Alan Teorisi

Uygulama 5 – Yük dağılımları: potansiyel, kuvvet, Gauss

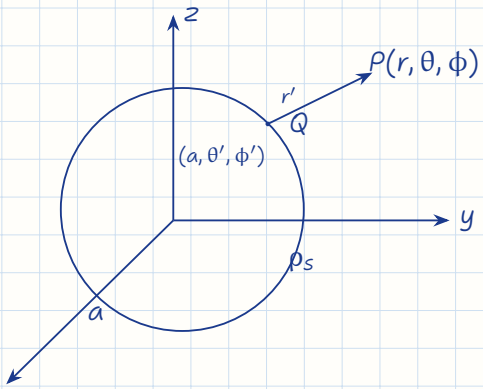
Orijinal not: UygulamaS.pdf

UYARI Bu belge, el yazısı ders notlarından Claude (Anthropic) yardımıyla dijitale aktarılmıştır. Yardımcı kaynak olarak hazırlanmıştır; hata veya eksik içerebilir. Lütfen orijinal notları ve ders kaynaklarını esas alın.

NOTICE *This document was transcribed from handwritten lecture notes with the help of Claude (Anthropic). It is meant as a supplementary resource and may contain errors or omissions. Always check against the original notes and official course material.*

orj. s. 1

S.27 Pr.10 ★ 4.65 P.2-3-10



a) $V = ?$

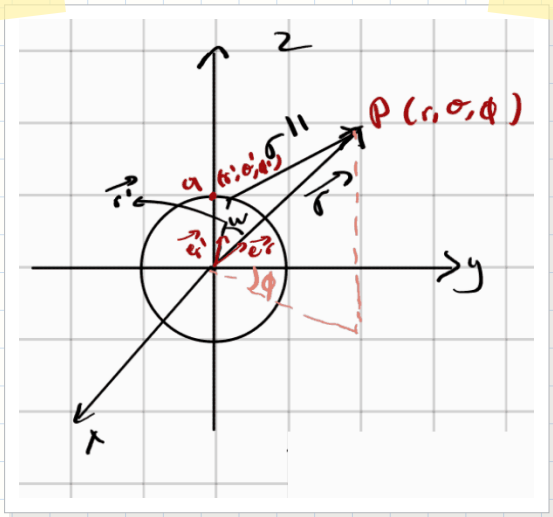
$$Q = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \rho_s \, ds, \qquad ds = a^2 \sin\theta \, d\phi \, d\theta$$

$$= \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \rho_s \, a^2 \sin\theta \, d\phi \, d\theta$$

$$\Rightarrow Q = 4\pi a^2 \rho_s \Rightarrow \rho_s = \frac{Q}{4\pi a^2}$$

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{r'} = \frac{1}{4\pi\varepsilon_0}\frac{\rho_s \, dS}{r'}$$

$$V = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} dV = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \frac{\rho_s}{4\pi\varepsilon_0 r'} a^2 \sin\theta' \, d\theta' \, d\phi'$$



orijinal çizim, s. 1

$\cos\psi$

$$r'' = \sqrt{a^2 + r^2 - 2ar\cos\psi}$$
$$= r\sqrt{1 + \frac{a^2}{r^2} - \frac{2a\cos\psi}{r}}$$

$$\vec{e}_r \cdot \vec{e}_{r'} = |\vec{e}_r| \cdot |\vec{e}_{r'}| \cos\psi = \cos\psi$$

$$\vec{e}_r = \sin\theta\cos\phi\,\vec{e}_x + \sin\theta\sin\phi\,\vec{e}_y + \cos\theta\,\vec{e}_z$$
$$\vec{e}_{r'} = \sin\theta'\cos\phi'\,\vec{e}_z + \sin\theta'\sin\phi'\,\vec{e}_y + \cos\theta'\,\vec{e}_z \quad [1]$$

$$\vec{e}_r \cdot \vec{e}_{r'} = \sin\theta\sin\theta'\cos\phi\cos\phi' + \sin\theta\sin\theta'\sin\phi\sin\phi' + \cos\theta\cos\theta'$$
$$\cos\psi = \sin\theta\sin\theta'\cos(\phi' - \phi) + \cos\theta\cos\theta'$$

orj. s. 2

Basitlik olması için P noktası z ekseni üzerinde olsun. $\theta = 0$ olur

$$\Rightarrow \cos\psi = \cos\theta'$$

$$V = \frac{Q}{16\pi^2\varepsilon_0}\,\frac{2\pi}{\varepsilon_0}\int_{\theta'=0}^{\pi} \frac{\sin\theta'\,d\theta'}{\sqrt{a^2 + r^2 - 2ar\cos\theta'}}\,d\theta' \quad [2]$$

$$V = \frac{Q}{16\pi^2\varepsilon_0}\,\frac{2\pi}{\varepsilon_0}\,\frac{1}{2ar}\int_{\theta'=0}^{\pi} \frac{2ar\sin\theta'\,d\theta'}{\sqrt{a^2 + r^2 - 2ar\cos\theta'}}\,d\theta'$$

$$= \frac{Q}{8\pi\varepsilon_0 ar}\left(\underbrace{\sqrt{a^2 + r^2 + 2ar}}_{\sqrt{(a+r)^2}} - \underbrace{\sqrt{a^2 + r^2 - 2ar}}_{\sqrt{(a-r)^2}}\right)$$

$$V = \frac{Q}{8\pi\varepsilon_0 ar}\Big(|a + r| - |a - r|\Big) \qquad \text{z ekseni üzerinde}$$

b) $\vec{E} = ?$ $\quad r > a$ $\quad V = \dfrac{Q}{4\pi\varepsilon_0 r}$

$$\vec{E} = -\operatorname{grad} V = -\frac{\partial V}{\partial r}\,\vec{e}_r$$

$$\boxed{\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\,\vec{e}_r}$$

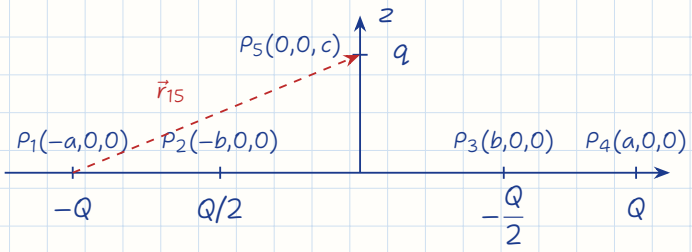
c) V ve $\vec{E}$'nin r'ye göre ? değişimi

$$V = \begin{cases} \dfrac{Q}{4\pi\varepsilon_0 r}, & r > a \\ \dfrac{Q}{4\pi\varepsilon_0 a}, & r < a \end{cases} \qquad \vec{E} = \begin{cases} \dfrac{Q}{4\pi\varepsilon_0 r^2}\,\vec{e}_r, \\ 0, \end{cases}$$

orj. s. 3

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[1] Claude notu: muhtemelen ilk terimdeki $\vec{e}_z$ yerine $\vec{e}_x$ olmalı.
[2] Claude notu: $2\pi/\varepsilon_0$ yerine muhtemelen 2π olmalı (sonraki satırlardaki sonuçla uyumlu).



a) q yüküne etkiyen $\vec{F}$ nedir?

$$\vec{F}_{15} = \frac{1}{4\pi\varepsilon_0}\frac{(-qQ)}{r_{15}^3}\vec{r}_{15}$$

$$\vec{r}_{15} = (0-(-a))\,\vec{e}_x + (c-0)\,\vec{e}_z = a\,\vec{e}_x + c\,\vec{e}_z$$

$$r_{15} = \sqrt{a^2+c^2}$$

$$\boxed{\vec{F}_{15} = \frac{Qq}{4\pi\varepsilon_0}\frac{1}{(a^2+c^2)^{3/2}}\left(-a\,\vec{e}_x - c\,\vec{e}_z\right)}$$

$\vec{r}_{25} = \vec{P}_5 - \vec{P}_2$

$$\vec{r}_{25} = b\,\vec{e}_x + c\,\vec{e}_z\,, \qquad r_{25} = \sqrt{b^2+c^2}$$

$$\boxed{\vec{F}_{25} = \frac{Qq}{8\pi\varepsilon_0}\frac{1}{(b^2+c^2)^{3/2}}\left(b\,\vec{e}_x + c\,\vec{e}_z\right)}$$

$$\vec{r}_{35} = -b\,\vec{e}_x + c\,\vec{e}_z\,, \qquad r_{35} = \sqrt{b^2+c^2}$$

$$\vec{F}_{35} = \frac{Qq}{4\pi\varepsilon_0}\frac{1}{(b^2+c^2)^{3/2}}\left(+\frac{b}{2}\,\vec{e}_x - \frac{c}{2}\,\vec{e}_z\right)$$

$$\vec{r}_{45} = -a\,\vec{e}_x + c\,\vec{e}_z\,, \qquad r_{45} = \sqrt{a^2+c^2}$$

$$\vec{F}_{45} = \frac{Qq}{4\pi\varepsilon_0}\frac{1}{(a^2+c^2)^{3/2}}\left(-a\,\vec{e}_x + c\,\vec{e}_z\right)$$

$$\vec{F} = \vec{F}_{15} + \vec{F}_{25} + \vec{F}_{35} + \vec{F}_{45}$$

$$\vec{F} = \frac{Qq}{4\pi\varepsilon_0}\left[\begin{aligned}&\frac{1}{(a^2+c^2)^{3/2}}\left(-a\,\vec{e}_x - \cancel{c\,\vec{e}_z}\right)\\ &+\frac{1}{(b^2+c^2)^{3/2}}\left(\frac{1}{2}b\,\vec{e}_x + \cancel{\frac{1}{2}c\,\vec{e}_z}\right)\\ &+\frac{1}{(b^2+c^2)^{3/2}}\left(\frac{1}{2}b\,\vec{e}_x - \cancel{\frac{1}{2}c\,\vec{e}_z}\right)\\ &+\frac{1}{(a^2+c^2)^{3/2}}\left(-a\,\vec{e}_x + \cancel{c\,\vec{e}_z}\right)\end{aligned}\right]$$

orj. s. 4

$$\boxed{\vec{F} = \frac{Qq}{4\pi\varepsilon_0}\left(\frac{-2a}{(a^2+c^2)^{3/2}} + \frac{b}{(b^2+c^2)^{3/2}}\right)}$$ [3]

[3] Claude notu: muhtemelen sonda $\vec{e}_x$ olmalı (c şıkkında aynı ifade $\vec{e}_x$ ile yazılmış).

b) q yükünün potansiyel enerjisi nedir?

$$V_1 = \frac{-Q}{4\pi\varepsilon_0}\frac{1}{\sqrt{a^2+c^2}} \qquad V_3 = \frac{-Q}{4\pi\varepsilon_0}\frac{1}{2\sqrt{b^2+c^2}}$$

$$V_2 = \frac{Q}{4\pi\varepsilon_0}\frac{1}{2\sqrt{b^2+c^2}} \qquad V_4 = \frac{Q}{4\pi\varepsilon_0}\frac{1}{\sqrt{a^2+c^2}}$$

$$V_T = 0$$

c) $a = 4b$ halinde Oy ekseni üzerinde hangi noktada hangi noktada $\vec{F} = 0$ olur? $P_q(0,y,0)$

$$\vec{F} = \frac{Qq}{4\pi\varepsilon_0}\left(\frac{-2a}{(a^2+y^2)^{3/2}} + \frac{b}{(b^2+y^2)^{3/2}}\right)\vec{e}_x$$

$$= \frac{Qq}{4\pi\varepsilon_0}\left(\underbrace{\frac{-8b}{(16b^2+y^2)^{3/2}} + \frac{b}{(b^2+y^2)^{3/2}}}_{0}\right)\vec{e}_x$$

$$= \frac{8\cancel{b}}{(16b^2+y^2)^{3/2}} = \frac{\cancel{b}}{(b^2+y^2)^{3/2}}$$

$$8(b^2+y^2)^{3/2} = (16b^2+y^2)^{3/2}$$

$$64(b^2+y^2)^3 = (16b^2+y^2)^3$$

$$4(b^2+y^2) = 16b^2 + y^2$$

$$\Rightarrow 3y^2 = 12b^2$$

$$y^2 = 4b^2$$

$\boxed{y = \pm 2b}$ noktalarında $\vec{F} = 0$ olur

orj. s. 5 Sayfa 32. 1,2,3 sorularını çözeceğiz

Gauss

$$\int_S \vec{D}\cdot ds = Q \qquad \vec{D} = \varepsilon_0\vec{E}$$



orijinal çizim, s. 5

$$Q = \int_V \rho_v\, dV$$

$\star$ $\operatorname{div} \vec{D} = \rho$

$\vec{D}$: deplansman vektörü

ε_0 : Boş uzayın elektriksel geçirgenliği

(dielectric permettivity)

$\Delta V = 0$ laplacian denklemi
kapalı yüzeyin içinde yük yok

$\Delta V = -\frac{\rho}{\varepsilon_0}$: poisson denklemi

Claude ile dijitalleştirildi · hata içerebilir / Digitised with Claude · may contain errors