Alan Teorisi

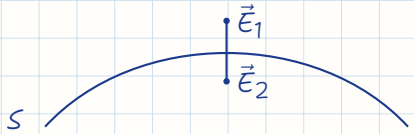
Delta fonksiyonu ve yük yoğunluğu ifadeleri

Orijinal not: Ders12.pdf

UYARI Bu belge, el yazısı ders notlarından Claude (Anthropic) yardımıyla dijitale aktarılmıştır. Yardımcı kaynak olarak hazırlanmıştır; hata veya eksik içerebilir. Lütfen orijinal notları ve ders kaynaklarını esas alın.

NOTICE *This document was transcribed from handwritten lecture notes with the help of Claude (Anthropic). It is meant as a supplementary resource and may contain errors or omissions. Always check against the original notes and official course material.*

orj. s. 1



$$\vec{F} = q\vec{E}$$

$$\vec{E}_s = \frac{\vec{E}_1|_S + \vec{E}_2|_S}{2}$$

$$d\vec{F} = \underbrace{\rho_s\, ds}_{dq}\,\vec{E} \Rightarrow \vec{F} = \int_S \rho_s \vec{E}_s\, ds = \frac{1}{2}\int_S \rho_s \left(\vec{E}_1|_S + \vec{E}_2|_S\right) ds$$

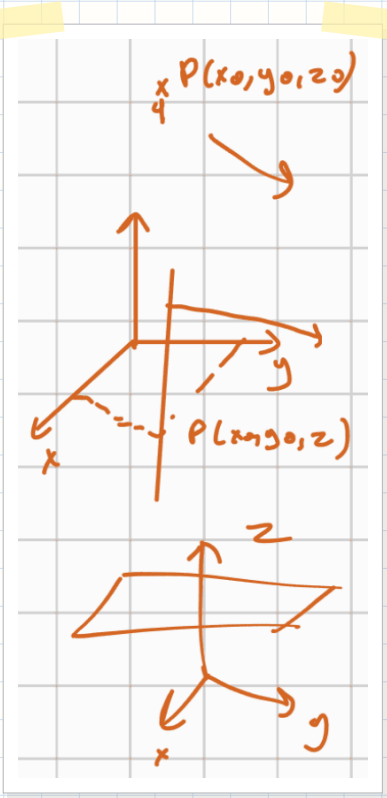
* $f(x)\,\delta(x-x_0) = f(x_0)\,\delta(x-x_0)$

$$\int_{-\infty}^{\infty} f(x)\,\delta(x-x_0)\,dx = \int_{-\infty}^{\infty} f(x)\,\delta(x-x_0)^{1}$$
$$= f(x_0)\int_{-\infty}^{\infty} \delta(x-x_0)\,dx = f(x_0) \;/\!/$$

$$\langle \delta(x-x_0),\, f(x)\rangle \overset{\triangle}{=} \int_{-\infty}^{\infty} \delta(x-x_0)\,f(x)\,dx = f(x_0)$$

$$\int_{-\infty}^{\infty} f(x)\,\delta'(x-x_0)\,dx$$

[1] Claude notu: muhtemelen $f(x_0)\,\delta(x-x_0)\,dx$ olmalı.



orijinal çizim, s. 1

$\rho = q\,\delta(x-x_0)\,\delta(y-y_0)\,\delta(z-z_0)$ $P(x_0,y_0,z_0)$

$\rho = \rho_L\,\delta(x-x_0)\,\delta(y-y_0)$ $P(x_0,y_0,z)$

$\rho = \rho_s\,\delta(z-z_0)$

$$\int_{-\infty}^{+\infty} \underbrace{f(x)}_{u}\ \underbrace{\delta'(x-x_0)\,dx}_{v'}$$



orijinal çizim, s. 1

$p = q\,d = sbt \qquad q \to \infty,\ d \to 0$

$$\rho = p\,\delta'(x-x_0)\,\delta(y-y_0)\,\delta(z-z_0)$$

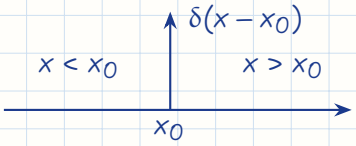
orj. s. 2

$$= f(x)\,\delta(x-x_0)\Big|_{x=-\infty}^{+\infty} - \underbrace{\int_{-\infty}^{\infty} f'(x)\,\delta(x-x_0)\,dx}_{f'(x_0)\,\delta(x-x_0)}$$

$$= f(x_0)\Big[\underbrace{\lim_{x\to\infty}\delta(x-x_0)}_{0} - \underbrace{\lim_{x\to-\infty}\delta(x-x_0)}_{0}\Big]$$

$$- f'(x_0)\underbrace{\int_{-\infty}^{+\infty}\delta(x-x_0)\,dx}_{1}$$

$$= -f'(x_0)$$



$$\int_{-\infty}^{\infty} f(x)\,\delta^{(n)}(x-x_0)\,dx = (-1)^n f^{(n)}(x_0)$$

$$\langle \delta^{(n)}(x-x_0),\, f(x)\rangle = (-1)^n f^{(n)}(x_0) \qquad n = 0,1,2,\ldots \quad n\in\mathbb{N}$$

$$\int_{-\infty}^{\infty} f(x)\,\underbrace{\frac{d^n\delta(x-x_0)}{dx^n}}_{\delta^{(n)}(x-x_0)}\,dx = (-1)^n f^{(n)}(x_0)$$

$$\int_{-\infty}^{\infty} f(x)\,\delta'(x-x_0)\,dx$$

$$= \int_{-\infty}^{+\infty} f(x)\Big[\underbrace{\lim_{\Delta\to0}\frac{\delta(x-x_0+\Delta)-\delta(x-x_0)}{\Delta}}_{\delta'(x-x_0)}\Big]dx$$

$$= \lim_{\Delta\to0}\frac{1}{\Delta}\Big[\underbrace{\int_{-\infty}^{+\infty} f(x)\,\delta\big(x-(x_0-\Delta)\big)\,dx}_{f(x_0+\Delta)^2}$$

$$- \int_{-\infty}^{\infty} f(x)\,\delta(x-x_0)\,dx\Big] = \lim_{\Delta\to0}\frac{f(x_0-\Delta)-f(x_0)}{\Delta} =$$

$$f'(x) = \lim_{\Delta\to0}\frac{f(x+\Delta)-f(x)}{\Delta}$$

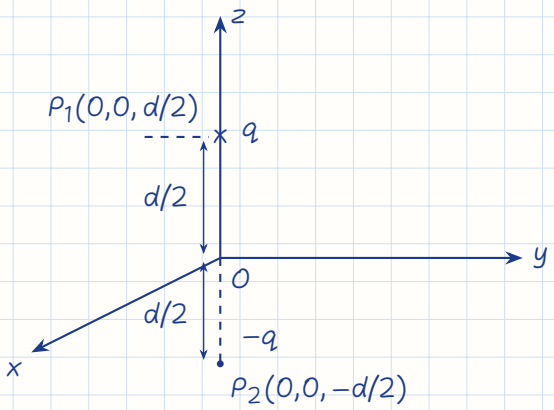
$$\delta'(x-x_0) = \lim_{\Delta\to0}\frac{\delta(x-x_0+\Delta)-\delta(x-x_0)}{\Delta}$$

[2] Claude notu: muhtemelen $f(x_0-\Delta)$ olmalı.

orj. s. 3

$$= -\lim_{\Delta\to 0}\frac{f(x_0-\Delta)-f(x_0)}{-\Delta} = -f'(x_0)$$

$$(-1)^a = e^{a\log(-1)} = e^{\pm i\pi a}$$



$$p = q\cdot d = \text{sabit}$$

$$\rho = q\,\delta(x)\,\delta(y)\,\delta(z-d/2) - q\,\delta(x)\,\delta(y)\,\delta(z+d/2)$$
$$= q\,\delta(x)\,\delta(y)\left[\frac{\delta(z-d/2)-\delta(z+d/2)}{d}\right]d$$
$$= -q\,d\,\delta(x)\,\delta(y)\left[\frac{\delta(z+d/2)-\delta(\overbrace{z-d/2}^{z'})}{d}\right]$$
$$= -p\,\delta(x)\,\delta(y)\left[\frac{\delta(z'+d)-\delta(z')}{d}\right]$$

$$\rho_{\text{dipol}} = \lim_{\substack{d\to 0\\ q\to\infty}}\rho = \lim_{\substack{d\to 0\\ q\to\infty}}(-p)\,\delta(x)\,\delta(y)\left[\frac{\delta(z'+d)-\delta(z')}{d}\right]$$
$$= -\delta(x)\,\delta(y)\underbrace{\lim_{\substack{d\to 0\\ q\to\infty}}p}_{p}\lim_{\substack{d\to 0\\ q\to\infty}}\left[\frac{\delta(z'+d)-\delta(z')}{d}\right]$$

orj. s. 4

$$= -\delta(x)\,\delta(y)\,p\,\delta'(z')$$
$$\rho_{\text{dipol}} = -p\,\delta(x)\,\delta(y)\,\delta'(z-d/2)$$

$d\to 0$'a gidiyordu

$$* \quad \rho_z^{\text{dipol}} = -p\,\delta(x)\,\delta(y)\,\delta'(z)$$
$$\rho_y^{\text{dipol}} = -p\,\delta(x)\,\delta'(y)\,\delta(z)$$
$$\rho_x^{\text{dipol}} = -p\,\delta'(x)\,\delta(y)\,\delta(z)$$

$$\rho_{dipol} = -\operatorname{div}\vec{p}$$
$$= -\operatorname{div}\left[\vec{p}\,\delta(x-x_0)\,\delta(y-y_0)\,\delta(z-z_0)\right]$$

örnek $\vec{p} = p\,\vec{e}_z$

$$= -\operatorname{div}\left[p\vec{n}\cdot\delta(x-x_0)\,\delta(y-y_0)\,\delta(z-z_0)\right]$$
$$= -\operatorname{grad}\left[\delta(x-x_0)\,\delta(y-y_0)\,\delta(z-z_0)\right]p\cdot\vec{n}$$

$$\operatorname{div}(\vec{A}\cdot f) = f\operatorname{div}\vec{A} + \vec{A}\cdot\operatorname{grad} f$$

$$= -\delta(x-x_0)\,\delta(y-y_0)\,\delta(z-z_0) \qquad \underbrace{\cancel{\operatorname{div}(p\cdot\vec{n})}}$$

div $\vec{n}$=0 birimvektör ? dür

$$-p\cdot\vec{n}\operatorname{grad}\left[\delta(x-x_0)\,\delta(y-y_0)\right.$$
$$= -p\cdot\vec{n}\operatorname{grad}\left[\delta(x-x_0)\,\delta(y-y_0)\,\delta(z-z_0)\right]$$

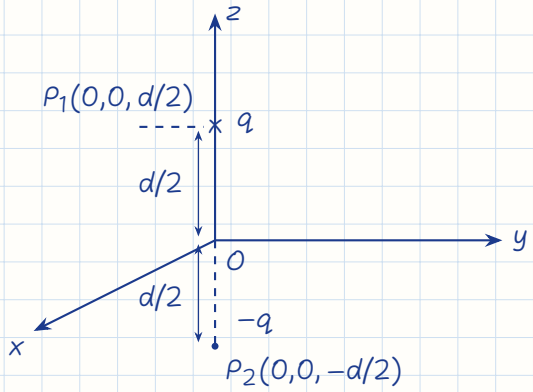
$$\operatorname{div}\vec{e}_x = \frac{\partial 1}{\partial x} = 0$$

orj. s. 5

$$\rho_x^{dipol} = -p\,\vec{e}_x$$

$$\underbrace{\operatorname{grad}\left[\delta(x-x_0)\,\delta(y-y_0)\,\delta(z-z_0)\right]}$$
$$\delta'(x-x_0)\,\delta(y-y_0)\,\delta(z-z_0)\,\vec{e}_x$$
$$+\,\delta(x-x_0)\,\delta'(y-y_0)\,\delta(z-z_0)\,\vec{e}_y$$
$$+\,\delta(x-x_0)\,\delta(y-y_0)\,\delta'(z-z_0)\,\vec{e}_z$$

$$\rho_x^{dipol} = -p\,\vec{e}_x\Big[\delta'(x-x_0)\,\delta(y-y_0)\,\delta(z-z_0)\,\vec{e}_x$$
$$+\,\delta(x-x_0)\,\delta'(y-y_0)\,\delta(z-z_0)\,\vec{e}_y$$
$$+\,\delta(x-x_0)\,\delta(y-y_0)\,\delta'(z-z_0)\,\vec{e}_z\Big]$$
$$= -p\,\delta'(x-x_0)\,\delta(y-y_0)\,\delta(z-z_0)$$



$$\rho_{dipol} = -p\,\vec{n}\,\mathrm{grad}\left[\delta(x)\,\delta(y)\,\delta(z)\right]$$

$$V = \frac{1}{4\pi\varepsilon_0}\iiint \frac{\rho_{dipol}}{r'}\underbrace{dv}_{dx'\,dy'\,dz'}$$

$$r' = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$$

orj. s. 6

$$V(x,y,z) = \frac{1}{4\pi\varepsilon_0}\underbrace{\int_{-\infty}^{+\infty}\int_{x'}\int_{y'}\int_{z'}}\frac{-p\cdot\vec{n}\,\mathrm{grad}\left[\delta(x')\,\delta(y')\,\delta(z')\right]}{\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}}\,dx'\,dy'\,dz'$$

$$V = \frac{-p\cdot\vec{n}}{4\pi\varepsilon_0}\Bigg[\vec{e}_x\underbrace{\int_{-\infty}^{\infty}\int_{x'}\int_{y'}\int_{z'}}\frac{\delta'(x')\,\delta(y')\,\delta(z')}{\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}}\,dx'\,dy'\,dz'$$

$$+\,\vec{e}_y\underbrace{\int_{-\infty}^{\infty}\int_{x'}\int_{y'}\int_{z'}}\frac{\delta(x')\,\delta'(y')\,\delta(z')}{\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}}\,dx'\,dy'\,dz'$$

$$+\,\vec{e}_z\underbrace{\int_{-\infty}^{\infty}\int_{x'}\int_{y'}\int_{z'}}\frac{\delta(x')\,\delta(y')\,\delta'(z')}{\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}}\,dx'\,dy'\,dz'$$

$$\delta(y')\,f(y') = f(0)\,\delta(y')$$

$$V = \frac{-p\cdot\vec{n}}{4\pi\varepsilon_0}\Big[\vec{e}_x \overbrace{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}}^{}\frac{\delta'(x')\,\delta(y')\,\delta(z')}{\sqrt{(x-x')^2+(y)^2+(z)^2}}\,dx'\,dy'\,dz'$$

$$+\vec{e}_y \overbrace{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}}^{}\frac{\delta(x')\,\delta'(y')\,\delta(z')}{\sqrt{(x)^2+(y-y')^2+(z)^2}}\,dx'\,dy'\,dz'$$

$$+\vec{e}_z \overbrace{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}}^{}\frac{\delta(x')\,\delta(y')\,\delta'(z')}{\sqrt{(x)^2+(y)^2+(z-z')^2}}\,dx'\,dy'\,dz'$$

$$\int_{x'=-\infty}^{\infty}\frac{\delta'(x')}{\sqrt{(x-x')^2+y^2+z^2}}\,dx'\cdot\int_{-\infty}^{\infty}\delta(y')\,dy'\int_{-\infty}^{\infty}\delta(z')\,dz'$$

$$V = \frac{-p\cdot\vec{n}}{4\pi\varepsilon_0}\Big[\vec{e}_x\int_{x'=-\infty}^{\infty}\left[(x-x')^2+y^2+z^2\right]^{-1/2}\delta'(x')\,dx'$$

orj. s. 7

$$+\vec{e}_y\int_{y'=-\infty}^{\infty}\left[x^2+z^2+(y-y')^2\right]^{-1/2}\delta'(y')\,dy'$$

$$+\vec{e}_z\int_{z'=-\infty}^{\infty}\left[x^2+y^2+(z-z')^2\right]^{-1/2}\delta'(z')\,dz'\Big]$$

$$V = \frac{-p\cdot\vec{n}}{4\pi\varepsilon_0}\Big[\vec{e}_x\cdot\frac{1}{2}\frac{2(x-x')(-1)}{\left[(x-x')^2+y^2+z^2\right]^{3/2}}\Big|_{x'=0}$$

$$+\vec{e}_y\frac{(y-y')(-1)}{\left[x^2+z^2+(y-y')^2\right]^{3/2}}\Big|_{y'=0}$$

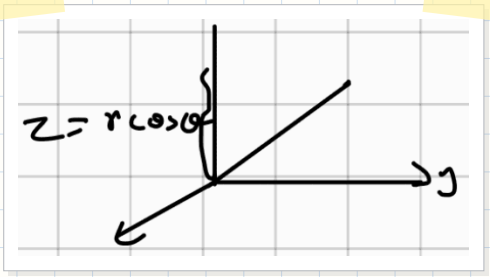
$$+\vec{e}_z\frac{(z-z')(-1)}{\left(x^2+y^2+(z-z')^2\right)^{3/2}}\Big|_{z'=0}\Big]$$

$$V = \frac{p\,\vec{n}}{4\pi\varepsilon_0}\left[\frac{x\,\vec{e}_x+y\,\vec{e}_y+z\,\vec{e}_z}{(x^2+y^2+z^2)^{3/2}}\right]$$

$$= \frac{p\cdot\vec{n}}{4\pi\varepsilon_0}\left[\frac{\vec{r}}{r^3}\right]$$

$$\vec{E} = -\operatorname{grad} V = -\frac{p}{4\pi\varepsilon_0}\,\vec{n}\,\operatorname{div}\left(\frac{\vec{r}}{r^3}\right)$$

$$\operatorname{grad}(\vec{a}\cdot\vec{b}) = \vec{a}\operatorname{div}\vec{b}\ \overbrace{+\vec{b}\operatorname{div}\vec{a}}^{0}\ {}^{3}$$



orijinal çizim, s. 7

$$\vec{e}_z \cdot \vec{r} = z = r\cos\theta$$

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$

$$\vec{n} = \vec{e}_z \Rightarrow V = \frac{p\,\vec{e}_z \cdot \vec{r}}{4\pi\varepsilon_0 r^3} = \frac{p\,z}{4\pi\varepsilon_0 r^3}$$
$$= \frac{p\,r\cos\theta}{4\pi\varepsilon_0 r^3}$$
$$= \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$$

orj. s. 8

$$\Delta V = -\frac{\rho}{\varepsilon_0} \Rightarrow \underbrace{V = \frac{1}{4\pi\varepsilon_0}\iiint \frac{\rho}{r'}\,dv}_{\text{Poisson denkleminin çözümü}}$$

$$\vec{n} = \vec{e}_y \Rightarrow V = \frac{p\,y}{4\pi\varepsilon_0 r^3} = \frac{p\sin\theta\sin\phi}{4\pi\varepsilon_0 r^2}$$

$$\vec{n} = \alpha\vec{e}_x + \beta\vec{e}_y + \gamma\vec{e}_z$$
$$|\vec{n}| = \sqrt{\alpha^2+\beta^2+\gamma^2} = 1$$

$$V = \frac{p\,\vec{n}}{4\pi\varepsilon_0}\left[\frac{\vec{r}}{r^3}\right] = \frac{p}{4\pi\varepsilon_0}\,\frac{\alpha x+\beta y+\gamma z}{r^3}$$
$$= \frac{p}{4\pi\varepsilon_0}\,\frac{1}{r^2}\Big[\alpha\sin\theta\cos\phi$$
$$+ \beta\sin\theta\sin\phi$$
$$+ \gamma\cos\theta\Big]$$

[3] Claude notu: genel özdeşlik $\operatorname{grad}(\vec{a}\cdot\vec{b}) = (\vec{a}\cdot\nabla)\vec{b} + (\vec{b}\cdot\nabla)\vec{a} + \vec{a}\times\operatorname{rot}\vec{b} + \vec{b}\times\operatorname{rot}\vec{a}$ biçimindedir; yazılan ifade bu haliyle genel değildir.