Alan Teorisi

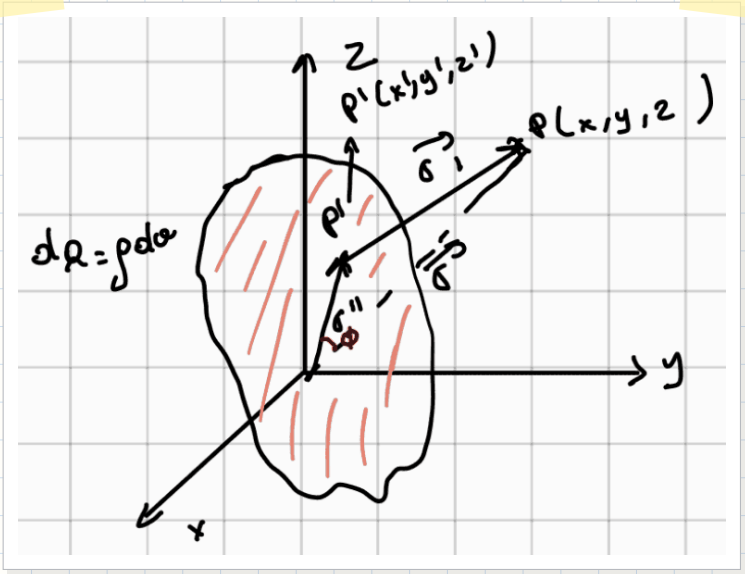
Elektrostatik potansiyel ve eş potansiyel yüzeyler

Orijinal not: Ders5.pdf

UYARI Bu belge, el yazısı ders notlarından Claude (Anthropic) yardımıyla dijitale aktarılmıştır. Yardımcı kaynak olarak hazırlanmıştır; hata veya eksik içerebilir. Lütfen orijinal notları ve ders kaynaklarını esas alın.

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orj. s. 1



orijinal çizim, s. 1

$$r' = \left[(x-x')^2 + (y-y')^2 + (z-z')^2\right]^{1/2}$$

$$\vec{r}'' = \overrightarrow{OP'} = x'\,\vec{e}_x + y'\,\vec{e}_y + z'\,\vec{e}_z$$

$$\vec{r} = x\,\vec{e}_x + y\,\vec{e}_y + z\,\vec{e}_z$$

$$\vec{E} = -\operatorname{grad} V$$

$$\vec{r}' = \vec{r} - \vec{r}'' = x\,\vec{e}_x + y\,\vec{e}_y + z\,\vec{e}_z - x'\,\vec{e}_x - y'\,\vec{e}_y - z'\,\vec{e}_z$$

$$\int_V dV = \frac{1}{4\pi\varepsilon_0}\int_V \frac{dQ}{r'}$$

$$V = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho}{r'}\,dv$$

$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho}{r'}\,dv \qquad \text{(coulomb potansiyel)}$$

potansiyel fonksiyonu

$$V(x,y,z) = \frac{1}{4\pi\varepsilon_0} \iiint_{x',y',z'=-\infty}^{\infty} \frac{\rho(x',y',z')}{\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}}\,dx'\,dy'\,dz' + V_0$$

$$\vec{E} = -\operatorname{grad} V = -\operatorname{grad}\left\{\frac{1}{4\pi\varepsilon_0}\iiint_{x',y',z'=-\infty}^{+\infty}\frac{\rho(x',y',z')}{r'}\,dx'\,dy'\,dz' + V_0\right\}$$

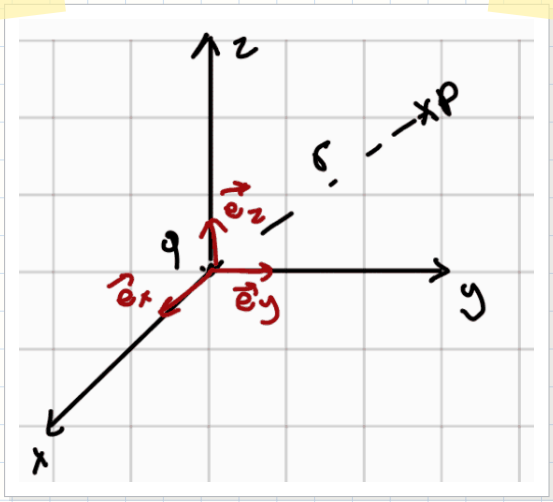
$$= -\operatorname{grad}_{(x,y,z)}\left\{\frac{1}{4\pi\varepsilon_0}\iiint\frac{\rho(x',y',z')}{r'}\,dx'\,dy'\,dz'\right\}$$

(Satırın sonunda $-\operatorname{grad} V_0$ yazılıp üstü çizilmiş.)

$$= -\frac{1}{4\pi\varepsilon_0}\iiint_{x',y',z'=-\infty}^{\infty}\rho(x',y',z')\,\underbrace{\operatorname{grad}\left(\frac{1}{r'}\right)}_{-\vec{r}'/r'^3}\,dx'\,dy'\,dz'$$

$$\Rightarrow \vec{E}(x,y,z) = \frac{1}{4\pi\varepsilon_0}\iiint\frac{\rho(x',y',z')\,\vec{r}'}{r'^3}\,dx'\,dy'\,dz'$$

orj. s. 2

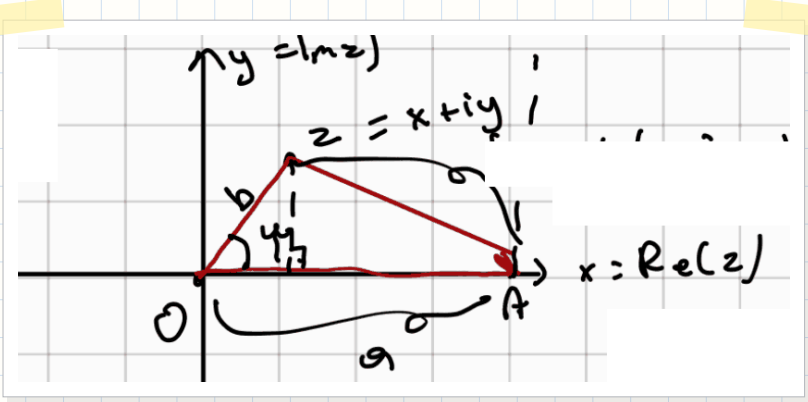


orijinal çizim, s. 2

$$\vec{e}_x \perp \vec{e}_y \perp \vec{e}_z, \qquad |\vec{e}_x| = |\vec{e}_y| = |\vec{e}_z| = 1$$

$$r' = \sqrt{r^2 + r''^2 - 2\cdot r\cdot r''\cos\varphi} \qquad \text{(cosinüs teoremi)}$$

$$b = |z| = |x+iy| = \sqrt{x^2+y^2}$$



orijinal çizim, s. 2

$$|z - z_A| = |x + iy - a| = |b\cos\psi + i\,b\sin\psi - a|$$
$$= |(b\cos\psi - a) + i\,b\sin\psi|$$

$$z_A = a > 0$$

$$b = |z|$$

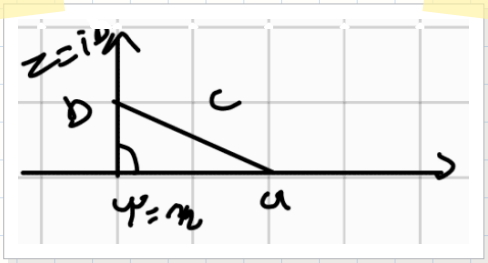
$$\psi = \arctan\left(\frac{y}{x}\right) \in [-\pi, \pi], \qquad \arg(z) = \psi \quad (\text{psi})$$

$$z = x + iy = \underbrace{b\cos\psi}_{x} + i\,\underbrace{b\sin\psi}_{y}$$

$$z = |z|\left(\cos(\underbrace{\arg z}_{\psi}) + i\sin(\underbrace{\arg z}_{\psi})\right)$$

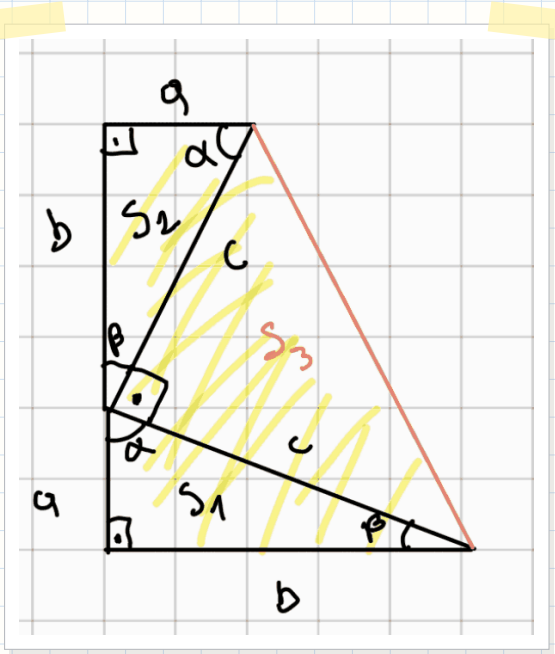
$$|z - z_A| = c = \sqrt{(b\cos\psi - a)^2 + (b\sin\psi)^2}$$
$$c = \sqrt{b^2\cos^2\psi + a^2 - 2ab\cos\psi + b^2\sin^2\psi}$$
$$c = \sqrt{b^2 + a^2 - 2ab\cos\psi}$$

$$\psi = \pi/2 \;\Rightarrow\; c = \sqrt{b^2 + a^2}$$



orijinal çizim, s. 2

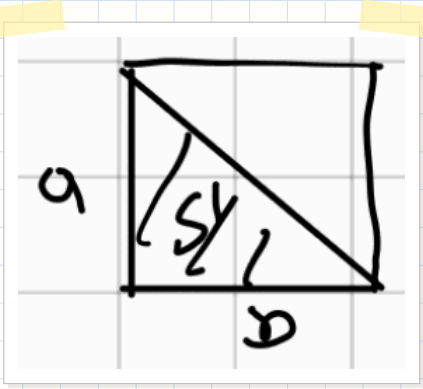
orj. s. 3



orijinal çizim, s. 3

$$\underbrace{\alpha + \beta}_{\pi/2} + \gamma = \pi$$

$$\gamma = \pi - \pi/2 = \pi/2$$



orijinal çizim, s. 3

$$S = a \cdot b, \qquad S' = \frac{S}{2}$$

$$S_1 = \frac{a \cdot b}{2} = S_2$$

$$S_3 = \frac{c \cdot c}{2} = \frac{c^2}{2}$$

$$S_1 + S_2 + S_3 = \frac{ab}{2} + \frac{ab}{2} + \frac{c^2}{2}$$

$$\underbrace{S_T}_{\text{trapez}} = ab + \frac{c^2}{2} = \frac{(a+b)(a+b)}{2}$$

$$2ab + c^2 = a^2 + b^2 + 2ab$$

$$c^2 = a^2 + b^2$$

OGR

$$\underbrace{e^{i\alpha} = \cos\alpha + i\sin\alpha}_{\text{Euler formülü}} \qquad e^{-i\alpha} = \cos\alpha - i\sin\alpha$$

$$\begin{aligned} e^{i(x+y)} &= \cos(x+y) + i\sin(x+y) \\ e^{ix+iy} &= e^{ix}\cdot e^{iy} = (\cos x + i\sin x)(\cos y + i\sin y) \\ &= \cos(x+y) + i\sin(x+y) \end{aligned}$$

$$\Downarrow$$

$$\underbrace{(\cos x\cos y - \sin x\sin y)}_{\cos(x+y)} + i\,\underbrace{(\sin x\cos y + \cos x\sin y)}_{\sin(x+y)}$$

orj. s. 4

Sonsuzda küresel cismin noktasal yük gibi davrandığını ispatlayalım

$$r' = \sqrt{r^2 + r''^2 - 2r\,r''\cos\psi}$$

$$\begin{aligned} &\frac{1}{4\pi\varepsilon_0}\iiint_v \frac{\rho(x',y',z')}{(r^2 + r''^2 - 2rr''\cos\psi)^{1/2}}\,dv \\ &= \frac{1}{4\pi\varepsilon_0}\iiint_{\substack{x',y',z' \\ (v)}} \frac{\rho(x',y',z')}{r\left[1 - \frac{2}{r}r''\cos\psi + \frac{r''^2}{r^2}\right]^{1/2}}\,dx'\,dy'\,dz' \\ &= \frac{1}{4\pi\varepsilon_0 r}\int_v \rho(x',y',z')\Big[\underbrace{1 - \frac{2}{r}r''\cos\psi + \left(\frac{r''}{r}\right)^2}_{\ll 1}\Big]^{-1/2}dv \end{aligned}$$

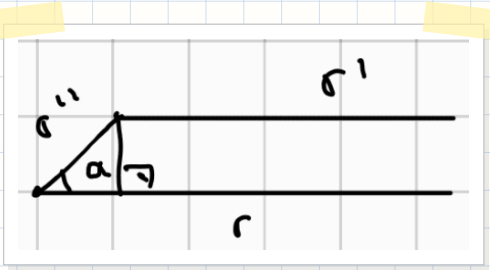
$$r \gg r'', \qquad \frac{r''}{r} \ll 1, \qquad \left(\frac{r''}{r}\right)^2 \lll 1$$

$$(1-x)^{-1/2} \approx 1 + \frac{1}{2}x - \frac{1}{8}?x^2 - \dots \qquad |x| \ll 1$$

$$= \frac{1}{4\pi\varepsilon_0 r}\int_v \rho(x',y',z')\,\underbrace{\Big[1 + \overbrace{\frac{r''}{r}\cos\psi}^{\ll 1} - \frac{r''^2}{2r^2}\Big]}_{1}$$

$$\approx \frac{1}{4\pi\varepsilon_0 r}\underbrace{\int_v \rho(x',y',z')\,dv}_{Q \text{ dersek}} = \frac{Q}{4\pi\varepsilon_0 r}$$

$$\underset{r\to\infty}{\frac{1}{r'}} = \frac{1}{r\sqrt{\underbrace{1 - \frac{2r''}{r}\cos\psi + \left(\frac{r''}{r}\right)^2}_{0}}} = \frac{1}{r\cdot\sqrt{1}} = \frac{1}{r}$$



orijinal çizim, s. 4

orj. s. 5

$$r = r' + r'' \cos\psi$$

$$r' \approx r - r'' \cos\psi \approx r\left[1 - \frac{r''}{r}\cos\psi\right]$$

$$1/r' \approx \frac{1}{r}\left[1 - \frac{r''}{r}\cos\psi\right]^{-1}$$

$$\approx \frac{1}{r}\left(1 + \frac{r''}{r}\cos\psi\right) = 1/r$$

$$e^{ir} \approx e^{ir'} = e^{i(r - r''\cos\psi)}$$

$$= e^{ir} \cdot e^{-ir''\cos\psi}$$

$f(x, y, z) = \text{sbt} \Rightarrow$ yüzey ailesi verir

$$r = \sqrt{x^2 + y^2 + z^2} = \text{sabit}$$

O zaman $V(x, y, z) = \text{sbt} \Rightarrow$ Eş potansiyel yüzeyleri

$$\vec{E} = -\operatorname{grad} V$$

$f(x, y, z)$: $x = x(s)$, $y = y(s)$, $z = z(s)$ (C eğrisi)

$$f(C) = f(x(s), y(s), z(s))$$

$$\Rightarrow \frac{\partial f}{\partial C} \widehat{=} \frac{\partial f}{\partial s}$$

$$= \frac{\partial f}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial s}$$

$f(x, y, z)$ fonksiyonun parametrik denklemleri $x = x(s)$, $y = y(s)$ ve $z = z(s)$ olan bir C eğrisi boyunca türevi

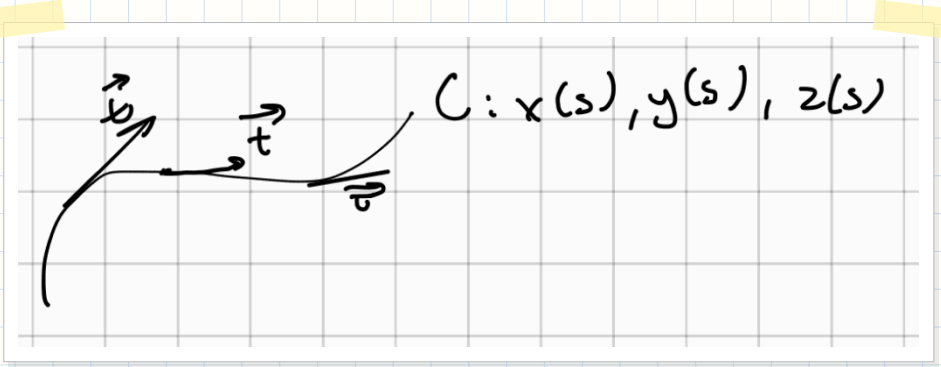
orj. s. 6

$$\frac{\partial f}{\partial C} = \underbrace{\left(\frac{\partial f}{\partial x}\vec{e}_x + \frac{\partial f}{\partial y}\vec{e}_y + \frac{\partial f}{\partial z}\vec{e}_z\right)}_{\operatorname{grad} f = \vec{\nabla} f} \underbrace{\left(\frac{\partial x}{\partial s}\vec{e}_x + \frac{\partial y}{\partial s}\vec{e}_y + \frac{\partial z}{\partial s}\vec{e}_z\right)}_{\vec{t}\ \text{(teğet denklem)}}$$

$$\frac{\partial f}{\partial C} = \text{grad} f \cdot \vec{t}$$

$$\frac{\partial V}{\partial C} = \text{grad} V \cdot \vec{t} = -\vec{E} \cdot \vec{t}$$

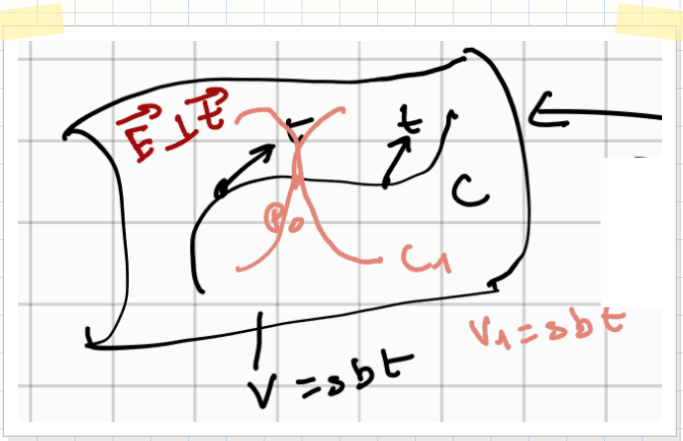
$$\vec{E} \cdot \vec{t} = -\frac{\partial V}{\partial C}$$



orijinal çizim, s. 6

Şayet V = sabit ise $\Rightarrow -\frac{\partial V}{\partial C} = 0$ olur

$\vec{E} \perp \vec{t}, \quad \vec{E} \perp \vec{t}_1, \quad \vec{E} \perp \vec{t}_2, \quad \ldots, \quad \vec{E} \perp \vec{t}_n$



orijinal çizim, s. 6

$\vec{E} \underset{\text{içinde}}{\subset} S$ olabilir

$$V(C) = sbt$$

$$\frac{\partial V}{\partial C} = \frac{\partial\, sbt}{\partial C} = 0$$

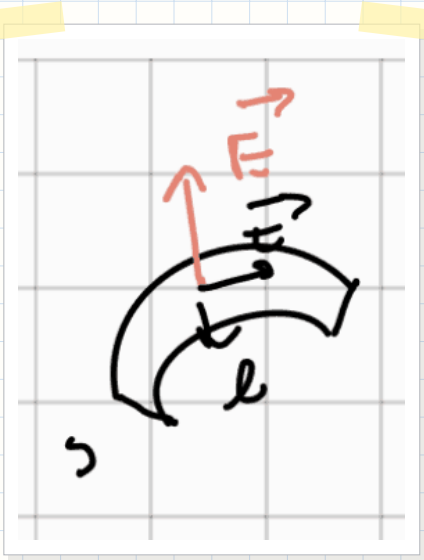
$\Rightarrow$ Eş potansiyel yüzey üzerinde

$$\vec{E} \cdot \vec{t} = 0 \qquad \vec{E} \perp \vec{t}$$

$$\Rightarrow \text{Eş pot. yüzeyi} \Rightarrow \frac{\partial V}{\partial C_1} = \vec{E} \cdot \vec{t}_1$$

$\Rightarrow \vec{E} \perp$?S (altında: ?S'ye dik)

orj. s. 7



orijinal çizim, s. 7

$\vec{E} \parallel \vec{t}_\ell$, ℓ alan çizgisidir

$$\vec{t}_\ell = \frac{dx}{ds}\vec{e}_x + \frac{dy}{ds}\vec{e}_y + \frac{dz}{ds}\vec{e}_z$$

$$\frac{E_x}{\frac{dx}{ds}} = \frac{E_y}{\frac{dy}{ds}} = \frac{E_z}{\frac{dz}{ds}}$$

$$\vec{E} = E_x\vec{e}_x + E_y\vec{e}_y + E_z\vec{e}_z$$

$$\vec{t}_\ell = \frac{dx}{ds}\vec{e}_x + \frac{dy}{ds}\vec{e}_y + \frac{dz}{ds}\vec{e}_z$$

$$\vec{E} \propto \vec{t}_\ell \Rightarrow \frac{E_x}{\frac{dx}{ds}} = \frac{E_y}{\frac{dy}{ds}} = \frac{E_z}{\frac{dz}{ds}}$$

Alan çizgileri:

$$\frac{E_x}{dx} = \frac{E_y}{dy} = \frac{E_z}{dz} = \text{sabit}$$

$$\vec{a} \parallel \vec{b}$$

$$\vec{b} = k\vec{a}$$

$$k = \frac{b_x}{a_x} = \frac{b_y}{a_y} = \frac{b_z}{a_z} = \text{sbt}$$