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Uygulama 7 (vize sonrası) – Eş merkezli küre/silindir yüzeyler

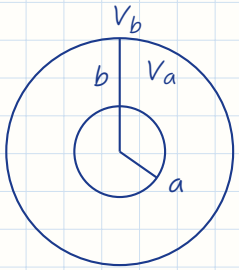
Orijinal not: Uygulama-7-Vize-sonrasi.pdf

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orj. s. 1

1)



$$a < r < b \quad V(r) = \frac{V_a - V_b}{\frac{1}{a} - \frac{1}{b}} \frac{1}{r} + \frac{aV_a - bV_b}{a - b}$$

$r < a \quad \vec{E} = 0 \quad \vec{E} = -\text{grad}\, V$

$V(r) = C \quad V(a) = V_a = C$

$$\boxed{V(r) = V_a}$$

$r > b \qquad \Delta V = 0 \qquad \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) = 0$

$$r^2 \frac{\partial V}{\partial r} = A \quad \Rightarrow \quad V(r) = \frac{A}{r} + B$$

$r \to \infty$: $V(r) = 0$ olmalıdır $\Rightarrow B = 0$

$r = b$: $V(b) = V_b = \frac{A}{b} \Rightarrow A = bV_b$

$$V(r) = \frac{bV_b}{r}$$

$$a < r < b: \quad \vec{E} = -\text{grad}\, V = \frac{V_a - V_b}{\frac{1}{a} - \frac{1}{b}} \frac{1}{r^2} \vec{e}_r$$

$$r < a: \quad \vec{E} = 0$$

$$r > b: \quad \vec{E} = -\text{grad}\, V = -\frac{\partial}{\partial r}\left(\frac{bV_b}{r}\right)\vec{e}_r = \frac{bV_b}{r^2} \vec{e}_r$$

$$r = a: \quad \vec{E} = \frac{\vec{E}^+(a) + \vec{E}^-(a)}{2}$$

$$= \frac{1}{2}\left[\frac{V_a - V_b}{\frac{1}{a} - \frac{1}{b}} \frac{1}{a^2}\right] \vec{e}_r$$

$$= \frac{1}{2} \frac{b}{a}\left[\frac{V_a - V_b}{b - a}\right] \vec{e}_r$$

orj. s. 2

$d\vec{F} = \rho_s \, ds \, \vec{E}$

$ds = a^2 \sin\theta \, d\phi \, d\theta$

$\operatorname{div} \vec{D} = \rho_s$

sınır koşulu: $\rho_s = \vec{n} \cdot \vec{D} = \vec{n} \cdot \varepsilon \vec{E}$

$= \vec{e}_r \, \varepsilon \, \vec{E}$

$$\rho_s = \frac{\varepsilon}{2} \frac{b}{a} \left[\frac{V_a - V_b}{b - a} \right]$$

$$d\vec{F} = \frac{\varepsilon}{2} \frac{b}{a} \left[\frac{V_a - V_b}{b - a} \right] (a^2 \sin\theta \, d\phi \, d\theta) \left[\frac{1}{2} \frac{b}{a} \left(\frac{V_a - V_b}{b - a} \right) \vec{e}_r \right]$$

$$\vec{F} = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \frac{\varepsilon}{4} b^2 \left[\frac{V_a - V_b}{b - a} \right]^2 \sin\theta \, d\phi \, d\theta \, \vec{e}_r$$

$$\vec{F} = \varepsilon \pi b^2 \left(\frac{V_a - V_b}{b - a} \right)^2 \vec{e}_r$$

$r = b$ için

$$\vec{E} = \frac{\vec{E}^+(b) + \vec{E}^-(b)}{2} = \frac{1}{2} \left[\frac{b V_b}{b^2} + \frac{a}{b} \left(\frac{V_a - V_b}{b - a} \right) \right] \vec{e}_r$$

$$= \frac{b V_b - 2a V_b + a V_a}{2b(b - a)} \vec{e}_r$$

$$\rho_s = \varepsilon \vec{n} \cdot \vec{E} = \varepsilon \vec{e}_r \cdot \vec{E}$$

$$\rho_s = \frac{\varepsilon}{2b(b - a)} \left[b V_b - 2a V_b + a V_a \right]$$

$ds = b^2 \sin\theta \, d\theta \, d\phi$

$d\vec{F} = \rho_s \, ds \, \vec{E}$

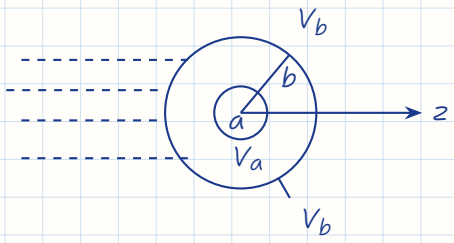
$$= \frac{\varepsilon}{2b(b - a)} \left[b V_b - 2a V_b + a V_a \right] (b^2 \sin\theta \, d\theta \, d\phi) \cdot \left[\frac{b V_b - 2a V_b + a V_a}{2b(b - a)} \right] \vec{e}_r$$

$$\vec{F} = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \frac{\varepsilon}{4} \left[\frac{b V_b - 2a V_b + a V_a}{b - a} \right]^2 \sin\theta \, d\theta \, d\phi \, \vec{e}_r$$

orj. s. 3

$$= \varepsilon \pi \left[\frac{b V_b - 2a V_b + a V_a}{b - a} \right]^2 \vec{e}_r$$

2)



$a < \rho < b$

$$V(\rho) = \frac{V_a - V_b}{\ln a - \ln b} \ln \rho - \frac{V_a \ln b - V_b \ln a}{\ln a - \ln b}$$

$\rho < a \quad \vec{E} = 0\,, \quad \vec{E} = -\operatorname{grad} V \Rightarrow V(\rho) = c$

$c = V_a \Rightarrow V(\rho) = V_a$

$\rho > b \quad \Delta V = 0$

$$\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) = 0, \qquad \rho \frac{\partial V}{\partial \rho} = A$$

$$\frac{\partial V}{\partial \rho} = \frac{A}{\rho} \Rightarrow V(\rho) = A \ln \rho + B$$

$$a < \rho < b: \quad \vec{E} = -\operatorname{grad} V = -\frac{V_a - V_b}{\ln a - \ln b}\frac{1}{\rho}\vec{e}_\rho$$

$$\rho < a: \quad \vec{E} = 0$$

$$\rho > b: \quad \vec{E} = -\operatorname{grad} V = -\frac{\partial}{\partial \rho}(A \ln \rho)\vec{e}_\rho$$

$$= -\frac{A}{\rho}\vec{e}_\rho$$

$$r = a: \quad \vec{E}(a) = \frac{\vec{E}^+(a) + \vec{E}^-(a)}{2}$$

$$= -\frac{1}{2}\frac{V_a - V_b}{\ln a - \ln b} \cdot \frac{1}{a}\vec{e}_\rho$$

$$d\vec{F} = \rho_s \, ds \, \vec{E}, \qquad ds = a \, d\phi \, dz, \qquad \rho_s = \vec{n} \cdot \varepsilon \vec{E}$$

$$= -\frac{\varepsilon}{2}\frac{V_a - V_b}{(\ln a - \ln b)}\frac{1}{a}$$

orj. s. 4

$$d\vec{F} = \frac{\varepsilon}{2}\frac{V_a - V_b}{\ln a - \ln b} \cdot \frac{1}{a}(a \, d\phi \, dz)\frac{1}{2}\frac{V_a - V_b}{\ln a - \ln b}\frac{1}{a}\vec{e}_\rho$$

Bir metrelik

$$\vec{F} = \int_{z=0}^{1}\int_{\phi=0}^{2\pi} \frac{\varepsilon}{4a}\frac{(V_a - V_b)^2}{(\ln a - \ln b)^2} \, d\phi \, dz \, \vec{e}_\rho$$

$$= \frac{\varepsilon \pi}{2a}\frac{(V_a - V_b)^2}{(\ln a - \ln b)^2}\vec{e}_\rho$$

$\rho = b$

$$\vec{E}(b) = \frac{\vec{E}^+(b) + \vec{E}^-(b)}{2}$$

$$= -\frac{1}{2}\left[\frac{A}{b} + \frac{V_a - V_b}{\ln a - \ln b}\frac{1}{b}\right]\vec{e}_\rho$$

$$= -\frac{1}{2}\frac{A(\ln a - \ln b) + (V_a - V_b)}{b(\ln a - \ln b)}\vec{e}_\rho$$

$$ds = b \, d\phi \, dz\,, \qquad \rho_s = \vec{e}_\rho \cdot \varepsilon \cdot \vec{E}$$

$$= -\frac{\varepsilon}{2} \cdot \frac{A(\ln a - \ln b) + (V_a - V_b)}{b(\ln a - \ln b)}$$

$$d\vec{F} = \frac{\varepsilon}{2} \cdot \frac{A(\ln a - \ln b) + (V_a - V_b)}{b(\ln a - \ln b)} (b\, d\phi\, dz) \cdot \frac{1}{2} \frac{A(\ln a - \ln b) + (V_a - V_b)}{b(\ln a - \ln b)} \vec{e}_\rho$$

$$\vec{F} = \int_{z=0}^{1} \int_{\phi=0}^{2\pi} \frac{\varepsilon}{4b} \left[\frac{A(\ln a - \ln b) + (V_a - V_b)}{\ln a - \ln b} \right]^2 \vec{e}_\rho\, d\phi\, dz$$

$$= \frac{\varepsilon\pi}{2b} \left[\frac{A(\ln a - \ln b) + (V_a - V_b)}{\ln a - \ln b} \right]^2 \vec{e}_\rho$$