Alan Teorisi

Yüklü küre, disk ve paralel düzlemlerin alanı

Orijinal not: Uygulama2.pdf

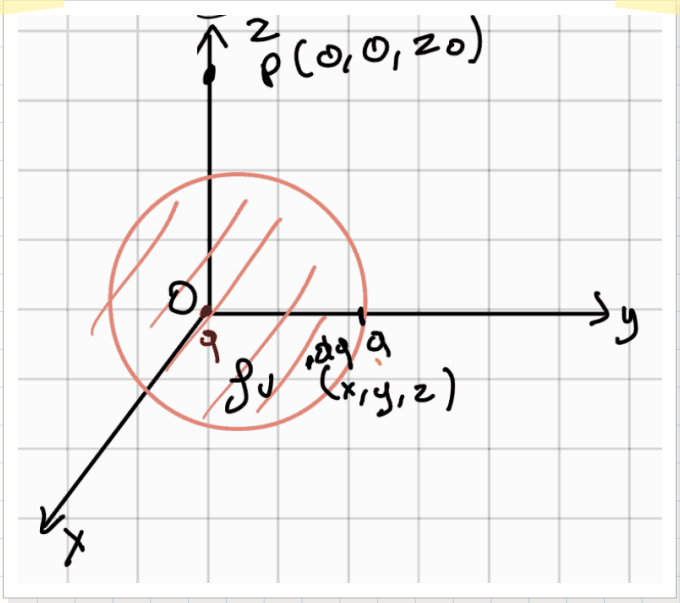
UYARI Bu belge, el yazısı ders notlarından Claude (Anthropic) yardımıyla dijitale aktarılmıştır. Yardımcı kaynak olarak hazırlanmıştır; hata veya eksik içerebilir. Lütfen orijinal notları ve ders kaynaklarını esas alın.

NOTICE *This document was transcribed from handwritten lecture notes with the help of Claude (Anthropic). It is meant as a supplementary resource and may contain errors or omissions. Always check against the original notes and official course material.*

orj. s. 1

S17 Pr.1

çözümlü sayfa 7



orijinal çizim, s. 1

Toplam yük: $q = \int_V \rho_v \, dv = \int_{r=0}^{a} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \rho_v \, r^2 \sin\theta \, dr \, d\phi \, d\theta$

$dv = r^2 \sin\theta \, dr \, d\theta \, d\phi$

$x = r \sin\theta \cos\phi$

$y = r \sin\theta \sin\phi$

$z = r \cos\theta$

Tek noktada olsaydı:

$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{qQ}{R^2} \vec{e}_R$$

ispatlayalım

$$dq = \rho_v \, dV = \rho_v \, r^2 \sin\theta \, dr \, d\phi \, d\theta$$

$$d\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{dq\,Q}{\mathcal{R}^2}\vec{e}_{\mathcal{R}} = \frac{1}{4\pi\varepsilon_0} Q\rho_v r^2 \sin\theta\, dr\, d\theta\, d\phi\, \frac{\vec{\mathcal{R}}}{\mathcal{R}^3}$$

$$\vec{\mathcal{R}} = -x\vec{e}_x - y\vec{e}_y + (z_0 - z)\vec{e}_z$$

$$= -r\sin\theta\cos\phi\,\vec{e}_x - r\sin\theta\sin\phi\,\vec{e}_y + (z_0 - r\cos\theta)\vec{e}_z$$

$$\mathcal{R} = \sqrt{r^2\sin^2\theta\cos^2\phi + r^2\sin^2\theta\sin^2\phi + z_0^2 - 2z_0 r\cos\theta + r^2\cos^2\theta}$$

$$= \sqrt{r^2\sin^2\theta + z_0^2 - 2z_0 r\cos\theta + r^2\cos^2\theta} \qquad = \sqrt{r^2 + z_0^2 - 2z_0 r\cos\theta}$$

$$= \mathcal{R}$$

$$\vec{F} = \iiint_V d\vec{F} = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{r=0}^{a} \frac{1}{4\pi\varepsilon_0} Q\rho_v r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$\frac{\vec{\mathcal{R}}}{\mathcal{R}^3} = \left[\frac{-r\sin\theta\cos\phi\,\vec{e}_x - r\sin\theta\sin\phi\,\vec{e}_y + (z_0 - r\cos\theta)\vec{e}_z}{(r^2 + z_0^2 - 2z_0 r\cos\theta)^{3/2}}\right]$$

0 ϕ integrali 0'dan 2π'ye

orj. s. 2

$$= \int_{\theta=0}^{\pi}\int_{r=0}^{a} \frac{1}{4\pi\varepsilon_0} Q\rho_v r^2 \sin\theta\, \frac{2\pi(z_0 - r\cos\theta)\vec{e}_z}{(r^2 + z_0^2 - 2z_0 r\cos\theta)^{3/2}}\, dr\, d\theta$$

$$(z_0 - r\cos\theta) = \frac{2z_0^2 - 2z_0 r\cos\theta}{2z_0}$$

$$= \frac{z_0^2 + \mathcal{R}^2 - r^2}{2z_0} \qquad \rightarrow \text{Allah bilir nasıl oldu}$$

$$= \frac{1}{2z_0}\left[\mathcal{R}^2 + (z_0^2 - r^2)\right]$$

$$\mathcal{R}^2 = r^2 + z_0^2 - 2rz_0\cos\theta$$

$$2\mathcal{R}\,d\mathcal{R} = 2rz_0\sin\theta\, d\theta$$

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{2\pi Q\vec{e}_z}{2z_0^2}\int_{r=0}^{a}\rho_v r\, dr \int_{\mathcal{R}=z_0-r}^{z_0+r}\left[\frac{z_0^2 + \mathcal{R}^2 - r^2}{\mathcal{R}^3}\right]\mathcal{R}\, d\mathcal{R}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{4\pi Q\vec{e}_z}{z_0^2}\int_{r=0}^{a}\rho_v r^2\, dr$$

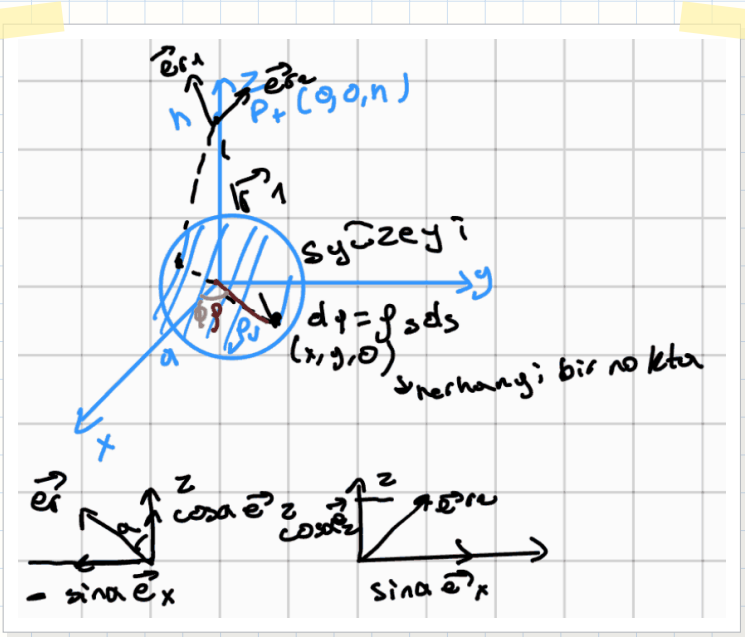
Neyse yeni soruya geçelim

S.19 pr.4 Mithat İdemen'in
S.22 k.soru elimizdeki baskıda

$x^2 + y^2 \le a^2,\ z = 0$ ρ_s yük yoğunluğu

$$\vec{E}(P_+) = e(h, a)\,\vec{e}_z$$

a)



orijinal çizim, s. 2

orj. s. 3

a) $\vec{e}_{r_1} + \vec{e}_{r_2} = 2\cos\alpha\,\vec{e}_z$
Herhangi bi nokta için geçerlidir

$$dq = \rho_s\,ds = \rho_s\,\rho\,d\rho\,d\phi$$

$$x = \rho\cos\phi$$
$$y = \rho\sin\phi$$
$$ds = \rho\,d\rho\,d\phi$$

$$\vec{r}_1 = -x\vec{e}_x - y\vec{e}_y + h\vec{e}_z$$

$$\vec{r}_1 = -\rho\cos\phi\,\vec{e}_x - \rho\sin\phi\,\vec{e}_y + h\vec{e}_z$$

$$r = \sqrt{\rho^2 + h^2} \quad //$$

$$d\vec{E}(P_+) = \frac{dq}{4\pi\varepsilon_0}\frac{\vec{r}_1}{r_1^3} = \frac{\rho_s}{4\pi\varepsilon_0}\frac{\rho\,d\rho\,d\phi}{(\rho^2+h^2)^{3/2}}\left[h\vec{e}_z - \rho\cos\phi\,\vec{e}_x - \rho\sin\phi\,\vec{e}_y\right]$$

$$\vec{E}(P_+) = \int_S d\vec{E}(P_+) = \frac{\rho_s}{4\pi\varepsilon_0}\int_{\rho=0}^{a}\int_{\phi=0}^{2\pi}\frac{\rho\,d\rho\,d\phi}{(\rho^2+h^2)^{3/2}}\left[h\vec{e}_z - \cancel{\rho\cos\phi\,\vec{e}_x}^{\,0} - \cancel{\rho\sin\phi\,\vec{e}_y}^{\,0}\right]$$

$$= \frac{\rho_s}{4\pi\varepsilon_0}\left[h\vec{e}_z\int_{\rho=0}^{a}\frac{\rho\,d\rho}{(\rho^2+h^2)^{3/2}}\underbrace{\int_{\phi=0}^{2\pi}d\phi}_{2\pi}\right]$$

$$= \frac{\rho_s\,h\vec{e}_z}{2\varepsilon_0}\int_{\rho=0}^{a}\frac{\rho\,d\rho}{(\rho^2+h^2)^{3/2}}$$

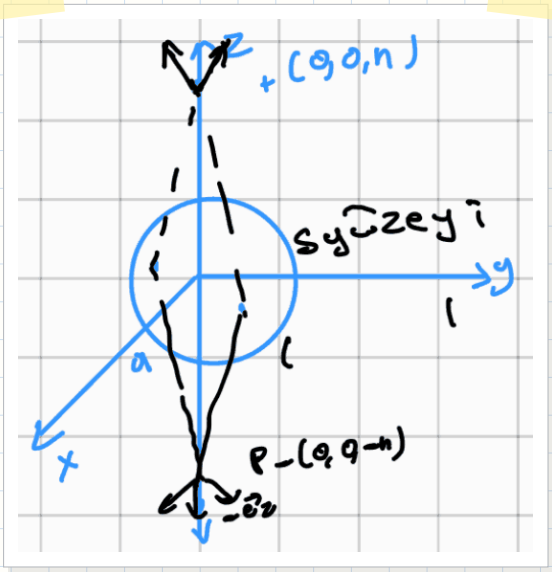
$$\boxed{\int_x \frac{x}{(x^2+a^2)^{3/2}} = \frac{-1}{\sqrt{x^2+a^2}} + C}$$

$$= \frac{\rho_s\,h\vec{e}_z}{2\varepsilon_0}\left(-\frac{1}{\sqrt{\rho^2+h^2}}\right)_{\rho=0}^{a} =$$

$$= \frac{\rho_s\,h\vec{e}_z}{2\varepsilon_0}\left(\frac{1}{h} - \frac{1}{\sqrt{a^2+h^2}}\right) = \frac{\rho_s\,\vec{e}_z}{2\varepsilon_0}\left(1 - \frac{h}{\sqrt{a^2+h^2}}\right)$$

$$\Rightarrow \vec{e}(h,a) = \frac{\rho_s \vec{e}_z}{2\varepsilon_0}\left(1 - \frac{h}{\sqrt{a^2+h^2}}\right)$$

b)



orijinal çizim, s. 3

$$\vec{E}(P_-) = -\vec{E}(P_+) = e(h,a)\left[-\vec{e}_z\right]$$
$$= -e(h,a)\,\vec{e}_z$$

orj. s. 4

c) $$\lim_{h\to 0} \vec{E}(P_+) = \frac{\rho_s \vec{e}_z}{2\varepsilon_0} \lim_{h\to 0} \underbrace{\left(1 - \frac{h}{\sqrt{a^2+h^2}}\right)}_{1}$$

$$= \frac{\rho_s \vec{e}_z}{2\varepsilon_0}$$

daireye yaklaştıkça a ve h bağlantısı azalır 0 olunca yok olur sonsuz düzlemden gibi olur.

$$\lim_{h\to 0} \vec{E}(P_-) = -\frac{\rho_s \vec{e}_z}{2\varepsilon_0}$$

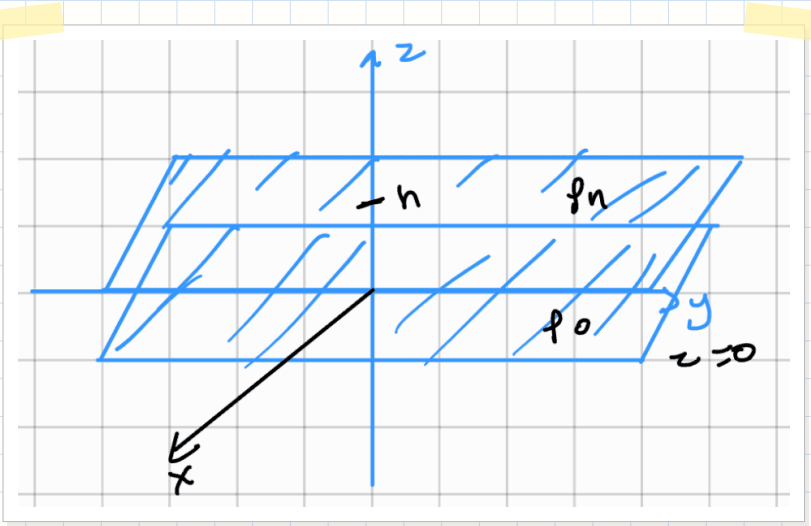
d) $a \to \infty \;\to$

$$\lim_{a\to\infty} E(P_+) = \frac{\rho_s \vec{e}_z}{2\varepsilon_0} \lim_{a\to\infty}\left(1 - \frac{h}{\sqrt{a^2+h^2}}\right)$$
$$= \frac{\rho_s \vec{e}_z}{2\varepsilon_0}$$

$$\lim_{a\to\infty} \vec{E}(P_-) = -\frac{\rho_s \vec{e}_z}{2\varepsilon_0}$$

Pr.5

> **Şekil:** Bu sayfada basılı bir soru metni (5., şıklar a, b, c) yer alıyor; dijitalleştirilmedi.
> *(orijinal notta, s. 4)*



orijinal çizim, s. 4

orj. s. 5

a) $\vec{E} = ?$ $z > h$ seçtiğimiz noktanın z'si

$$\vec{E}_I = \vec{E}_0^+ + \vec{E}_h^+ = \frac{\rho_0 \vec{e}_z}{2\varepsilon_0} + \frac{\rho_h \vec{e}_z}{2\varepsilon_0}$$

$$= \frac{(\rho_0 + \rho_h)\vec{e}_z}{2\varepsilon_0}$$

$$0 < z < h \Rightarrow \vec{E}_0^+ + \vec{E}_h^- = \frac{(\rho_0 - \rho_h)\vec{e}_z}{2\varepsilon_0}$$

$$z < 0 \Rightarrow \vec{E}_0^- + \vec{E}_h^- = -\frac{(\rho_0 + \rho_h)\vec{e}_z}{2\varepsilon_0}$$

b) $\rho_0 = \rho_h$

$$\vec{E}_I = \frac{2\rho_0 \vec{e}_z}{2\varepsilon_0} = \frac{\rho_0 \vec{e}_z}{\varepsilon_0}$$

$$\vec{E}_{II} = 0$$

$$\vec{E}_{III} = -\frac{\rho_0 \vec{e}_z}{\varepsilon_0}$$

c) $\rho_0 = -\rho_h$

$$\vec{E}_I = 0$$

$$\vec{E}_{II} = \frac{\rho_0 \vec{e}_z}{\varepsilon_0}$$

$$\vec{E}_{III} = 0$$