# Alan Teorisi

## Elektriksel dipol, kompleks sayılar, Gauss teoremi

Orijinal not: Ders7.pdf

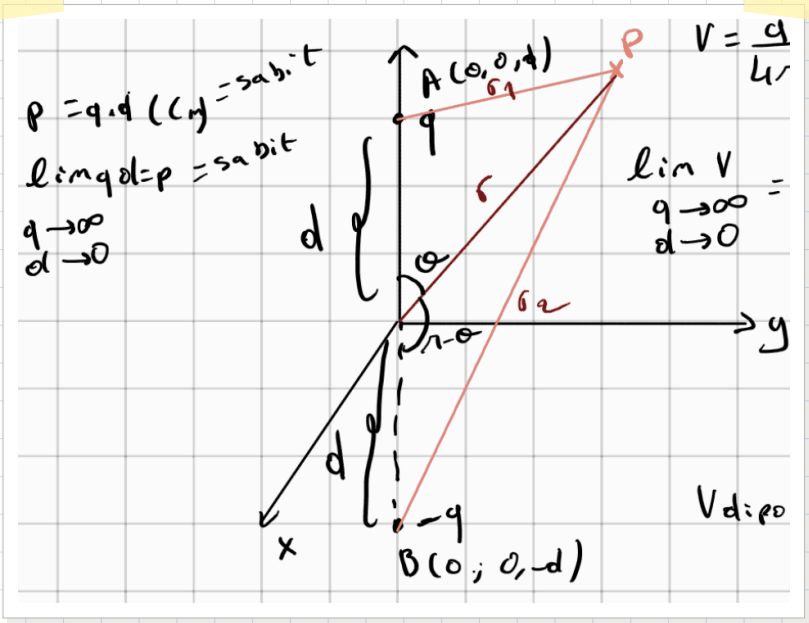
**UYARI** Bu belge, el yazısı ders notlarından Claude (Anthropic) yardımıyla dijitale aktarılmıştır. Yardımcı kaynak olarak hazırlanmıştır; hata veya eksik içerebilir. Lütfen orijinal notları ve ders kaynaklarını esas alın.

**NOTICE** *This document was transcribed from handwritten lecture notes with the help of Claude (Anthropic). It is meant as a supplementary resource and may contain errors or omissions. Always check against the original notes and official course material.*

orj. s. 1

## Dipol (Elektriksel)

$p = q \cdot d \quad (C \cdot m) = \text{sabit}$ $\qquad \lim_{\substack{q\to\infty \\ d\to 0}} qd = p = \text{sabit}$




orijinal çizim, s. 1

$$\lim_{d\to 0}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)=\frac{1}{r}-\frac{1}{r}=0$$

$$V=\frac{q}{4\pi\varepsilon_0}\frac{1}{r_1}-\frac{q}{4\pi\varepsilon_0}\frac{1}{r_2}$$

$$\lim_{\substack{q\to\infty \\ d\to 0}} V = V_{dipol} = \frac{1}{4\pi\varepsilon_0}\lim_{\substack{q\to\infty \\ d\to\infty}}\frac{qd}{d}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)^1$$

$$= \frac{1}{4\pi\varepsilon_0} \underbrace{\lim_{\substack{q\to\infty \\ d\to 0}} q\,d}\ \lim_{\substack{q\to\infty \\ d\to 0}} \frac{1}{d}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$V_{dipol} = \frac{p}{4\pi\varepsilon_0} \cdot \lim_{d\to 0} \frac{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}{d}$$

$\widehat{OAP}$ $r_1 = \sqrt{r^2 + d^2 - 2rd\cos\theta}$

$\widehat{OBP}$ $r_2 = \sqrt{r^2 + d^2 - 2rd\cos(\pi - \theta)} = \sqrt{r^2 + d^2 + 2rd\cos\theta}$

$$V_{dipol} = \frac{p}{4\pi\varepsilon_0} \lim_{d\to 0} \frac{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}{d}$$

$$= \frac{p}{4\pi\varepsilon_0} \lim_{d\to 0} \frac{\frac{1}{\sqrt{r^2 + d^2 - 2rd\cos\theta}} - \frac{1}{\sqrt{r^2 + d^2 + 2rd\cos\theta}}}{d}$$

$$= \frac{p}{4\pi\varepsilon_0} \lim_{d\to 0} \frac{d}{d(d)} \overset{0/0}{\left[(r^2 + d^2 - 2rd\cos\theta)^{-1/2} + (r^2 + d^2 + 2rd\cos\theta)^{-1/2}\right]}^{2}$$

$$V_{dipol} = \frac{p}{4\pi\varepsilon_0} \lim_{d\to 0} \left[-\frac{1}{2}\frac{2d - 2r\cos\theta}{(r^2 + d^2 - 2rd\cos\theta)^{3/2}} + \frac{1}{2}\frac{2d + 2r\cos\theta}{(r^2 + d^2 + 2rd\cos\theta)^{3/2}}\right]$$

$$V_{dipol} = \frac{-p}{4\pi\varepsilon_0} \lim_{d\to 0} \left[\frac{d - r\cos\theta}{(r^2 + d^2 - 2rd\cos\theta)^{3/2}} - \frac{d + r\cos\theta}{(r^2 + d^2 - 2rd\cos\theta)^{3/2}}\right]^{3}$$

$2p = 2qd$
$= q(2d)$

$$= -\frac{p}{4\pi\varepsilon_0}\left[-\frac{r\cos\theta}{r^3} - \frac{r\cos\theta}{r^3}\right] = \frac{p\cos\theta}{2\pi\varepsilon_0 r^2} \qquad V = \frac{2p\cos\theta}{4\pi\varepsilon_0 r^2}$$

$p$ ← dipolun moment yoğunluğu  $2p$ (çemberlenmiş) ← kitaptaki $p$

$z$ ekseni boyunca oluşturulmuş $p = qd$ moment yoğunluk bir noktasal dipolun potansiyeli

orj. s. 2

$p$ = dipol moment yoğunluğu  $[p] = C\,m$

$[V_{dipol}] = V$

$f(r, \theta, \phi)$

$$\text{grad}\, f = \frac{\partial f}{\partial r}\vec{e}_r + \frac{1}{r}\frac{\partial f}{\partial \theta}\vec{e}_\theta + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\vec{e}_\phi$$

[1] Claude notu: muhtemelen $d \to 0$ olmalı.
[2] Claude notu: muhtemelen iki terim arasında $-$ olmalı (bir sonraki satır buna uygun).
[3] Claude notu: ikinci paydada muhtemelen $+2rd\cos\theta$ olmalı.

$$\vec{E} = -\text{grad}\, V = -\frac{\partial V_{dipol}}{\partial r}\vec{e}_r - \frac{1}{r}\frac{\partial V_{dipol}}{\partial \theta}\vec{e}_\theta$$

$$- \frac{1}{r\sin\theta}\frac{\partial V_{dipol}}{\partial \phi}\vec{e}_\phi$$

$$= \frac{2p\cos\theta}{4\pi\varepsilon_0 r^3}\vec{e}_r + \frac{1}{r}\frac{p}{4\pi\varepsilon_0 r^2}\sin\theta\,\vec{e}_\theta$$

kitaptaki p ↓

$$= \frac{p}{4\pi\varepsilon_0}\left[2\cos\theta\,\vec{e}_r + \sin\theta\,\vec{e}_\theta\right]$$

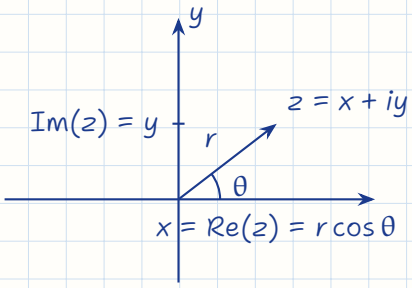
Yada en baştan

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\frac{\vec{e}_{r_1}}{r_1^2} - \frac{q}{4\pi\varepsilon_0}\frac{1}{r_2^2}\vec{e}_{r_2}$$

ama çok daha ?zooooooor!!

$$\star\ \text{rot}\,\vec{E}_{dipol} = 0$$

## Hadi kompleks sayılardan bahsedelim



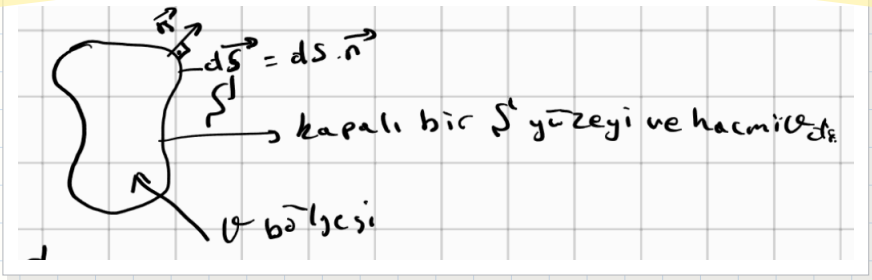

$$z = \text{Re}(z) + i\,\text{Im}(z)$$
$$r = \sqrt{x^2 + y^2} = |z|$$
$$z = r\cos\theta + i\,r\sin\theta$$
$$= r[\cos\theta + i\sin\theta]$$
$$= |z|[\cos(\arg z) + i\sin(\arg z)]$$

orj. s. 3

## Gauss Teoremi

$$\vec{A} = A_x\vec{e}_x + A_y\vec{e}_y + A_z\vec{e}_z$$

$A_x,\ A_y,\ A_z\,(x, y, z)$




orijinal çizim, s. 3

v S tarafından kuşatılan bölge

Gauss–Ostrogradski teoremi

$$\int_S \vec{A} \cdot d\vec{S} = \int_V \operatorname{div} \vec{A} \cdot dv$$

$$\iint_S \vec{A} \cdot d\vec{S} = \iiint_V \operatorname{div} \vec{A} \cdot dv$$