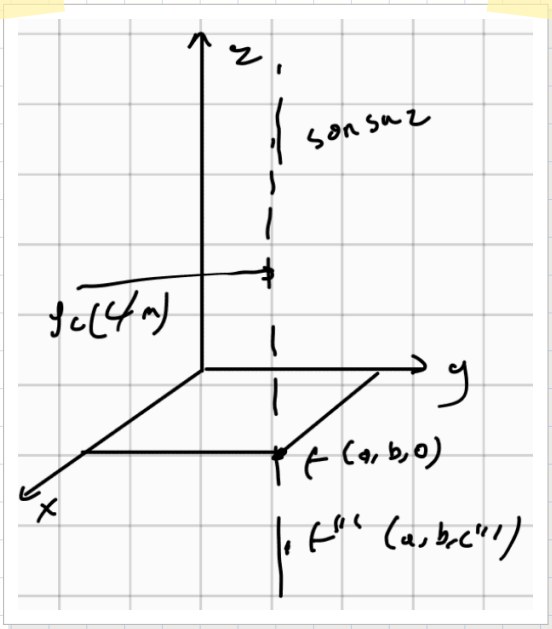
$$\rho = \left\{ \begin{array}{l} \rho_1 \\ \rho_2 \end{array} \right\} + \rho_s \delta(S)$$



orijinal çizim, s. 5

$$\rho = q\,\underbrace{\delta(x-a)}_{1/m}\,\underbrace{\delta(y-b)}_{1/m}\,\underbrace{\delta(z-c)}_{1/m} \quad (C/m^3)$$

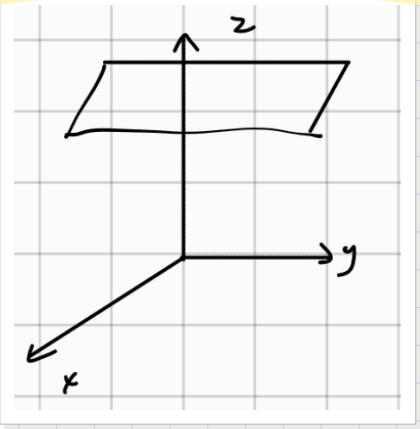
$$\rho = \frac{dq}{dv} = \infty$$



orijinal çizim, s. 5

$$\rho = \rho_s\,\delta(x-a)\,\delta(y-b)$$ [4]

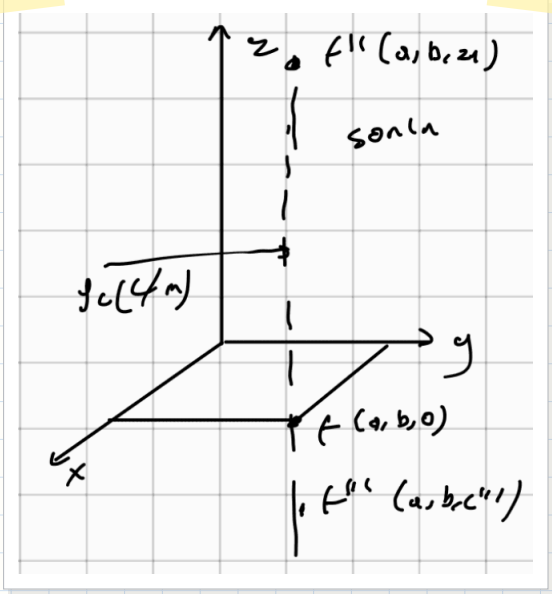
[4] Claude notu: sonsuz doğru yükü için çizimdeki ρ_c (C/m) beklenir; ρ_s yüzey yük yoğunluğudur.



orijinal çizim, s. 5

$$\rho = \rho_s\,\delta(z-a) \quad (C/m^3)$$

orj. s. 6



orijinal çizim, s. 6

$$\rho = \rho_c\,\delta(x-a)\,\delta(y-b)\,[u(z-z_1)+u(z-z_2)]$$ [5]

$$= \rho_c\,\delta(x-a)\,\delta(y-b) \qquad z_2 < z < z_1$$

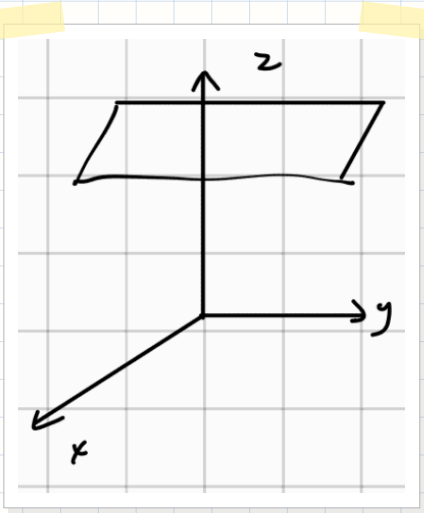
$$\sin x\,\delta(x) = \sin 0\,\delta(x) = 0$$

$$\cos x\,\delta(x) = \cos\,\delta(x) = \delta(x)$$ [6]

$$\tan(x)\,\delta(x-\pi)$$

$$= \tan(\pi)\,\delta(x-\pi) = 0 \quad /\!/$$

[5] Claude notu: $z_2 < z < z_1$ için muhtemelen $[u(z-z_2)-u(z-z_1)]$ olmalı.
[6] Claude notu: cos 0 olmalı.



orijinal çizim, s. 6

$$\rho = \rho_s \, \delta(z-a) \left[u(y-y_1) - u(y-y_2) \right] \left[u(x-x_2) - u(x-x_1) \right]$$

$$y \; ? < \; y_1 < y_2$$