Analog Haberleşme

Bozulmasız iletim, filtreler ve modülasyon

Orijinal not: Ders6.pdf

UYARI Bu belge, el yazısı ders notlarından Claude (Anthropic) yardımıyla dijitale aktarılmıştır. Yardımcı kaynak olarak hazırlanmıştır; hata veya eksik içerebilir. Lütfen orijinal notları ve ders kaynaklarını esas alın.

NOTICE *This document was transcribed from handwritten lecture notes with the help of Claude (Anthropic). It is meant as a supplementary resource and may contain errors or omissions. Always check against the original notes and official course material.*

Bozulmasız iletim

orj. s. 1

(Distortionless Transmission)

$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t) = k\,x(t - t_d)$

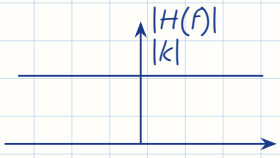
kanal

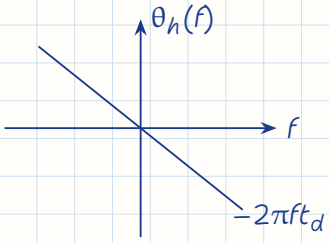
$$y(t) = K \cdot x(t - t_d)$$
$$Y(f) = k\,X(f)\,e^{-j2\pi f t_d}$$
$$H(f) = \frac{Y(f)}{X(f)} = k\,e^{-j2\pi f t_d}$$
$$= |H(f)|\,e^{j\theta_h(f)}$$

$$\begin{cases} |H(f)| = |k| \ \checkmark \\ \theta_h(f) = -2\pi f t_d \ \checkmark \end{cases} \qquad \mathcal{F}^{-1} \downarrow$$

$$\star \ \boxed{h(t) = k\,\delta(t - t_d)}$$

Bozulmasız sistem



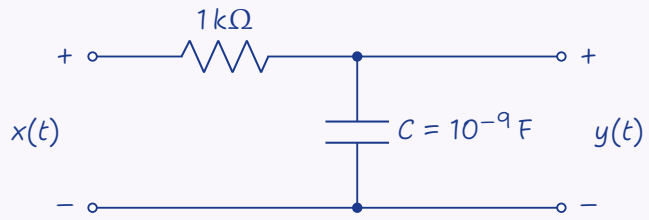


$$\text{Grup gecikmesi} = t_d(f) = -\frac{1}{2\pi}\frac{d\theta_h(f)}{df} \quad \text{(Group / envelope delay)}$$

$$\theta_h(f) = 2\pi f t_d \Rightarrow t_d(f) = \underset{\star \text{ sabit}}{t_d}$$ [1]

[1] Claude notu: önceki satıra göre muhtemelen $\theta_h(f) = -2\pi f t_d$ olmalı.

Örnek



Bu filtreden işaretin bozulmadan geçmesi için $x(t)$'nin bant genişliği ne olmalıdır? (%2'lik genlik cevabı toleransı, %5'lik grup gecikmesi toleransı mevcuttur.)

Alçak geçiren filtre

$$H(f) = \frac{Y(f)}{X(f)} = \frac{\frac{1}{j2\pi fC}}{R + \frac{1}{j2\pi fC}} = \frac{1}{1 + j2\pi fRC} = \frac{1}{1 + j\frac{f}{a}}$$
$$= \frac{a}{a + jf}$$

$$a = \frac{1}{2\pi RC} = \frac{10^6}{2\pi}$$

orj. s. 2

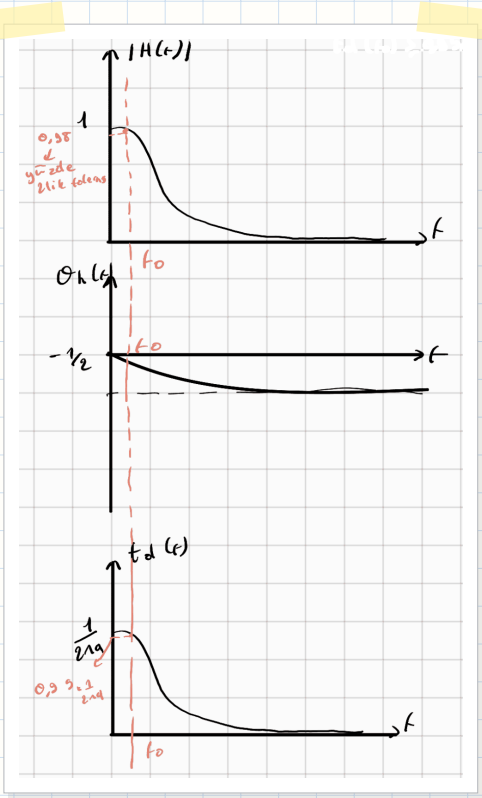
$$\Rightarrow |H(f)| = \frac{a}{\sqrt{a^2 + f^2}}$$
$$\theta_h(f) = 0 - \arctan\left(\frac{f}{a}\right)$$
$$t_d(f) = -\frac{1}{2\pi}\frac{d\theta_h(f)}{df} = \frac{1}{2\pi}\frac{a}{a^2 + f^2}$$

$$|H(0)| = 1 \rightarrow |H(f_0)| \geq 0{,}98 \qquad t_d(0) = \frac{1}{2\pi a}$$

$$t_d(f_0) \geq 0{,}95 \cdot \frac{1}{2\pi a}$$

$z = \frac{y}{x}$: $y \rightarrow |y|e^{j\theta_y}$, $x \rightarrow |x|e^{j\theta_x}$, $z \rightarrow |z|e^{j\theta_z}$

$$|z| = \frac{|y|}{|x|} \qquad \theta_z = \theta_y - \theta_x$$



orijinal çizim, s. 2

$$|H(f_0)| \geq 0{,}98$$

$$\frac{a}{\sqrt{a^2 + f_0^2}} \geq 0{,}98 \Rightarrow \underline{f_0 \leq 32{,}32\,\text{kHz}}$$

$$t_d(f_0) \geq 0{,}95 \times \frac{1}{2\pi a}$$

$$\frac{1}{2\pi}\frac{a}{a^2 + f_0^2} \geq \frac{0{,}95}{2\pi a} \Rightarrow \underline{f_0 \leq 36{,}5\,\text{kHz}}$$

$$\Rightarrow f_0 = 32{,}32\,\text{kHz}$$

Bu bantta ($f < f_0$),

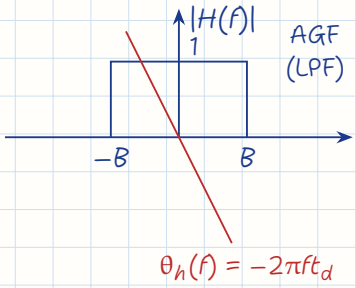
$$|H(f)| \cong 1 \qquad t_d \cong \frac{1}{2\pi a} = 10^{-6}\ \text{sn}$$

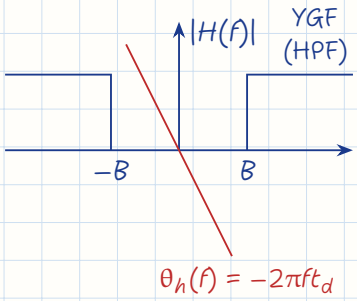
$$y(t) = k\,x(t - t_d)$$

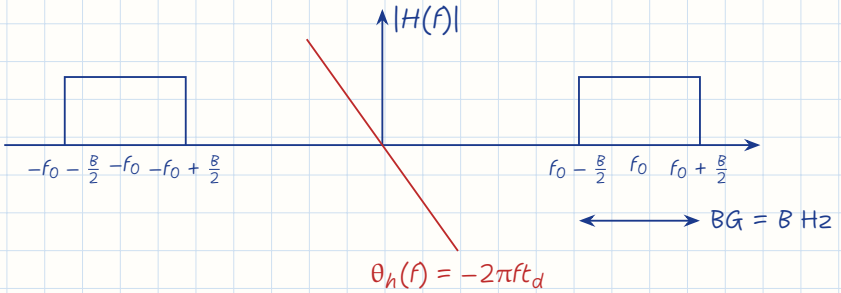
$$y(t) \cong x(t - t_d) = x(t - 10^{-6})$$

orj. s. 3

İdeal ve pratik filtreler







$$BG\{x(t)\} \le B \Rightarrow y(t) = x(t - t_d)$$

$$|H(f)| = \Pi\left(\frac{f}{2B}\right)$$

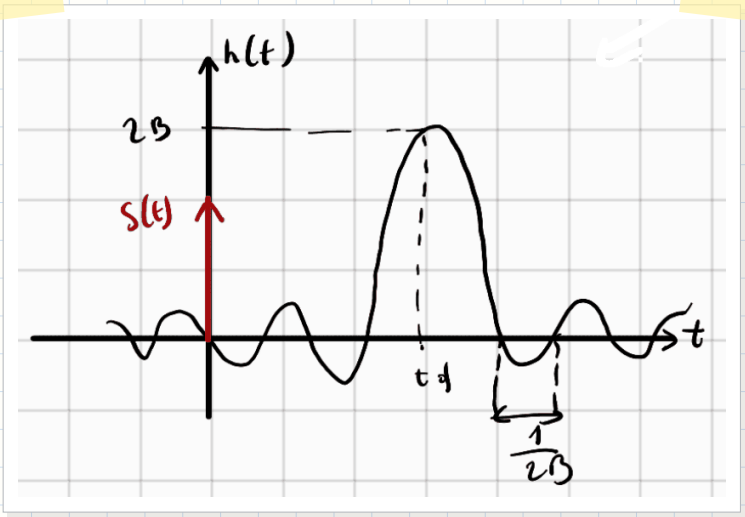
$$H(f) = |H(f)| \cdot e^{-j\theta_h(f)2}$$

$$H(f) = \Pi\left(\frac{f}{2B}\right) e^{-j2\pi f t_d}$$

$$\downarrow \mathcal{F}^{-1}$$

$$h(t) = 2B\,\mathrm{sinc}\left[2B(t - t_d)\right]$$

$$t = \frac{n}{2B} \qquad 2Bt = n \in \mathbb{Z} - \{0\}$$



orijinal çizim, s. 3

[2] Claude notu: sayfa 1'deki $H(f) = |H(f)|e^{j\theta_h(f)}$ ile tutarlı olması için muhtemelen $e^{j\theta_h(f)}$ olmalı.

Bu sistem nedensel değildir çünkü $h(t) = 0,\ t < 0$ şartı sağlanmıyor!!

$t_d \gg$ (çok büyükse) $h(t)$ nedenseldir

$t_d \approx 3 \cdot \frac{1}{2B}$ ise $h(t)$ ideale yakın ve nedensel bir gerçekleştirimdir.

orj. s. 4

$$\text{Ses işareti}: \quad B = 20\,\text{kHz} \Rightarrow t_d \simeq 3 \cdot \frac{1}{2 \cdot 20 \cdot 10^3} = 0{,}75 \cdot 10^{-4}$$
$$= 7{,}5 \cdot 10^{-5}$$
$$\approx 10^{-4} = 0{,}1\ \text{ms}$$

– $\hat{h}(t) = h(t) \cdot u(t)$

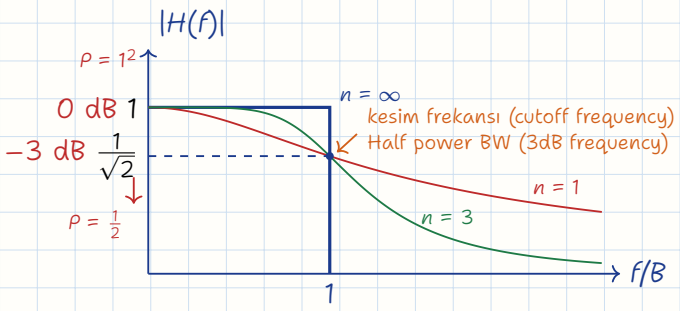
Maliyeti Bant genişliği arttırır
O'daki keskin geçişinden dolayı yüksek frekanslı bileşenleri var

diğer çözüm $w(t)$ pencereleme ile çarparız
$\hat{h}(t) = h(t) \cdot w(t)$

Pratik filtre: Butterworth

$$H(f) = \frac{1}{\sqrt{1 + (f/B)^{2n}}}$$

n: Filtrenin derecesi



n arttıkça filtrenin karmaşıklığı artar ve gecikme artar hem de maliyeti artıyor



orj. s. 5

Bir haberleşme kanalında işaret Bozulması

- Lineer Bozulma
  (Lineer Distortion)

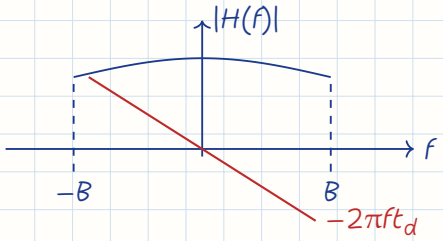
kanal ideal ⇒ Bozulmasız (Distortionless)

$$\begin{aligned} |H(f)| &= |k| \\ \theta_h(f) &= -2\pi f t_d \end{aligned} \quad \Rightarrow \quad \begin{aligned} &\text{Biri sağlanmıyorsa kanal} \\ &\text{bozulmalı (with distortion)} \end{aligned}$$

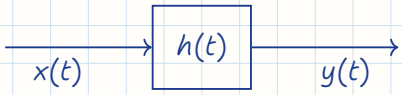
kanal bozulmalı ise $x(t)$ zamanda yayılır (time dispersion)

Örnek
(Lathi)

$$H(f) = \begin{cases} (1 + k\cos(2\pi f T))\, e^{-j2\pi f t_d}, & |f| < B \\ 0, & \text{Else} \end{cases}$$



A LPF transfer function is given in the figure. A pulse $x(t)$ band-limited to B Hz is applied at the input of this filter. Find the output $y(t)$



$$Y(f) = X(f)H(f) = X(f)\left[1 + k \underbrace{\cos(2\pi f T)}_{\frac{1}{2}\left[e^{j2\pi fT} + e^{-j2\pi fT}\right]}\right] e^{-j2\pi f t_d}$$

$$Y(f) = X(f)\, e^{-j2\pi f t_d} + k X(f) \cos(2\pi f T)\, e^{-j2\pi f t_d}$$

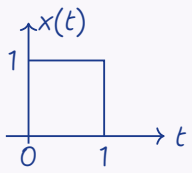
orj. s. 6

$\updownarrow F^{-1}$

$$y(t) = x(t - t_d) + \frac{k}{2}\left[x(t + T - t_d) + x(t - T - t_d)\right]$$

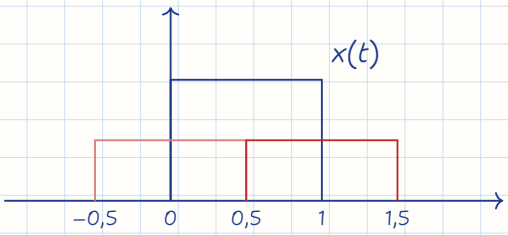
$$x(t) + \frac{k}{2}\left[x(t + T) + x(t - T)\right] \xrightarrow{t_d \text{ sabit}}$$

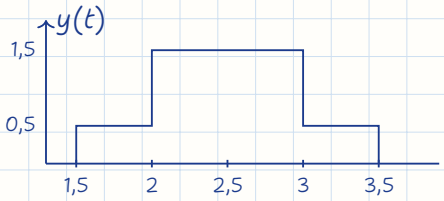
Örnek
e.g



$k = 1,\ T = 0{,}5,\ t_d = 2$

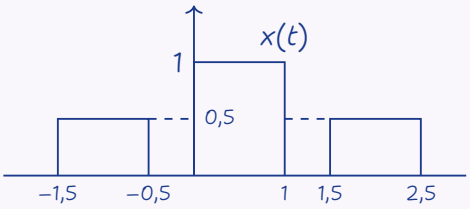
Burada ?yankılık var //





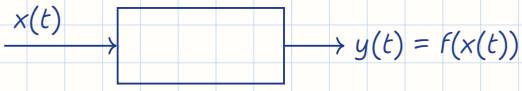
Örnek

eg $k = 1,\ T = 1{,}5,\ t_d = 2$



orj. s. 7

- Linear olmayan bozulma
  (Non linear distortion)



Belleksiz lineer olmayan kanal
(memoryless non linear channel)

Maclaurin serisine açılırsa

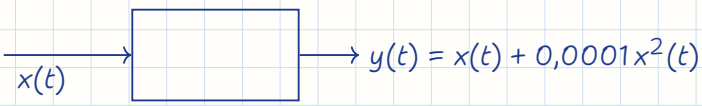
$$y(t) = f(x(t)) = a_0 + a_1 x(t) + a_2 x^2(t) + a_3 x^3(t) + \cdots + a_k x^k(t)$$

$$BG\{x(t)\} = B \qquad BG\{x^k(t)\} = k \cdot B \text{ Hz}$$

$\Rightarrow$ Frekansta yayılma
(spectral dispersion)

Örnek

(Lathi)



$$x(t) = 2000\,\text{sinc}(2000t)$$

$y(t) = ?$ $\quad Y(f) = ?$ $\quad BW\{y(t)\} = ?$

Can the signal $x(t)$ be recovered without distortion from the output $y(t)$?

$$A\tau\,\mathrm{sinc}(t\tau) \overset{F}{\longleftrightarrow} A\,\Pi\left(\frac{f}{\tau}\right)$$

$$\underbrace{2000}_{\tau}\,\mathrm{sinc}(\underbrace{2000}_{\tau}\,t) \overset{F}{\longleftrightarrow} X(f) = \Pi\left(\frac{f}{2000}\right)$$

$$\begin{aligned} y(t) &= x(t) + 0{,}0001\,x^2(t) \\ &= x(t) + 400\,\mathrm{sinc}^2(2000t) \end{aligned}$$

$$A\tau\,\mathrm{sinc}^2(t\tau) \overset{F}{\longleftrightarrow} A\,\Lambda\left(\frac{f}{\tau}\right)$$

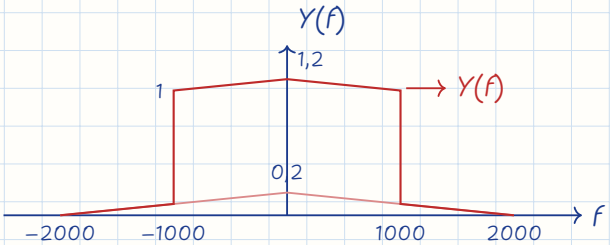
orj. s. 8

$$400\,\mathrm{sinc}^2(2000t) \overset{F}{\longleftrightarrow} 0{,}2\,\Lambda\left(\frac{f}{2000}\right)$$

? kırmızı not: $A = 400/2000$

$$y(t) = 2000\,\mathrm{sinc}(2000t) + 400\,\mathrm{sinc}^2(2000t)$$

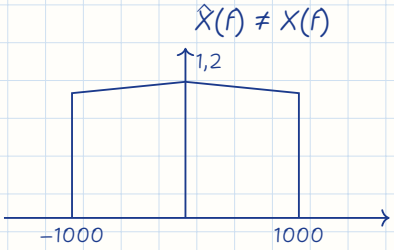
$$Y(f) = \Pi\left(\frac{f}{2000}\right) + 0{,}2\,\Lambda\left(\frac{f}{2000}\right) \quad \checkmark$$



$$BW\{y(t)\} = 2000\ \text{Hz}$$



$$BG = BG\{x(t)\} = B\ \text{Hz}$$



Bozulma

- Lineer bozulma → zamanda yayılma → TDM'de karışım (interference)
- Lineer olmayan bozulma → Frekansta yayılma → FDM'de karışım
- ⋆ Çok yollu bozulma (multipath distortion)

Claude ile dijitalleştirildi · hata içerebilir / Digitised with Claude · may contain errors

Modülasyon (Modulation)

Temel band işareti (Base band signal) → Baseband comm.
Temel bant haberleşmeyi (dijitalde yapılır)

Voice(ses) : 0-4 kHz
Video(görüntü) : 0-5/6 MHz (siyah-beyaz)
} ≡ Baseband analog signals ≡ message signals

orj. s. 9

Modülasyon → Carrier communication
(Taşıyıcı haberleşme)

Alçak geçiren veya t.b. taki işaretin bg. hale geçirmektir.

$$BG\{m(t)\} = B, \quad M(f) = 0, \quad |f| > B \Rightarrow$$ Temel band işareti (Low pass signal)

Taşıyıcı işaret (Carrier signal)

$$c(t) = \underbrace{A_c}_{\text{Genlik (Amplitude)}} \cos\left(2\pi \underbrace{f_c}_{\text{Frekans (Freq)}} t + \underbrace{\theta_c}_{\text{Faz (phase)}}\right)$$

(Frekans ve Faz) → Açı modülasyonu

Modülasyonun Sebepleri

1) Frekans bölmeli çoklama (FDM) yapmak için
(Frequency Division multiplexing (FDM))
BTK haberleşme bandını söylüyor
2) Verici alıcı yapısını basitleştirmek için.
Dalga boyu küçülür

$$\underbrace{l}_{\text{length of antenna}} = \text{Anten boyu} \propto \frac{\lambda}{4} \rightarrow \text{Dalga boyu (wavelength)}$$

Ses BG ≈ 3 kHz $\qquad c = f \cdot \lambda \Rightarrow \lambda = \frac{c}{f}$

$$= \frac{3 \cdot 10^8 \text{ m/sn}}{3 \cdot 10^3 \underbrace{\text{Hz}}_{1/\text{sn}}}$$

orj. s. 10

$$= 10^5 \text{ m} = 100 \text{ km} \rightarrow l \approx \frac{\lambda}{4} = 25 \text{ km}$$

Genlik modülasyonu (?yaptık) : $f = 1\,\text{MHz}$

$$\lambda = \frac{3 \cdot 10^8 \text{ m/sn}}{10^6 \text{ Hz}} = 300 \text{ m} \rightarrow l \approx \frac{\lambda}{4} = 75 \text{ m}$$

3) Mesaj işaretinin band genişliğini arttırarak gürültü ve karışımı bastırma özelliğini arttırmak için
(noise and interference immunity)
→ Açı modülasyonu