Alan Teorisi

Elektrik alan ve potansiyel: grad, div, rot

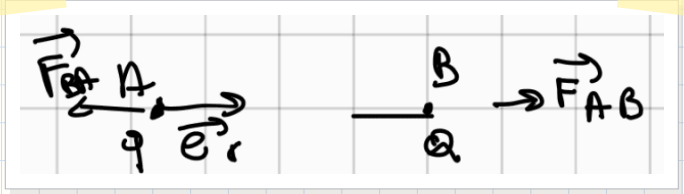
Orijinal not: Ders4.pdf

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orj. s. 1

Sayfa 17 Problem 1



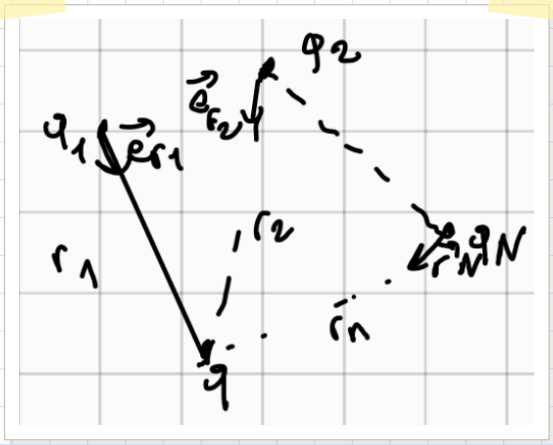
orijinal çizim, s. 1

$$\vec{e}_r = \frac{\vec{r}}{r} = \frac{\overrightarrow{AB}}{|\overrightarrow{AB}|}$$

$$\vec{F}_{AB} = \frac{1}{4\pi\varepsilon_0}\frac{q \cdot Q}{r^2}\vec{e}_r \quad \left(\vec{e}_r = \frac{\vec{r}}{r}\right) = -\vec{F}_{BA}$$

$$\vec{F}_{AB} = \frac{1}{4\pi\varepsilon_0}\frac{q \cdot Q}{r^3}\vec{r} = -\vec{F}_{BA}$$

N+1 yük olma durumu

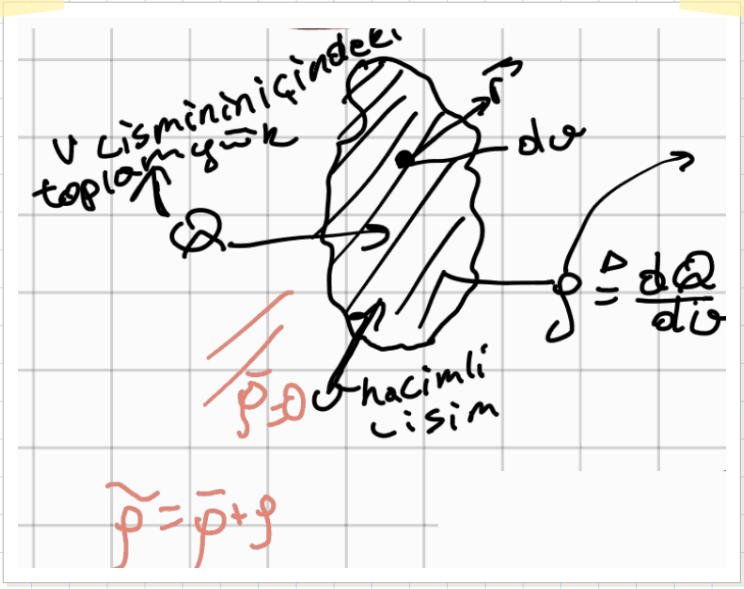


orijinal çizim, s. 1

$$\vec{r}_n = r_n \cdot \vec{e}_n$$

$$\vec{F}_q = \frac{q}{4\pi\varepsilon_0} \sum_{n=1}^{N} \frac{q_n}{(r_n)^2} \vec{e}_n = \frac{q}{4\pi\varepsilon_0} \sum_{n=1}^{N} \frac{q_n}{(r_n)^3} \vec{r}_n$$

Yükler ayrık değil de bir bölgeye dağılmışsa



orijinal çizim, s. 1

hacimsel yük yoğunluğu $[\rho] = \frac{C}{m^3}$

$$\rho\, dv = dQ$$

dQ: hacim elemanının içindeki yük

$$d\vec{F}_q = \frac{1}{4\pi\varepsilon_0} \frac{q \cdot dQ}{r^3} \vec{r}$$

$$\vec{F}_q = \int_v d\vec{F}_q = \frac{q}{4\pi\varepsilon_0}\int_v \frac{dQ}{r^3}\vec{r} = \frac{q}{4\pi\varepsilon_0}\int_v \frac{\rho}{r^3}\cdot\vec{r}\,dv$$

orj. s. 2

$$\equiv \frac{q}{4\pi\varepsilon_0}\int_x\int_y\int_z \frac{\rho}{r^3}\vec{r}\,\overbrace{dx\,dy\,dz}^{dv} \triangleq q\cdot\vec{E}$$

$\vec{E}$: elektrostatik alan vektörü; ?okunamadı zaman bağlı değil

ρ genelleştirebilirsek tümünü bir formülde bulabiliriz, (ilerde yapacağız)

$$\vec{E} = \underbrace{\frac{1}{4\pi\varepsilon_0}\int \frac{\rho}{r^3}\vec{r}\,dv}_{\text{tüm uzayda}}$$

Sorduğum hocanın soruya cevabı

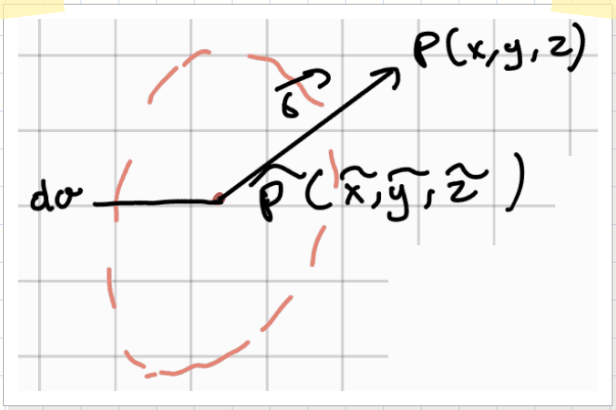
$$\vec{E} = \vec{E}_1 + \vec{E}_2 \qquad \text{(superposition)}$$



orijinal çizim, s. 2

$\rho_1 + \rho_2 = \rho$ yazarsak genel formülü uygulayabiliriz

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orijinal çizim, s. 2

$$\vec{r} = \overrightarrow{\tilde{P}P} = (x-\tilde{x})\,\vec{e}_x + (y-\tilde{y})\,\vec{e}_y + (z-\tilde{z})\,\vec{e}_z$$

$$|\vec{r}| = r = \sqrt{(x-\tilde{x})^2 + (y-\tilde{y})^2 + (z-\tilde{z})^2}$$

⋆ gradiant x, y, z'ye göre

$$\text{grad}\left(\frac{1}{r}\right) = \text{grad}\left(r^{-1}\right)$$

$$= \text{grad}\left[\left[(x-\tilde{x})^2 + (y-\tilde{y})^2 + (z-\tilde{z})^2\right]^{-1/2}\right]$$

$$= -\frac{1}{2}\,2\,(x-\tilde{x})\left[(x-\tilde{x})^2 + (y-\tilde{y})^2 + (z-\tilde{z})^2\right]^{-3/2}\vec{e}_x$$
$$-\frac{1}{2}\,2\,(y-\tilde{y})\left[\text{——}\right]^{-3/2}\vec{e}_y$$
$$-\frac{1}{2}\,2\,(z-\tilde{z})\left[\text{——}\right]^{-3/2}\vec{e}_z$$

$$\text{grad}(1/r) = -\frac{1}{r^3}\left[(x-\tilde{x})\,\vec{e}_x + (y-\tilde{y})\,\vec{e}_y + (z-\tilde{z})\,\vec{e}_z\right]$$

$$= -\frac{\vec{r}}{r^3}$$

$$\boxed{\text{grad}\left(\frac{1}{r}\right) = -\frac{\vec{r}}{r^3}}$$

orj. s. 3

$\vec{E}$'yi böyle yazabiliriz

$$\vec{E} = -\frac{1}{4\pi\varepsilon_0}\int \rho \underbrace{\left(-\frac{\vec{r}}{r^3}\right)}_{\text{grad}(1/r)} dv$$

ρ sabit sayarsak

$$\vec{E} = -\frac{1}{4\pi\varepsilon_0}\int_{\tilde{x}}\int_{\tilde{y}}\int_{\tilde{z}} \rho(\tilde{x},\tilde{y},\tilde{z})\ \text{grad}(1/r)\,d\tilde{x}\,d\tilde{y}\,d\tilde{z}$$

⋆

$$\vec{E} = -\,\text{grad}\left[\frac{1}{4\pi\varepsilon_0}\underbrace{\int_{\tilde{x}}\int_{\tilde{y}}\int_{\tilde{z}} \frac{\rho(\tilde{x},\tilde{y},\tilde{z})}{r}\,d\tilde{x}\,d\tilde{y}\,d\tilde{z}}_{\Downarrow}\right]$$

⋆ skaler potansiyel fonksiyonu

$$V(x,y,z) = \frac{1}{4\pi\varepsilon_0}\int_{\tilde{x}}\int_{\tilde{y}}\int_{\tilde{z}} \frac{\rho(\tilde{x},\tilde{y},\tilde{z})}{r}\,d\tilde{x}\,d\tilde{y}\,d\tilde{z} + \underbrace{V_0}_{\text{sabit}}$$

sonsuza giderken $V_0 = 0$'dır

⋆

$$\boxed{\vec{E}(x,y,z) = -\,\text{grad}\,V(x,y,z)}$$

Noktasal için bakarsak

$$\vec{E} = -\,\text{grad}\,V = -\,\text{grad}\left[\frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{r}\right]$$

$$= -\frac{q}{4\pi\varepsilon_0}\underbrace{\text{grad}(1/r)}_{-\vec{r}/r^3} = \frac{q}{4\pi\varepsilon_0}\frac{\vec{r}}{r^3}$$

$$\operatorname{rot}\vec{E} = \underbrace{-\operatorname{rot}\operatorname{grad} V}_{=0} = 0$$

$$\vec{\nabla} = \frac{\partial}{\partial x}\vec{e}_x + \frac{\partial}{\partial y}\vec{e}_y + \frac{\partial}{\partial z}\vec{e}_z$$

$$\Delta V = \operatorname{div}\operatorname{grad} V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0 \quad (\rho = 0\text{'nın dışında})$$

(Δ: laplacian (laplasien); $\operatorname{div}\operatorname{grad} V$: skaler)

$$\Delta \triangleq \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \triangleq \vec{\nabla}\cdot\vec{\nabla} = |\vec{\nabla}|^2 \star$$

$$\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\Delta \vec{A} = \frac{\partial^2}{\partial x^2}\vec{A} + \frac{\partial^2 \vec{A}}{\partial y^2} + \frac{\partial^2 \vec{A}}{\partial z^2}$$

$$\operatorname{rot}\operatorname{grad}\overset{\text{skaler}}{f} = \vec{\nabla}\times(\vec{\nabla} f) = (\vec{\nabla}\times\vec{\nabla}) f = 0$$

orj. s. 4

$$\vec{\nabla}\times\vec{\nabla} = \left(\frac{\partial}{\partial x}\vec{e}_x + \frac{\partial}{\partial y}\vec{e}_y + \frac{\partial}{\partial z}\vec{e}_z\right) \times \left(\frac{\partial}{\partial x}\vec{e}_x + \frac{\partial}{\partial y}\vec{e}_y + \frac{\partial}{\partial z}\vec{e}_z\right)$$

($\vec{e}_x \times \vec{e}_y = \vec{e}_z$, $\vec{e}_x \times \vec{e}_z = -\vec{e}_y$, $\vec{e}_y \times \vec{e}_z = \vec{e}_x$)

$$= \frac{\partial^2}{\partial x^2}(\vec{e}_x \times \vec{e}_x) + \frac{\partial^2}{\partial x\,\partial y}\vec{e}_z - \frac{\partial^2}{\partial x\,\partial y} + \frac{\partial^2}{\partial y\,\partial z}\vec{e}_x + \frac{\partial^2}{\partial x\,\partial z}\vec{e}_y - \frac{\partial^2}{\partial y\,\partial z}\vec{e}_x$$

$$= 0$$

$\star$

$$\operatorname{rot}\vec{E} = -\operatorname{rot}\operatorname{grad} V = 0$$

Örnek

$\vec{E} = 3xy\,\vec{e}_y$ sağlayan yükleri bulun

$$\operatorname{rot}\vec{E} = \vec{\nabla}\times\vec{E} = \left(\frac{\partial}{\partial x}\vec{e}_x + \frac{\partial}{\partial y}\vec{e}_y + \frac{\partial}{\partial z}\vec{e}_z\right) \times (3xy\,\vec{e}_y)$$

$$= \frac{\partial}{\partial x}(3xy)\,\vec{e}_z - \frac{\partial(3xy)}{\partial z}\vec{e}_x$$

$$= 3y\,\vec{e}_z + 0 = 3y\,\vec{e}_z \neq 0$$

o zaman bu elektrostatik alan değildir. Böyle bir yük dağılımı yoktur.

$$\operatorname{div}\operatorname{rot}\ \overset{\text{vektör}}{\vec{A}} = 0 \qquad \text{(her zaman geçerlidir)}$$

$$= \vec{\nabla} \cdot \underbrace{(\vec{\nabla} \times \vec{A})}_{\substack{\text{burdan çıkan vektör} \\ \vec{\nabla} \text{ ve } \vec{A}\text{'ya diktir}}}$$

dikler skaler çarpımı 0'dır

$$\operatorname{rot}\operatorname{rot}\vec{A} = \operatorname{grad}\operatorname{div}\vec{A} - \Delta\vec{A}$$