# Alan Teorisi

## Ders 14 – Görüntü kavramı ve yöntemi

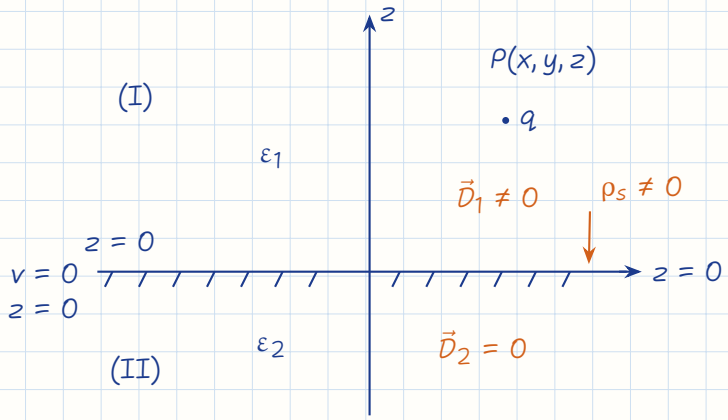
Orijinal not: Ders14.pdf

**UYARI** Bu belge, el yazısı ders notlarından Claude (Anthropic) yardımıyla dijitale aktarılmıştır. Yardımcı kaynak olarak hazırlanmıştır; hata veya eksik içerebilir. Lütfen orijinal notları ve ders kaynaklarını esas alın.

**NOTICE** *This document was transcribed from handwritten lecture notes with the help of Claude (Anthropic). It is meant as a supplementary resource and may contain errors or omissions. Always check against the original notes and official course material.*

orj. s. 1

## Görüntü kavramı ve yöntemi




$$\rho_s = \vec{e}_z(\vec{D}_1 - \underbrace{\vec{D}_2}_{0})\Big|_{z=0}$$

$$= \vec{e}_z \vec{D}_1\Big|_{z=0} = -\vec{e}_z\, \varepsilon_1 \operatorname{grad} V_1\Big|_{z=0}$$

$z < 0$ bölgesi ($\varepsilon_2$ dielektrik sabitli bölge)

$V_2 = 0$ sabit
$\vec{E}_2 = 0$ ; $\vec{D}_2 = 0$

II nolu bölgede yük olmadığından ve $z = 0$ yüzeyi üzerinde $v = 0$ sabit olduğundan bölge iletkendir ve potansiyeli $V_2 = 0$ olur.

$z > 0$ bölgesi ($\varepsilon_1$ dielektrik sabitli bölge)

$V_1 = ?$ $\quad \vec{E} = -\operatorname{grad} V_1$ ; $\quad \vec{D}_1 = \varepsilon_1 \vec{E}_1 = -\varepsilon_1 \operatorname{grad} V_1$

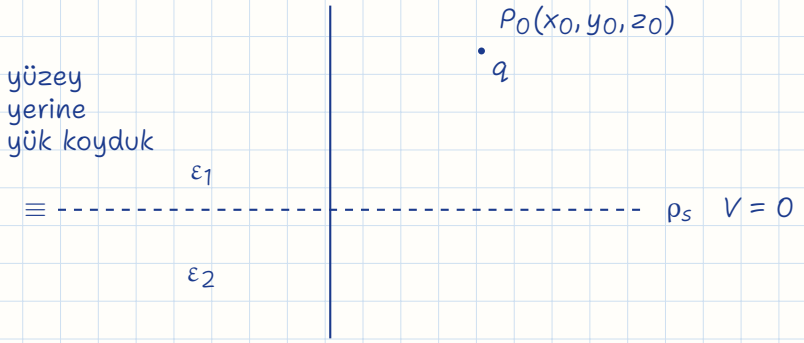
$z > 0$ bölgesinin $\rho = q\,\delta(x - x_0)\,\delta(y - y_0)\,\delta(z - z_0) \neq 0$

I nolu bölge ise II gibi iletken bir bölgede değil, $V_1 \neq 0$

(I) $V_1$'in sağladığı diferansiyel denklem ve sınır koşul dikkate alınmalı.

$$z > 0 \text{ da } \begin{cases} \Delta V_1 = -\dfrac{\rho}{\varepsilon_1} = -\dfrac{q\,\delta(x-x_0)\,\delta(y-y_0)\,\delta(z-z_0)}{\varepsilon_1} \\ V_1\big|_{z=0} = 0 \qquad \lim\limits_{z\to\infty} V_1 \text{ için } < \infty \end{cases}$$

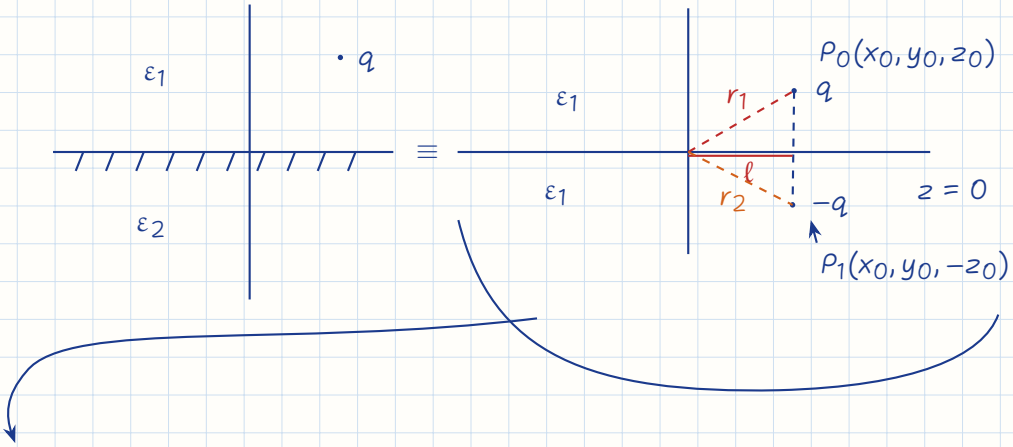
Ek örnek problem




orj. s. 2

$z > 0$ bölgesi bakımından bir birine özdeştir

altı baştan biliyoruz yukarısı özdeştir



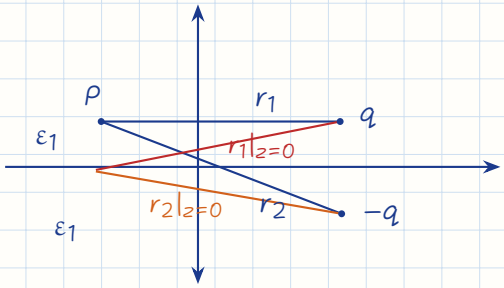

$$z > 0 \text{ da } \Delta V_1 = -\frac{\rho_1}{\varepsilon_1} = -\frac{q\,\delta(x-x_0)\,\delta(y-y_0)\,\delta(z-z_0)}{\varepsilon_1}$$

$$V_q\big|_{z=0} = \frac{q}{4\pi\varepsilon_1 r_1} \qquad V_{-q}\big|_{z=0} = \frac{-q}{4\pi\varepsilon_1 r_2}$$

$r_1 = r_2 = \sqrt{z_0^2 + \ell^2}$

* Tüm uzay homojense Coulomb yasası geçerli

$$V\big|_{z=0} = V_q\big|_{z=0} + V_{-q}\big|_{z=0} = \frac{q}{4\pi\varepsilon_1 r_1} - \frac{q}{4\pi\varepsilon_1 r_1} = V_1\big|_{z=0} = 0$$




$$V_P = \frac{q}{4\pi\varepsilon_1 r_1} - \frac{q}{4\pi\varepsilon_1 r_2}$$

$$V_P = \frac{q}{4\pi\varepsilon_1}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$V_1\Big|_{z=0} = V_P\Big|_{z=0} = \frac{q}{4\pi\varepsilon_1}\left(\frac{1}{r_1|_{z=0}} - \frac{1}{r_2|_{z=0}}\right) = 0$$

orj. s. 3

$$\Rightarrow \quad z > 0 \quad V_1 = \frac{q}{4\pi\varepsilon_1}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

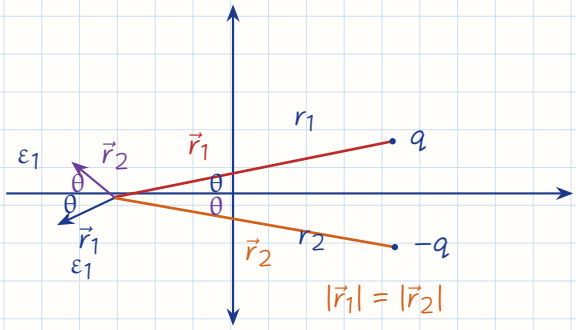
$$z < 0 \quad V_2 = 0$$

$$\Rightarrow \vec{E}_1 = -\operatorname{grad} V_1 = \frac{q}{4\pi\varepsilon_1}\frac{\vec{r}_1}{r_1^3} - \frac{q}{4\pi\varepsilon_1}\frac{\vec{r}_2}{r_2^3}$$

$$\vec{D}_1 = \varepsilon_1\vec{E}_1 = \frac{q}{4\pi}\left(\frac{\vec{r}_1}{r_1^2} - \frac{\vec{r}_2}{r_2^3}\right) \quad [1]$$

$$\rho_s\Big|_{z=0} = \vec{e}_z(\vec{D}_1 - \vec{D}_2)\Big|_{z=0} = \vec{e}_z \cdot \vec{D}_1\Big|_{z=0}$$

$$\rho_s = \frac{q}{4\pi}\vec{e}_z\left(\frac{\vec{r}_1}{r_1^3}\Big|_{z=0} - \frac{\vec{r}_2}{r_2^3}\right)\Big|_{z=0}$$




$$\vec{r}_1\Big|_{z=0} = r_1 \cdot \vec{n}_1$$

$$\vec{r}_1 = r_1\Big|_{z=0} n_1\Big|_{z=0} = r_1[\cos\theta\,\vec{e}_z - \sin\theta\,\vec{e}_z] \quad [2]$$

[1] Claude notu: muhtemelen $r_1^3$ olmalı.

$$\vec{r}_2\big|_{z=0} = r_2\big|_{z=0}\, n_2\big|_{z=0} = r_2\left[\cos\theta\,\vec{e}_z + \sin\theta\,\vec{e}_z\right]$$

$$r_1 = r_2 = r$$

$$\Rightarrow \rho_s = \frac{q}{4\pi}\vec{e}_z\left(\underbrace{\frac{\vec{r}_1|_{z=0}}{(r_1|_{z=0})^3}}_{r^3} - \underbrace{\frac{\vec{r}_2|_{z=0}}{(r_2|_{z=0})^3}}_{r^3}\right)$$

$$= \frac{q}{4\pi}\frac{\vec{e}_z}{r^3}\left(\vec{r}_1|_{z=0} - \vec{r}_2|_{z=0}\right)$$

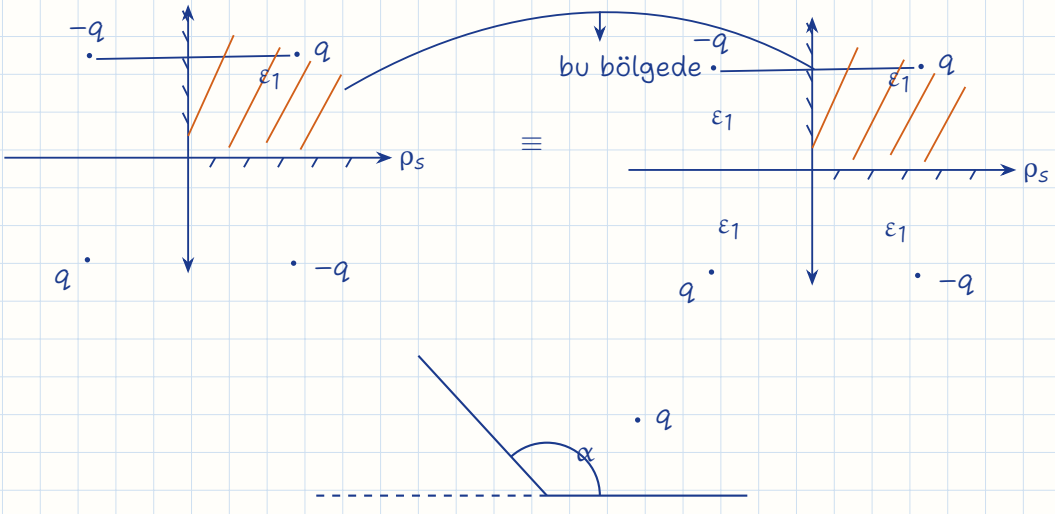
$$= \frac{q}{4\pi r^3}\vec{e}_z\left[r\cos\theta\,\vec{e}_z - r\sin\theta\,\vec{e}_z - r\cos\theta\,\vec{e}_\ell - r\sin\theta\,\vec{e}_z\right]$$

$$= \frac{-q\sin\theta}{2\pi r^2} \qquad \text{ve } \sin\theta = \frac{\text{Karşı}}{\text{hipo}} = \frac{z_0}{r}$$

yüzeyin üzerindeki yük 0 değildir ve değişkendir

$$= \frac{-q z_0}{2\pi r^3} \qquad (C/m^2)$$

orj. s. 4

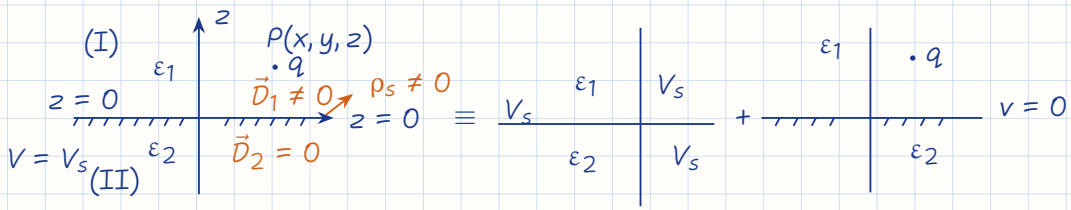




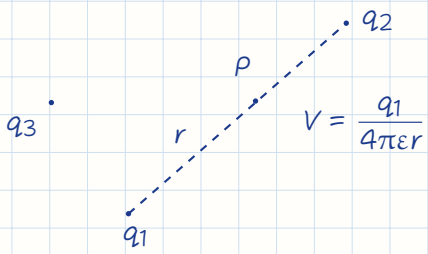


$$n_i = \frac{2\pi}{\alpha} - 1 \qquad \text{görüntü yük sayısı}$$

tam sayı gelmezse sonsuz görüntü var demektir.




[2] Claude notu: cos θ ile çarpılan birim vektör muhtemelen düzlem içi yöndeki birim vektör olmalı ($\vec{e}_z$ değil); sin θ ile çarpılan ise $\vec{e}_z$. Aşağıdaki satırlar için de aynı.



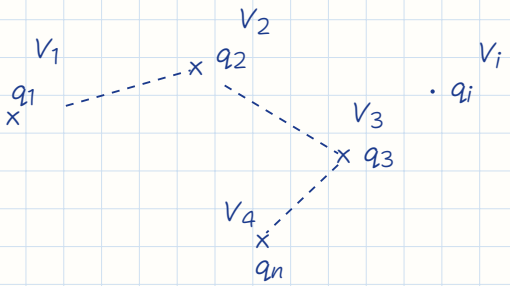

$q_1$ baştan vardı
$q_2$ uzaktan getirildi

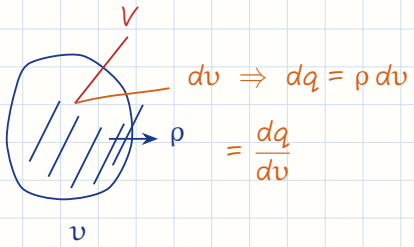
$$W = q_2(V_1 - \underbrace{V_\infty}_{0}) = q_2 \frac{q_1}{4\pi\varepsilon r}$$

$$W = \frac{1}{2}\sum_{i=1}^{n} q_i V_i$$
$$= \frac{1}{2}\left[q_1 V_1 + q_2 V_2 + \cdots + q_n V_n\right]$$

$$W_1 = W_{13} + W_{23} + W_{12}$$
$$= \frac{q_3 q_1}{4\pi\varepsilon r_{13}} + \frac{q_3 \cdot q_2}{4\pi\varepsilon r_{23}} + \frac{q_1 \cdot q_2}{4\pi\varepsilon r_{12}}$$

orj. s. 5



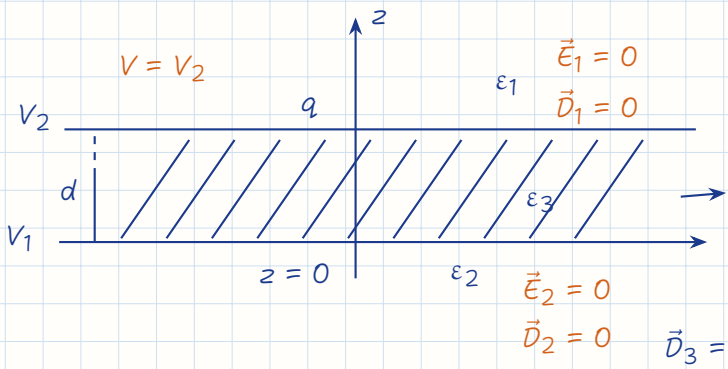




$$dV = \frac{1}{2}\rho\, d\upsilon \cdot V$$ [3]

$$\Rightarrow W = \frac{1}{2}\int_\upsilon \rho V\, d\upsilon$$
$$= \frac{1}{2}\int \underbrace{\rho}_{\operatorname{div}\vec{D}} V\, d\upsilon \qquad = \frac{1}{2}\int$$

$$\vec{E} = -\operatorname{grad} V = -\frac{\partial V}{\partial n}\vec{n}$$
$$= \frac{1}{2}\iint \rho V\, d\upsilon = \frac{1}{2}\int \vec{E}\cdot\underbrace{\vec{D}}_{\varepsilon\vec{E}}\, d\upsilon = \frac{1}{2}\int \varepsilon |\vec{E}|^2\, d\upsilon$$

[3] Claude notu: muhtemelen $dW$ olmalı.




$$\vec{E} = -\operatorname{grad} V = \frac{V_2 - V_1}{d}\vec{e}_z$$ [4]

$$\vec{D}_3 = \varepsilon_3 \vec{E} = \varepsilon_3 \frac{V_1 - V_2}{d}\vec{e}_z$$

$$\Delta V = 0$$

$$V = Az + B$$

$$B = V|_{z=0} = V_1 \qquad V|_{z=d} = V_2 = Ad + V_1$$

$$A = \frac{V_2 - V_1}{d}$$

orj. s. 6

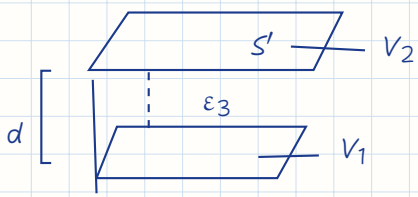
$$W = \frac{1}{2}\int \varepsilon |\vec{E}|^2 \, dv$$

$$W = \frac{1}{2}\int_{v_1} \varepsilon_1 |\vec{E}_1|^2 \, dv_1 + \frac{1}{2}\int_{v_3} \varepsilon_3 |\vec{E}_3|^3 \, dv_3$$ [5]

$$+ \underbrace{\frac{1}{2}\int_{v_2} \varepsilon_2 |\vec{E}_2|^2 \, dv_2}_{0}$$

$$= \frac{\varepsilon_3}{2}\frac{(V_2 - V_1)^2}{d^2} \underbrace{\int_{v_3} dv_3}_{S'd} = \frac{\varepsilon_3}{2}\frac{(V_2 - V_1)^2}{d} S' \not{d}$$

$$= \frac{\varepsilon_3}{2d} S' (V_2 - V_1)^2$$

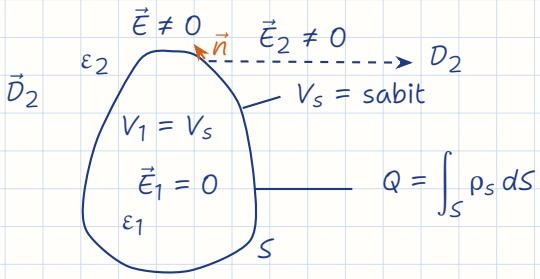



← kapasite

[4] Claude notu: eksi işareti eksik görünüyor; $\vec{E} = -A\vec{e}_z = \frac{V_1 - V_2}{d}\vec{e}_z$ olmalı (alttaki $\vec{D}_3$ satırıyla tutarlı).

[5] Claude notu: üs muhtemelen 2 olmalı.

# Kapasite problemi




$$\vec{D}_2 = \varepsilon_2(-\operatorname{grad} V_2) = \varepsilon_2 \operatorname{grad} V_2 \quad [6]$$

$$\rho_s = \vec{n}(\vec{D}_2 - \vec{D}_1)\Big|_S = \vec{n} \cdot \vec{D}_2\Big|_S$$

$$= -\vec{n}\,\varepsilon_2 \operatorname{grad} V_2\Big|_S \qquad = -\varepsilon_2 \frac{\partial V_2}{\partial n}\Big|_S$$

$$\frac{\partial V_2}{\partial n} = -\operatorname{grad} V_2 \cdot \vec{n}\Big|_S \quad [7]$$

$$= -\varepsilon_2 \int_S \frac{\partial V_2}{\partial n} \, dS$$

orj. s. 7

$$V_2 = V_s \, \Phi(x, y, z)$$

$$V_2\Big|_S = V_s \, \Phi(x, y, z)\Big|_S = V_s$$

$$\Rightarrow \cancel{V_s} \, \Phi(x, y, z)\Big|_S = \cancel{V_s}$$

$$\Rightarrow \Phi(x, y, z)\Big|_S = 1$$

geometriye bağlı fonksiyondur.

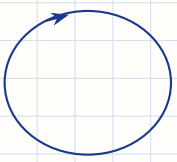
$$\Delta V_2 = V_s \, \Delta\Phi = 0 \qquad = -\frac{\rho}{\varepsilon_2} = 0$$

$$\Delta\Phi = 0$$

$$\Phi\Big|_S = 1$$

$$Q = -\varepsilon_2 V_s \int_S \frac{\partial \Phi}{\partial n} \, dS$$

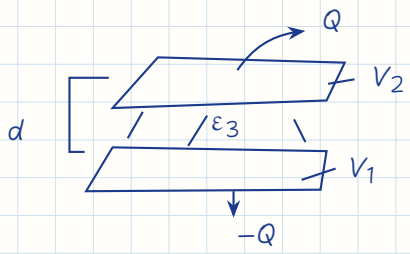
$$\frac{Q}{V_s} \triangleq \underbrace{C}_{\text{iletkenin yapısı}} = -\varepsilon_2 \int_{S'} \frac{\partial \Phi}{\partial n} \, dS$$




[6] Claude notu: muhtemelen $-\varepsilon_2 \operatorname{grad} V_2$ olmalı.

[7] Claude notu: eksi işareti muhtemelen fazla; bir önceki satırla tutarlı olması için $\frac{\partial V_2}{\partial n} = \operatorname{grad} V_2 \cdot \vec{n}\big|_S$ olmalı.

$$\Rightarrow Q = C V_s$$



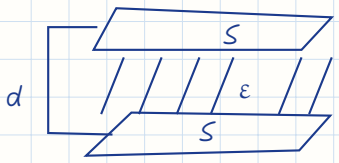

$$Q = C \cdot (V_2 - V_1)$$
$$-Q = C \cdot (V_1 - V_2)$$

orj. s. 8

$$W = \frac{1}{2} Q (V_2 - V_1) = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} C (V_2 - V_1)^2$$
$$= \frac{\varepsilon_2}{2d} S' (V_2 - V_2)^2$$ [8]

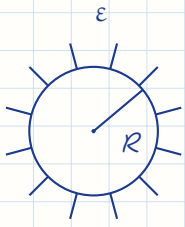
$$\Rightarrow C = \frac{\varepsilon_3 S'}{d}$$

$$\Rightarrow$$




$$C = \frac{\varepsilon \cdot S'}{d} \qquad \frac{(\text{Farad/m})\,(\text{m}^2)}{\text{m}} = \text{Farad}$$

Bir kürenin kapasitesi




$$C = 4\pi\varepsilon R$$
$$V = \frac{V_s \cdot R}{r} = V_s \Phi \Rightarrow \Phi = \frac{R}{r}$$

$$dS = r^2 \sin\theta \, d\theta \, d\varphi$$

$$C = -\varepsilon \int_{S_R} \frac{-R}{r^2}$$
$$= C \varepsilon R \int_{\theta=0}^{\pi} \int_{\varphi=0}^{2\pi} \sin\theta \, d\theta \, d\varphi$$ [9]

[8] Claude notu: el yazısında $\varepsilon_2$ ve $(V_2 - V_2)$ gibi okunuyor; önceki sayfadaki ifadeye göre muhtemelen $\varepsilon_3$ ve $(V_2 - V_1)$ olmalı.

$$= 4\pi\varepsilon R$$

silindir için

$$V = A \ln \rho$$

$$V = A \ln R = V_s \Rightarrow A = \frac{V_s}{\ln R}$$

$$\Rightarrow V = \frac{V_s}{\ln R} \ln \rho = V_s \Phi \Rightarrow \Phi = \frac{\ln \rho}{\ln R}$$

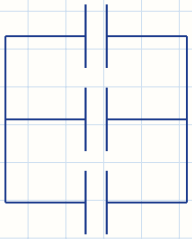
$dn = d\rho$

$$= -\varepsilon \int_{S'} \frac{\partial \Phi}{\partial n} dS = -\varepsilon \int \frac{\ln \rho}{\ln R} \underbrace{\rho \, d\varphi \, dz}_{dS} \quad [10]$$

$$= -\varepsilon \int_{\varphi=0}^{2\pi} \int_0^{\ell} \frac{1}{\ln R} \frac{1}{\rho} \rho \, d\varphi \, dz = \frac{-\varepsilon}{\ln R} 2\pi\ell$$

orj. s. 9

$$= \frac{2\pi\varepsilon\ell}{-\ln R} = \frac{2\pi\varepsilon\ell}{\ln(1/R)}$$



$$C = C_1 + C_2 + C_3$$

[9] Claude notu: baştaki $C$ muhtemelen fazla; sonuç $= 4\pi\varepsilon R$.

[10] Claude notu: $\frac{\partial \Phi}{\partial \rho} = \frac{1}{\rho \ln R}$ olmalı (bir sonraki satırla tutarlı).