Alan Teorisi

Uygulama 9 – Görüntü yöntemi (köşe, dielektrik arayüz)

Orijinal not: Uygulama-9.pdf

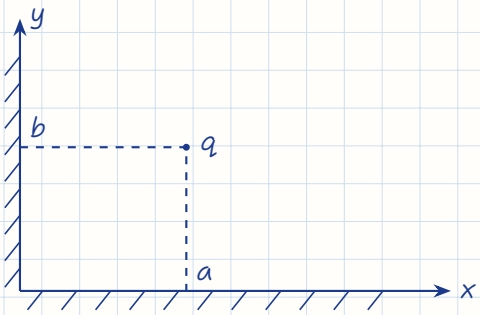
UYARI Bu belge, el yazısı ders notlarından Claude (Anthropic) yardımıyla dijitale aktarılmıştır. Yardımcı kaynak olarak hazırlanmıştır; hata veya eksik içerebilir. Lütfen orijinal notları ve ders kaynaklarını esas alın.

NOTICE *This document was transcribed from handwritten lecture notes with the help of Claude (Anthropic). It is meant as a supplementary resource and may contain errors or omissions. Always check against the original notes and official course material.*

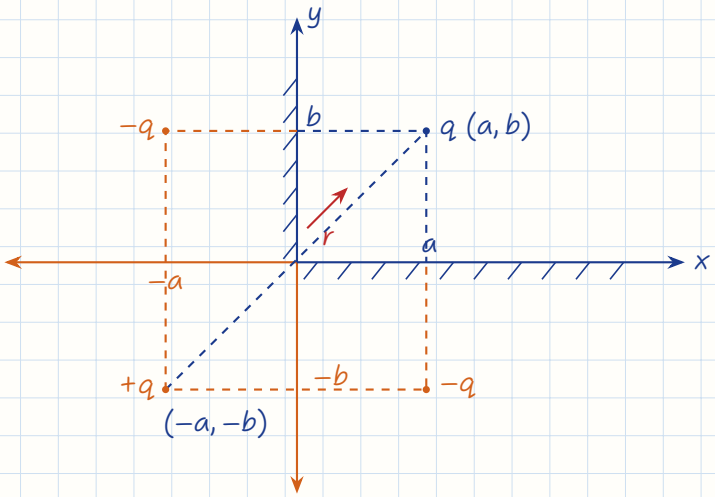
orj. s. 1

görüntü teoremi

S65 Pr.6



q yüküne
etki eden kuvvet



$$\vec{F} = -\frac{q^2}{4\pi\varepsilon\, 4a^2}\vec{e}_x - \frac{q^2}{4\pi\varepsilon\, 4b^2}\vec{e}_y + \frac{q^2}{4\pi\varepsilon\, 2\sqrt{a^2+b^2}}\left(\frac{a\vec{e}_x + b\vec{e}_y}{\sqrt{a^2+b^2}}\right)$$ [1]

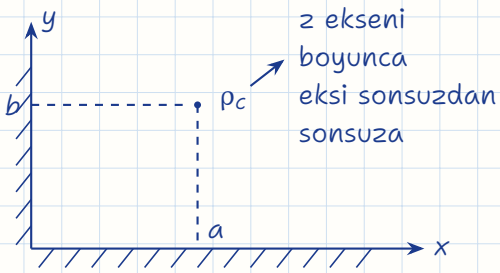
$$|\vec{r}| = 2\sqrt{a^2+b^2}$$

[1] Claude notu: muhtemelen payda $4\pi\varepsilon\left(2\sqrt{a^2+b^2}\right)^2$ olmalı (son satırdaki $16\pi\varepsilon$ ile uyumlu).

$$\vec{e}_r = \frac{\vec{r}}{|\vec{r}|} = \frac{2a\,\vec{e}_x + 2b\,\vec{e}_y}{2\sqrt{a^2+b^2}} = \frac{a\,\vec{e}_x + b\,\vec{e}_y}{\sqrt{a^2+b^2}}$$

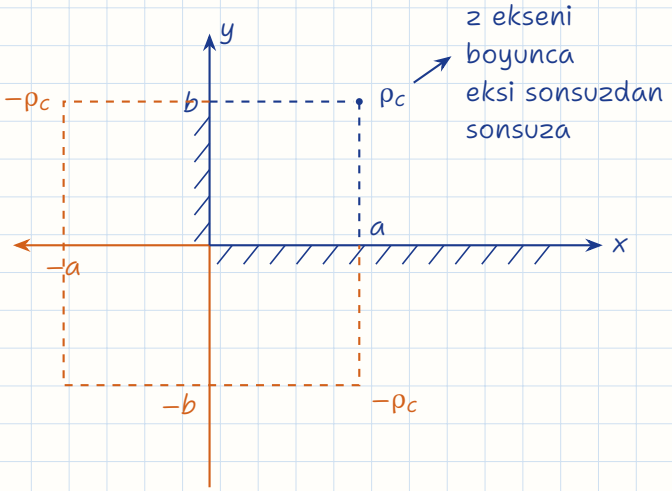
$$\vec{F} = \frac{q^2}{16\pi\varepsilon}\left[\vec{e}_x\left(-\frac{1}{a^2} + \frac{a}{(a^2+b^2)^{3/2}}\right) + \vec{e}_y\left(-\frac{1}{b^2} + \frac{b}{(a^2+b^2)^{3/2}}\right)\right]$$

S66 Pr.7



ρ_c'nin birim uzunluğuna etki eden kuvvet nedir?

orj. s. 2



Sonsuz uzun çizgisel yük dağılımı için

$$\vec{E} = \frac{\rho_c}{2\pi\varepsilon\rho}\,\vec{e}_\rho$$

$$\vec{E} = -\frac{\rho_c}{2\pi\varepsilon\,2a}\,\vec{e}_x - \frac{\rho_c}{2\pi\varepsilon\,2b}\,\vec{e}_y + \frac{\rho_c}{2\pi\varepsilon\,2\sqrt{a^2+b^2}}\left(\frac{a\vec{e}_x + b\vec{e}_y}{\sqrt{a^2+b^2}}\right)$$

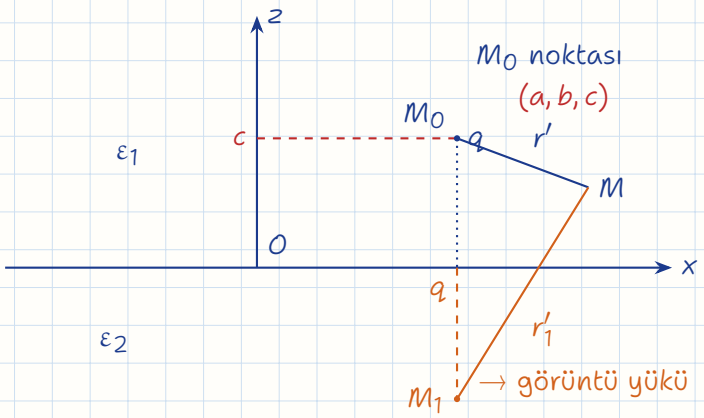
$$\vec{E} = \frac{\rho_c}{4\pi\varepsilon}\left(\vec{e}_x\left(-\frac{1}{a} + \frac{a}{a^2+b^2}\right) + \vec{e}_y\left(-\frac{1}{b} + \frac{b}{a^2+b^2}\right)\right)$$

$$= -\frac{\rho_c}{4\pi\varepsilon(a^2+b^2)}\left[+\vec{e}_x\frac{b^2}{a} + \vec{e}_y\frac{a^2}{b}\right]$$

$$\vec{F} = \int_{z=0}^{z=1} \rho_c\,\vec{E}\,dz = \rho_c\vec{E}$$

$$= -\frac{\rho_c^2}{4\pi\varepsilon(a^2+b^2)}\left[+\vec{e}_x\frac{b^2}{a} + \vec{e}_y\frac{a^2}{b}\right]$$

S66 Pr.8



q yükünün oluşturduğu pot. fonksiyonu

$$V(M) = \begin{cases} \dfrac{q}{4\pi\varepsilon_1 r'} + \dfrac{q(\varepsilon_1 - \varepsilon_2)}{4\pi\varepsilon_1(\varepsilon_1 + \varepsilon_2) r_1'}, & z > 0 \\ \dfrac{2q\varepsilon_2}{4\pi\varepsilon_2(\varepsilon_1 + \varepsilon_2) r'}, & z < 0 \end{cases}$$

orj. s. 3 olduğunu gösteriniz

$z = 0$ düzleminde biriken yükü bulunuz.

Denk problem çözülmeden yazılır

$z > 0$ $\quad \Delta V = -\dfrac{\rho_v}{\varepsilon_1} = -\dfrac{q}{\varepsilon_1}\delta(x-a)\,\delta(y-b)\,\delta(z-c)$

$z < 0$ $\quad \Delta V = 0$

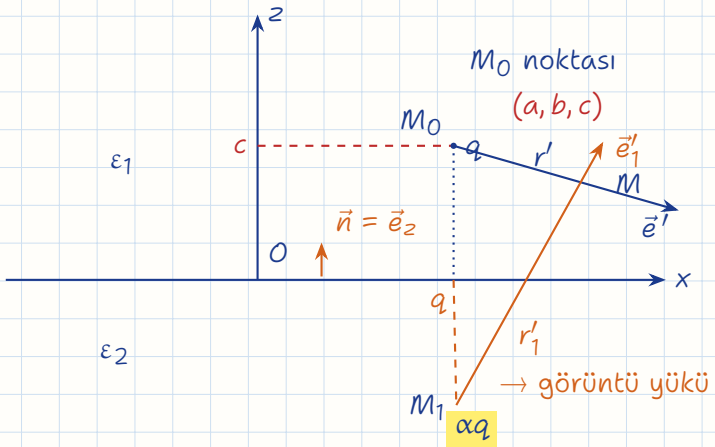
$z = 0$ sınır problemleri:

$$V_1 = V_2 \qquad \vec{n}\cdot\vec{D}_1 = \vec{n}\cdot\vec{D}_2$$

$$\vec{e}_z\cdot\vec{D}_1 = \vec{e}_z\cdot\vec{D}_2$$

Şimdi de denk problemden bakalım

$z > 0$ $\quad \varepsilon_2 = \varepsilon_1$ seçelim

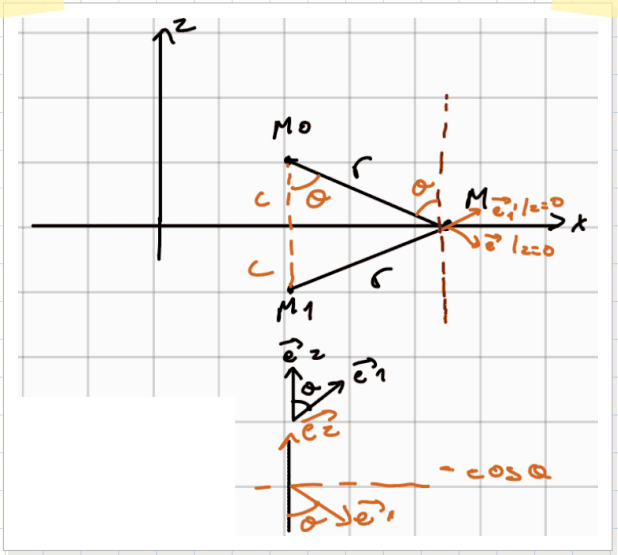


M'nin $z > 0$ iken M noktasında kaynaklanan potansiyel

$$V_1 = \frac{q}{4\pi\varepsilon_1 r'} + \frac{\alpha q}{4\pi\varepsilon_1 r_1'}$$

$$\vec{E}_1 = -\operatorname{grad} V_1 = \frac{q}{4\pi\varepsilon_1}\left(\frac{1}{r'^2}\vec{e}' + \frac{\alpha}{r_1'^2}\vec{e}_1'\right)$$

$$\begin{aligned} V_1\big|_{z=0} &= \frac{q}{4\pi\varepsilon_1}\left(\frac{1}{r} + \frac{\alpha}{r}\right) \\ &= \frac{q}{4\pi\varepsilon_1 r}\left(\frac{1}{r} + \frac{\alpha}{r}\right)^{2} \\ &= \frac{q}{4\pi\varepsilon_1 r}(1+\alpha) \end{aligned}$$



orijinal çizim, s. 3

$$\vec{e}_z \cdot \vec{D}_1\big|_{z=0} \overset{\varepsilon_1 \text{ gider}}{=} \frac{q}{4\pi}\left(\frac{\vec{e}_z \cdot \vec{e}'|_{z=0}}{r^2} + \frac{\alpha\,\vec{e}_z \cdot \vec{e}_1'|_{z=0}}{r^2}\right)$$

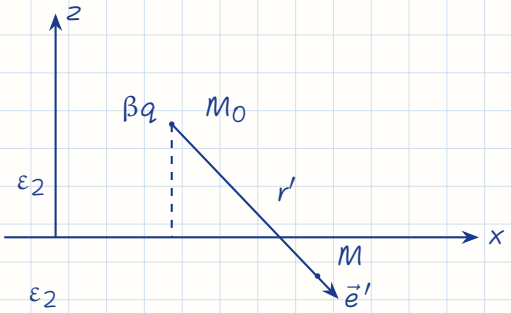
orj. s. 4

$$= \frac{q}{4\pi r^2}(-\cos\theta + \alpha\cos\theta)$$

$$= \frac{qc}{4\pi r^3}(\alpha - 1)$$

$z < 0$ bölgesinde $\varepsilon_1 = \varepsilon_2 =$ olduğunu varsayarak M_0'a bir βq yükü yerleştirerek denk problemi tanımlayalım

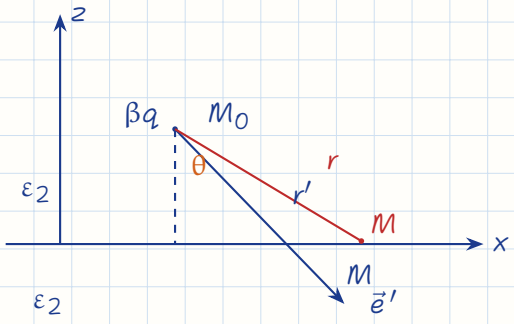
[2] Claude notu: bu satır bir sonraki satırla tutarlı değil; muhtemelen parantez içi $(1+\alpha)$ olmalı.



$$V_2 = \frac{\beta q}{4\pi\varepsilon_2 r'}$$

$$\vec{E} = -\operatorname{grad} V = -\frac{\partial V}{\partial r}\vec{e}\,' = \frac{\beta q}{4\pi\varepsilon_2 r'^2}\vec{e}\,'$$

$$z = 0 \qquad V_2\Big|_z = \frac{\beta q}{4\pi\varepsilon_2 r'}$$



$$\vec{e}_z \cdot \vec{D}_2\Big|_{z=0} = \frac{\beta q}{4\pi r^2}\,\vec{e}_z \cdot \vec{e}\,'\Big|_{z=0} = -\frac{\beta q}{4\pi r^2}\cos\theta = -\frac{\beta q c}{4\pi r^3}$$

$$V_1\Big|_{z=0} = V_2\Big|_{z=0}$$

$$\frac{q}{4\pi\varepsilon_1 r}(1+\alpha) = \frac{\beta q}{4\pi\varepsilon_2 r}$$

$$\Rightarrow \boxed{\varepsilon_2(1+\alpha) = \varepsilon_1\beta}$$

$$\vec{e}_z \cdot \vec{D}_1\Big|_{z=0} = \vec{e}_z \cdot \vec{D}_2\Big|_{z=0}$$

$$\Rightarrow \frac{qc}{4\pi r^3}(\alpha - 1) = \frac{-\beta qc}{4\pi r^3}$$

orj. s. 5

$$(\alpha - 1) = -\beta \Rightarrow \boxed{1 - \alpha = \beta}$$

$$\Rightarrow \alpha = \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2}$$

$$\beta = \frac{2\varepsilon_2}{\varepsilon_1 + \varepsilon_2} \qquad V_1 = \frac{q}{4\pi\varepsilon_1 r'} + \frac{\alpha q}{4\pi\varepsilon_1 r_1'}$$

$$V(M) = \begin{cases} V_1, & z > 0 \\ V_2, & z < 0 \end{cases} = \begin{cases} \dfrac{q}{4\pi\varepsilon_1 r'} + \dfrac{q(\varepsilon_1 - \varepsilon_2)}{4\pi\varepsilon_1(\varepsilon_1 + \varepsilon_2) r_1'}, & z > 0 \\ \dfrac{2q\varepsilon_2}{4\pi\varepsilon_2(\varepsilon_1 + \varepsilon_2) r'}, & z < 0 \end{cases}$$

$z = 0$ düzleminde biriken yükü bulalım

$$\Delta V = -\frac{\rho_v(x, y, z)}{\varepsilon} \qquad V\Big|_{z=0} = V_0$$

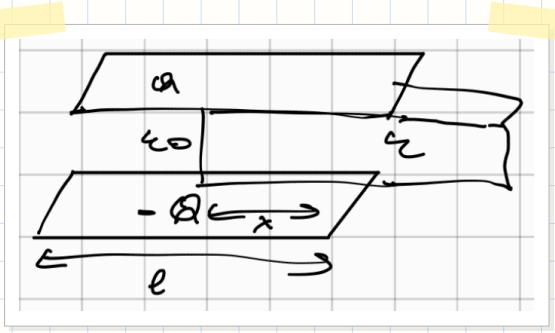
$$V = f(x, y, z) - f(x, y, -z) + V_0$$

$$\Delta V = \Delta f(x, y, z) - \overset{0}{\cancel{\Delta f(x, y, -z)}} + \overset{0}{\cancel{\Delta V_0}}$$

sadece $z > 0$? potansiyelin etkisi

$$\rho(x, y, z) = -\varepsilon_1 \Delta V\Big|_{z>0}$$

Enerji yoğunlukları
Sayfa 74 Pr.7



orijinal çizim, s. 5

Salı, 1.3