Analog Haberleşme

Frekans karıştırıcı, DSB-SC, AM modülasyonu: güç ve verim

Orijinal not: Ders8.pdf

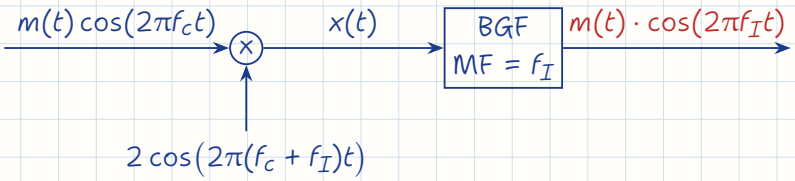
UYARI Bu belge, el yazısı ders notlarından Claude (Anthropic) yardımıyla dijitale aktarılmıştır. Yardımcı kaynak olarak hazırlanmıştır; hata veya eksik içerebilir. Lütfen orijinal notları ve ders kaynaklarını esas alın.

NOTICE *This document was transcribed from handwritten lecture notes with the help of Claude (Anthropic). It is meant as a supplementary resource and may contain errors or omissions. Always check against the original notes and official course material.*

orj. s. 1

Örnek: Frekans karıştırıcı (Frequency mixer)

Changes carrier frequency f_c (yüksek) to f_I (düşük)
i.e, $m(t)\cos(2\pi f_c t) \rightarrow m(t)\cos(2\pi f_I t)$

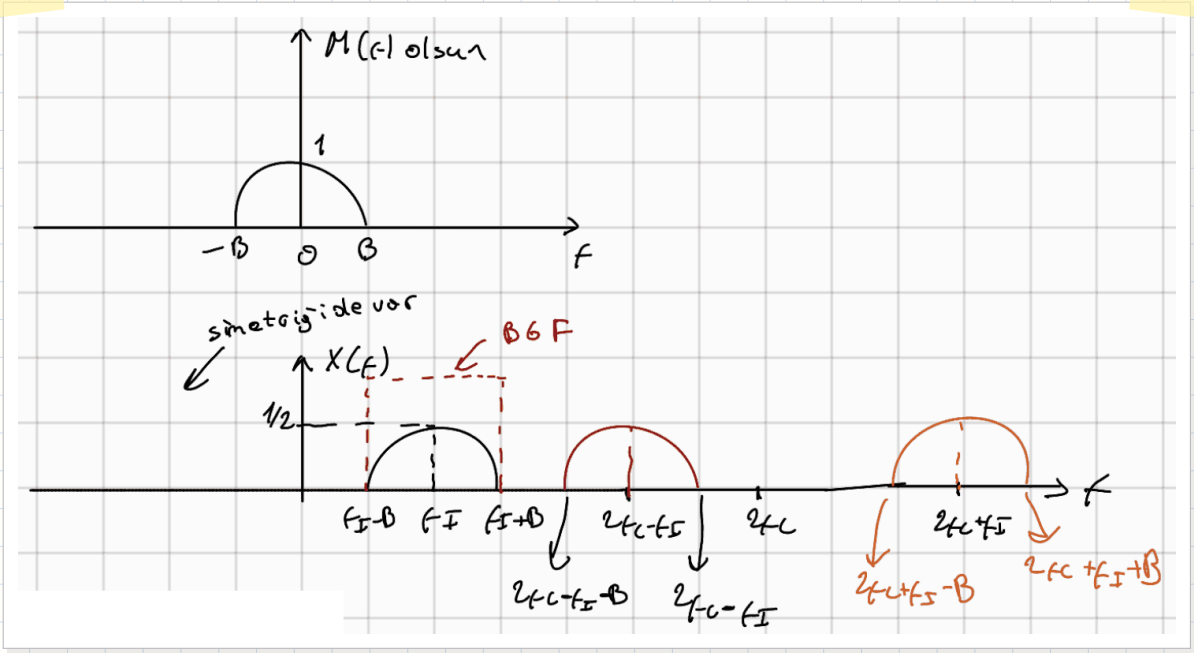


$$x(t) = 2m(t)\cos(2\pi f_c t)\cos\big(2\pi(f_c \pm f_I)t\big)$$
$$f_k = f_c - f_I \quad \text{veya} \quad f_k = f_c + f_I$$
$$= m(t)\big[\cos(2\pi(f_c - f_k)t) + \cos(2\pi(f_c + f_k)t)\big]$$

$$f_k = f_c - f_I \Rightarrow x(t) = m(t)\Big[\cos(2\pi f_I t) + \cos\big(2\pi(2f_c - f_I)t\big)\Big]$$

$$f_k = f_c + f_I \Rightarrow x(t) = m(t)\Big[\underbrace{\cos(2\pi f_I t)}_{\text{Bant geçiren filtreden geçerler}} + \underbrace{\cos\big(2\pi(2f_c + f_I)t\big)}_{\text{BGF'den geçmez}}\Big]$$

$f_I = 450\ \text{kHz}$
$f_c = 1000\ \text{kHz}$
Çok yüksek frekanslarda devre tasarlamak zor o yüzden ? *düşürüyoruz*



orijinal çizim, s. 1

$\star\ f_I - B \geq 0 \Rightarrow f_I \geq B$
Saf olarak elde edebilmek için

$$2f_c - f_I - B \geq f_I + B \Rightarrow 2(f_c - f_I) \geq 2B \Rightarrow (f_c - f_I) \geq B$$

orj. s. 2

4.2.4 kağıttaki soruları çözelim

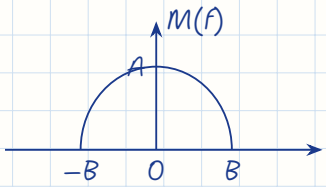
Örnek 4.2-4 (basılı soru)

Design a DSB-SC modulator to generate $km(t)\cos\omega_c t$, where $m(t)$ is band-limited to B Hz, using the modulator of Fig. P4.2-4 whose carrier generator gives $\cos^3\omega_c t$ instead of $\cos\omega_c t$ (any filter allowed). (a) What kind of filter is required? (b) Determine the signal spectra at points b and c and the frequency bands occupied. (c) What is the minimum usable value of ω_c? (d) Would this scheme work if the carrier were $\cos^2\omega_c t$? (e) Would it work for $\cos^n\omega_c t$, any integer $n \geq 2$?

Şekil: Basılı Şekil P4.2-4: $m(t)$ girişi (a noktası) → çarpıcı ⊗ (altından $\cos^3\omega_c t$ giriyor) → (b) → Filter → (c) → $km(t)\cos\omega_c t$; yanda (b) spektrumu $M(\omega)$: $-2\pi B$ ile $2\pi B$ arasında, tepe değeri A olan yarım elips. El yazısı eklemeler: çıkışın altına $\cos(2\pi f_c t)$, çarpıcıya giren taşıyıcının altına $\cos^3(2\pi f_c t)$ yazılmış.
(orijinal notta, s. 2)

DSB-SC-AM modulator

$BG\{m(t)\} = B$ Hz



a) $x(t) = m(t)\cos^3(2\pi f_c t)$

$$\cos^3(\alpha) = \cos(\alpha)\cdot\cos^2(\alpha)$$
$$= \cos\alpha\ \tfrac{1}{2}\left[1+\cos(2\alpha)\right]$$
$$= \tfrac{1}{2}\cos(\alpha) + \tfrac{1}{2}\cos(\alpha)\cos(2\alpha)$$
$$= \tfrac{1}{2}\cos(\alpha) + \tfrac{1}{4}\left[\cos(\alpha)+\cos(3\alpha)\right]$$
$$\cos^3\alpha = \tfrac{3}{4}\cos(\alpha) + \tfrac{1}{4}\cos(3\alpha)$$

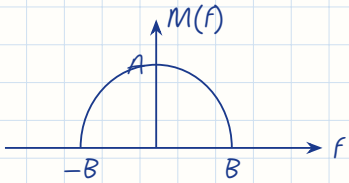
$$x(t) = m(t)\cdot\cos^3(2\pi f_c t)$$
$$= \underbrace{\tfrac{3}{4}m(t)\cos(\underbrace{2\pi f_c t}_{\alpha})}_{\text{o zaman filtremiz bu işareti geçirmeli}} + \tfrac{1}{4}m(t)\cos(2\pi\, 3 f_c\, t)$$

$\alpha \searrow$ $2B$ bant genişliğine sahip

O zaman BGF
$MF = f_c$
$BG = 2B$

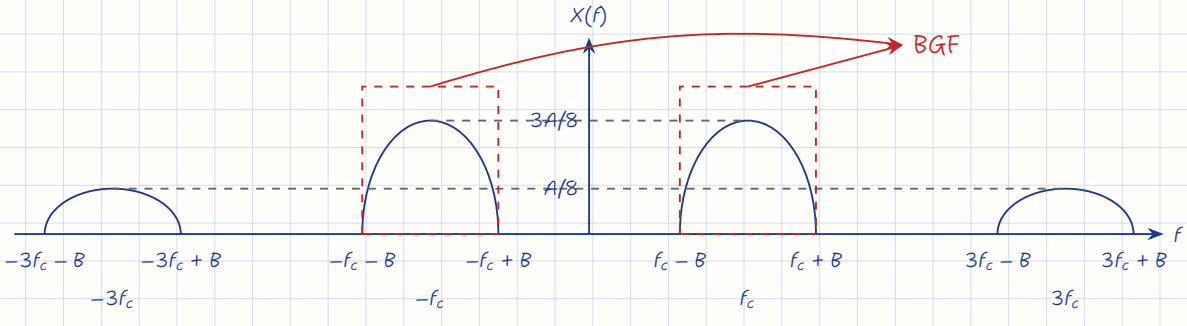
orj. s. 3

b)
ⓐ a)



ⓑ

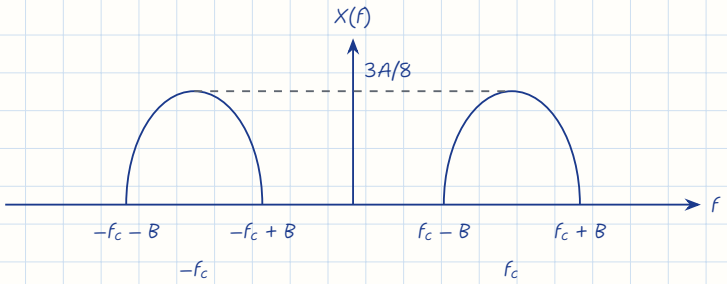
$$X(f) = \tfrac{3}{8}\left[M(f-f_c)+M(f+f_c)\right] + \tfrac{1}{8}\left[M(f-3f_c)+M(f+3f_c)\right]$$



ⓒ[1]

$$y(t) = \tfrac{3}{4}m(t)\cos(2\pi f_c t)$$
$$Y(f) = \tfrac{3}{8}\left[M(f-f_c)+M(f+f_c)\right]$$

[1] Şekildeki eksen etiketi el yazısında $X(f)$ olarak görünüyor; Claude notu: muhtemelen $Y(f)$ olmalı.



c) $f_c - B \geq 0 \Rightarrow \boxed{f_c \geq B}$

$$3f_c - B \geq f_c + B \Rightarrow 2f_c \geq 2B$$
$$\Rightarrow f_c \geq B$$

d) $\cos^2(2\pi f_c t) = \frac{1}{2}[1 + \cos(2\pi\, 2f_c t)]$

$$x(t) = m(t)\cos^2(2\pi f_c t)$$
$$= \tfrac{1}{2} m(t) + \tfrac{1}{2} m(t)\cos(2\pi\, 2f_c\, t)$$

Hayır.

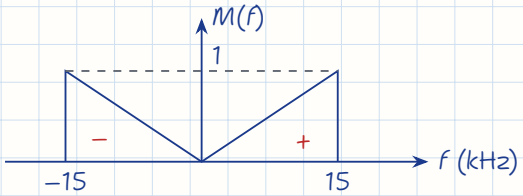
orj. s. 4

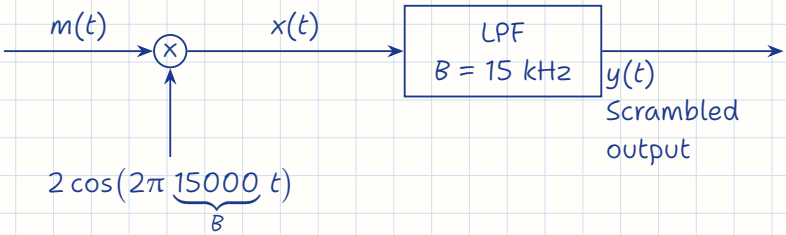
e) $\cos^n(2\pi f_c t)$, $n \geq 2$
Evet, n tek ise

Problem 4-2-9 Lathi

Örnek 4.2-9 (basılı soru)

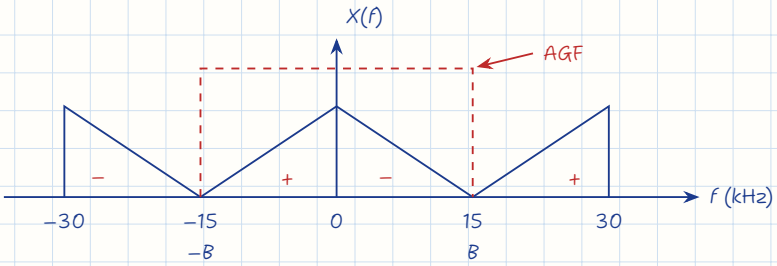
System shown in Fig. P4.2-9 is used for scrambling audio signals. The output $y(t)$ is the scrambled version of the input $m(t)$. (a) Find the spectrum of the scrambled signal $y(t)$. (b) Suggest a method of descrambling $y(t)$ to obtain $m(t)$. (Fig. P4.2-9: $m(t)$ with $M(\omega)$ between −15 kHz and 15 kHz → multiplier with $2\cos 30{,}000\pi t$ → low-pass filter 0–15 kHz → $y(t)$, scrambled output.)

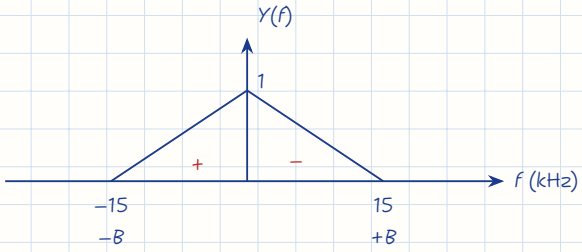




a) $x(t) = 2\, m(t)\cos(2\pi B t)$

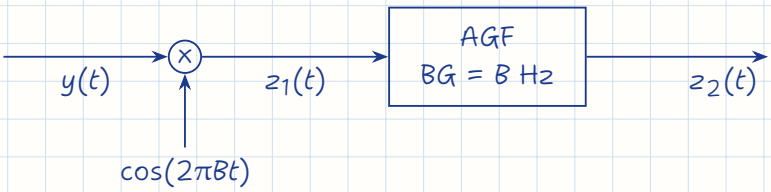
$$X(f) = M(f - B) + M(f + B)$$



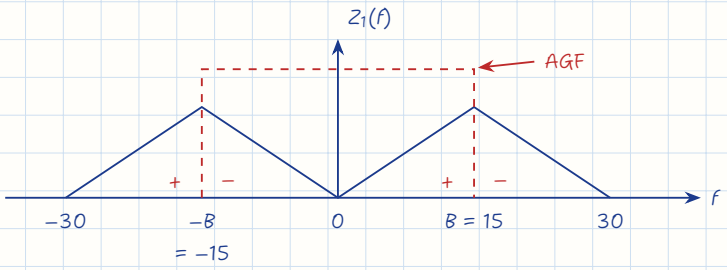


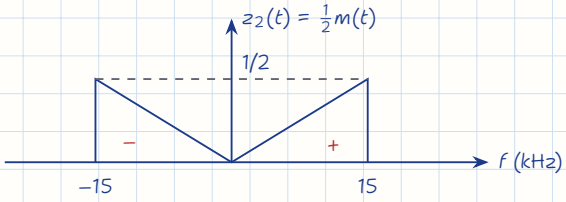
orj. s. 5

b) Descrambling: $y(t) \rightarrow m(t)$



$$z_1(t) = y(t)\cos(2\pi Bt) \Rightarrow Z_1(f) = \frac{1}{2}\left[Y(f-B) + Y(f+B)\right]$$





Bu soruyu niye yaptık ??
Mesaj işaretini şifrelemek için mesaj işareti karıştırılır. Böylece mesajı okumak için deşifre etmek lazım.

DSB-Large carrier (LC)-AM / Büyük taşıyıcılı (BT) - ÇYB-GM

Conventional AM / Commercial AM ≡ AM (Ticari GM)

Faz tahminsiz demodülasyon yapılabilir. Burada maksat alıcıyı basitleştirmek. Alıcı daha ucuz olur.

İlk sayısal haberleşme s. telegraf, analogta ise radyo.

Verici karmaşık olur alıcı ise ucuz olur.

orj. s. 6

$$y(t) = A_c\left[\underbrace{1 + \mu m_n(t)}_{\text{modüle eden işaret}}\right]\cos(2\pi f_c t)$$

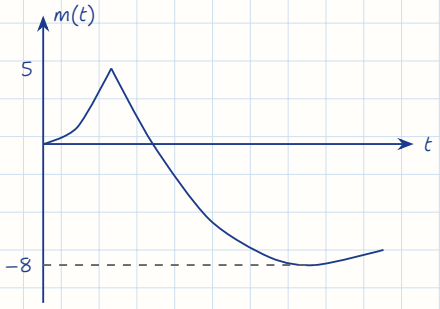
$0 < \mu < 1$ modülasyon indeksi

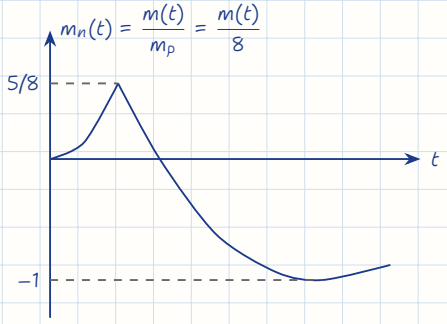
$$\underbrace{m_p}_{\text{peak value}} = \text{maks}\,|m(t)| \Rightarrow \underbrace{m_n(t)}_{\text{normalleştirilmiş mesaj işareti}} = \frac{m(t)}{m_p}$$

bütün t'ler için $\forall t \quad |m_n(t)| \le 1$

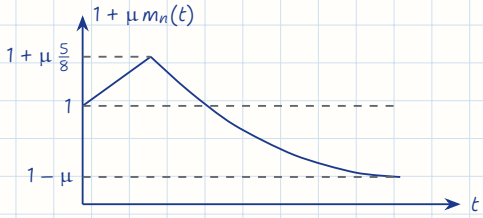
$$-1 \le m_n(t) \le 1$$

$\forall t \quad -1 \le m_n(t) \le 1$
$\Rightarrow \forall t \quad 0 < 1 + \mu m_n(t) < 2$
$\Rightarrow \forall t \quad 1 + \mu m_n(t) \ge 0$ modüle eden işaret her zaman pozitif



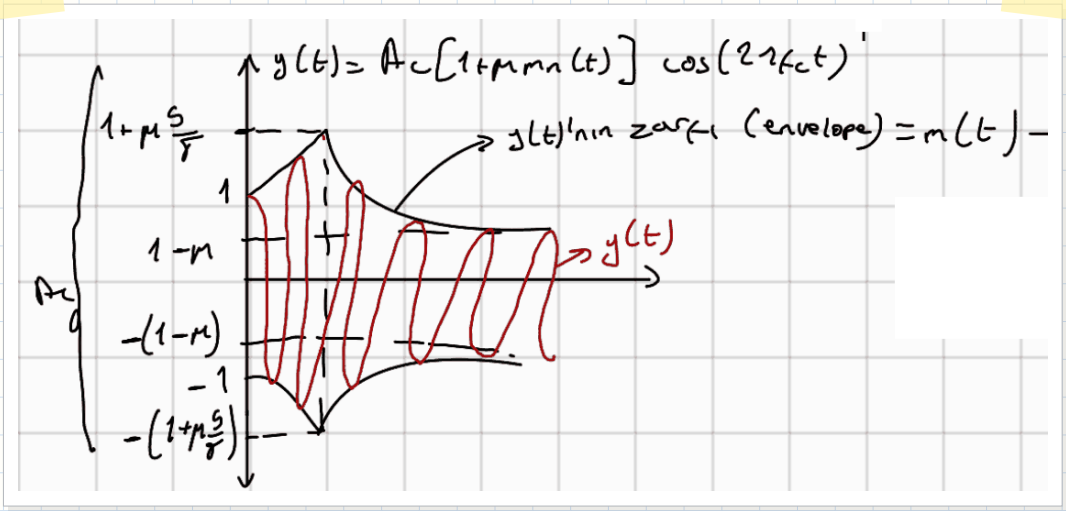


$m_p = \text{maks}\,|m(t)| = 8$



μ 1 olmasını tercih etmiyoruz

$0 < \mu < 1$ arasında seçilirse
$\Rightarrow \forall t \quad 1 + \mu m_n(t) > 0$



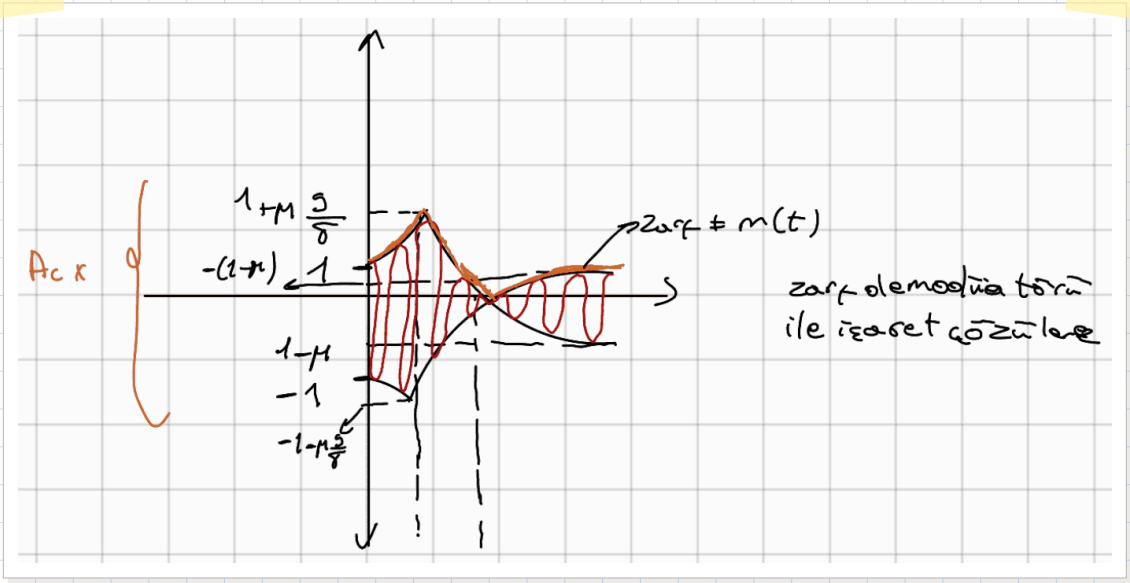
orijinal çizim, s. 6

$y(t)$'nin zarfı (envelope) = $m(t)$[2] ⟶ Faz tahminsiz demodülasyon, zarf demodülatörü (Envelop detector)

mesaj işareti zarfta gizli

orj. s. 7

Eğer $\mu > 1$



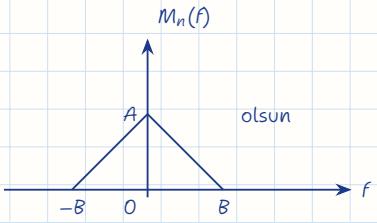
orijinal çizim, s. 7

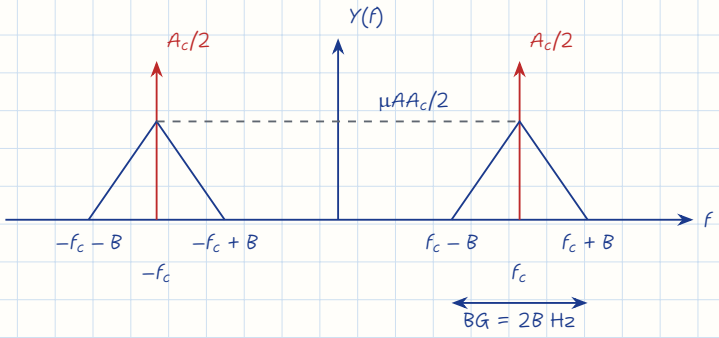
⋆ Eğer $\mu > 1$ (Aşırı modülasyon)
Over modulation

$$y(t) = A_c\left[1 + \mu\, m_n(t)\right]\cos(2\pi f_c t)$$
$$= \underbrace{A_c\cos(2\pi f_c t)}_{\substack{\text{BT}\\ \text{Büyük taşıyıcı}}} + \underbrace{\mu A_c\, m_n(t)\cos(2\pi f_c t)}_{\text{TB-ÇYB-GM}}$$

$$Y(f) = \frac{A_c}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right] + \frac{\mu A_c}{2}\left[M_n(f-f_c) + M_n(f+f_c)\right]$$

[2] Claude notu: zarf aslında $A_c[1 + \mu\, m_n(t)]$ ifadesidir (mesajla orantılı değişir); muhtemelen kastedilen budur.





orj. s. 8

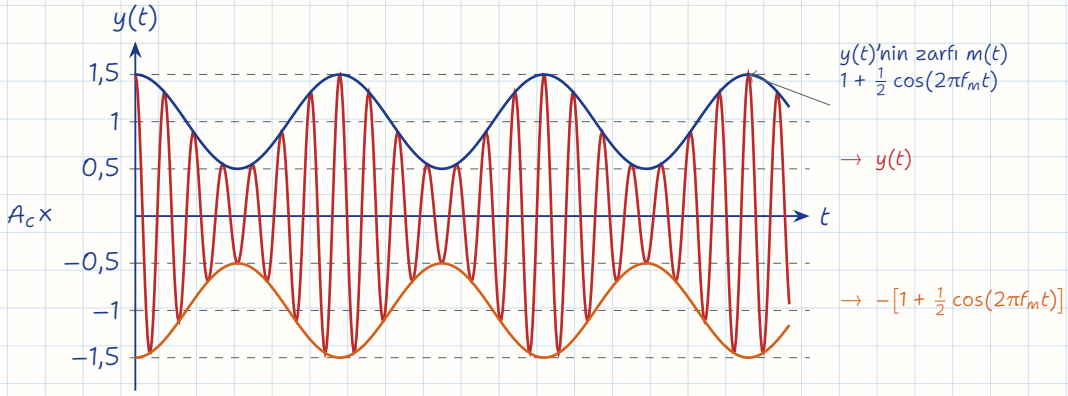
Örnek

Ton modülasyonu $\Rightarrow m(t) = c\cos(2\pi f_m t)$, $f_m \ll f_c$
for $\mu = \frac{1}{2}$, sketch DSB-LC-AM signal $g(t)$
" $Y(f)$

$$m(t) = \underbrace{c}_{c>0}\cos(2\pi f_m t) \Rightarrow m_p = c \Rightarrow m_n(t) = \frac{m(t)}{m_p} = \cos(2\pi f_m t)$$

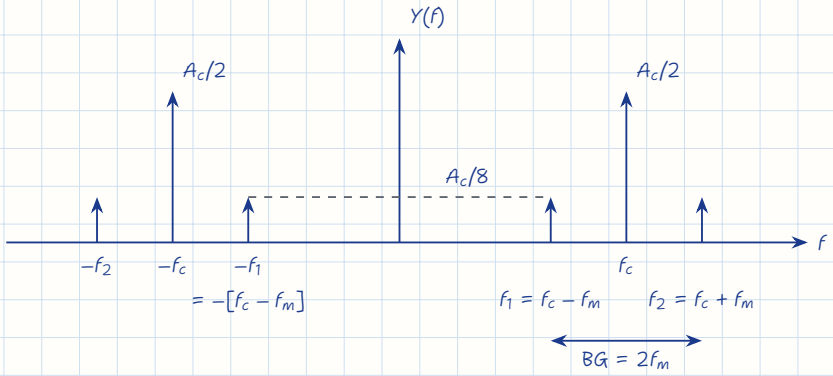
$$\begin{aligned} y(t) &= A_c\left[1 + \mu m_n(t)\right]\cos(2\pi f_c t) \\ &= A_c\left[1 + \tfrac{1}{2}\cos(2\pi f_m t)\right]\cos(2\pi f_c t) \end{aligned}$$

$f_m \ll f_c \Rightarrow T_m \gg T_c$



$$\begin{aligned} y(t) &= A_c\left[1 + \tfrac{1}{2}\cos(2\pi f_m t)\right]\cos(2\pi f_c t) \\ &= A_c\cos(2\pi f_c t) + \frac{A_c}{2}\cos(2\pi f_m t)\cos(2\pi f_c t) \\ &= A_c\cos(2\pi f_c t) + \frac{A_c}{4}\left[\cos\left(2\pi(f_c - f_m)t\right) + \cos\left(2\pi(f_c + f_m)t\right)\right] \end{aligned}$$

$$\begin{aligned} Y(f) = \frac{A_c}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right] + \frac{A_c}{8}\big[&\delta(f - f_1) + \delta(f + f_1) \\ &+ \delta(f - f_2) + \delta(f + f_2)\big] \end{aligned}$$



orj. s. 9

BT - ÇYB-GM'nin Gücü

$$y(t) = A_c\left[1 + \mu m_n(t)\right]\cos(2\pi f_c t)$$
$$= \underbrace{A_c \cos(2\pi f_c t)}_{\text{Taşıyıcı (carrier)}} + \underbrace{\mu A_c\, m_n(t)\cos(2\pi f_c t)}_{\text{yan bandlar (side bands)}}$$

$$\text{Taşıyıcının gücü} = P_c = \frac{A_c^2}{2}$$

$$\text{Yan bandın gücü} = P_s = \frac{\mu^2 A_c^2}{2}\cdot P_{m_n}$$

$$P_{m_n} = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} |m_n(t)|^2\, dt$$

⋆ m_n'nin ortalaması pratikte 0 o yüzden $\frac{1}{2}m_n(t)$'nin gücü yaklaşık 0 o yüzden iki işaretin çarpımından gelen katkı $(a+b)^2 = a^2 + b^2 + 2ab$ görmezden gelinir.

$$P_y = P_c + P_s = \frac{A_c^2}{2} + \frac{\mu^2 A_c^2}{2} P_{m_n}$$

$$\underset{\text{(Efficiency)}}{\text{Verim}} = V = \frac{P_s}{P_y} = \frac{P_s}{P_c + P_s} = \frac{\mu^2 \frac{A_c^2}{2} P_{m_n}}{\frac{A_c^2}{2} + \mu^2 \frac{A_c^2}{2} P_{m_n}}$$

$$V = \frac{\mu^2 P_{m_n}}{1 + \mu^2 P_{m_n}}$$

Ton modülasyonu $\rightarrow m(t) = c\cos(2\pi f_m t) \rightarrow m_n(t) = \cos(2\pi f_m t)$

$P_{m_n} = \frac{1}{2}$

$$\Rightarrow V = \frac{\mu^2 \cdot \frac{1}{2}}{1 + \mu^2 \frac{1}{2}} = \frac{\mu^2}{2 + \mu^2}$$

ton modülasyonu verimi $V = \dfrac{\mu^2}{2 + \mu^2}$

$\mu = 1 \rightarrow V_{max} = \dfrac{1}{3} = \%33$

Pratikte verim $V \simeq \%25$ olur.

Neden yapıyoruz alıcı ucuzluyor verici pahalı oluyor, böyle herkes alabiliyor alıcıyı.

⚠ Dikkat!
Verim sorabilir

Örnek

$m(t) = 5\cos(2\pi f_m t)$ $\quad \mu = \frac{1}{2},\ A_c = 10$
$y(t) = ?\quad P_c = ?\quad P_s = ?\quad V = ?$

orj. s. 10

$m_p = 5 \Rightarrow m_n(t) = \cos(2\pi f_m t)$

$$y(t) = A_c\left[1 + \mu m_n(t)\right]\cos(2\pi f_c t)$$
$$= 10\left[1 + \tfrac{1}{2}\cos(2\pi f_m t)\right]\cos(2\pi f_c t)$$

$$P_c = \frac{A_c^2}{2} = \frac{10^2}{2} = 50\ \text{W}$$
$$P_s = \frac{\mu^2 A_c^2}{2}\cdot P_{m_n} = \frac{(\frac{1}{2})^2 \cdot 10^2}{2} = \frac{1}{2}^3 = \frac{100}{16} = 6{,}25\ \text{W}$$

$$P_y = P_c + P_s = 56{,}25\ \text{W}$$

$$V = \frac{P_s}{P_y} = \frac{P_s}{P_c + P_s} = \frac{6{,}25}{50 + 6{,}25} = \frac{6{,}25}{56{,}25} = \frac{1}{9}$$
$$\text{veya}\quad V = \frac{\mu^2}{2 + \mu^2} = \frac{\frac{1}{4}}{2 + \frac{1}{4}} = \frac{1}{9}$$

Örnek (Proakis)

$m(t) = 3\cos(200\pi t) + \sin(600\pi t)$ is used to modulate the carrier $c(t) = \cos(2\pi \cdot 10^5 t)$. The modulation Index is $\mu = 0{,}85$. Determine the power in the carrier component and in the sideband components of the modulated signal.

$m(t) = 3\cos(200\pi t) + \sin(600\pi t)$

Max değeri bulmak için türev alıp 0'a eşitleriz

$$m'(t) = -3 \cdot 200\pi \sin(200\pi t) + 600\pi\cos(600\pi t) = 0$$
$$600\pi\sin(200\pi t) = 600\pi\cos(600\pi t)$$
$$\sin(200\pi t) = \cos(600\pi t)$$
$$\cos\left(\tfrac{\pi}{2} - 200\pi t\right) = \cos(600\pi t)$$
$$\Rightarrow \tfrac{\pi}{2} - 200\pi t = 600\pi t$$
$$\tfrac{\pi}{2} = 800\pi t \Rightarrow t = \frac{1}{1600}$$

$$\sin\alpha = \cos\left(\tfrac{\pi}{2} - \alpha\right)$$

$$m(t)\Big|_{t=\frac{1}{1600}} = m\left(\frac{1}{1600}\right) = 3{,}6955 = m_p$$

[3] Claude notu: muhtemelen $\cdot \frac{1}{2}$ ($P_{m_n} = \frac{1}{2}$ ile çarpım) olmalı.

$$m_n(t) = \frac{m(t)}{3{,}6955} = 0{,}8118\cos(200\pi t) + 0{,}2706\sin(600\pi t)$$

$$P_{m_n} = \tfrac{1}{2}\left[0{,}8118^2 + 0{,}2706^2\right] = 0{,}3661 \text{ W}$$

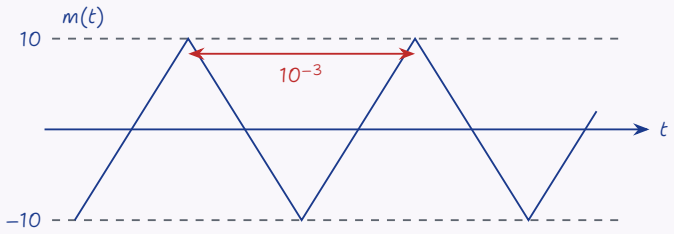
$P_c = \frac{1}{2}$

orj. s. 11

$$P_s = \frac{\mu^2 A_c^2}{2} P_{m_n} = \frac{(0{,}85)^2\,(1)^2\,0{,}3661}{2} = 0{,}1323 \text{ W}$$

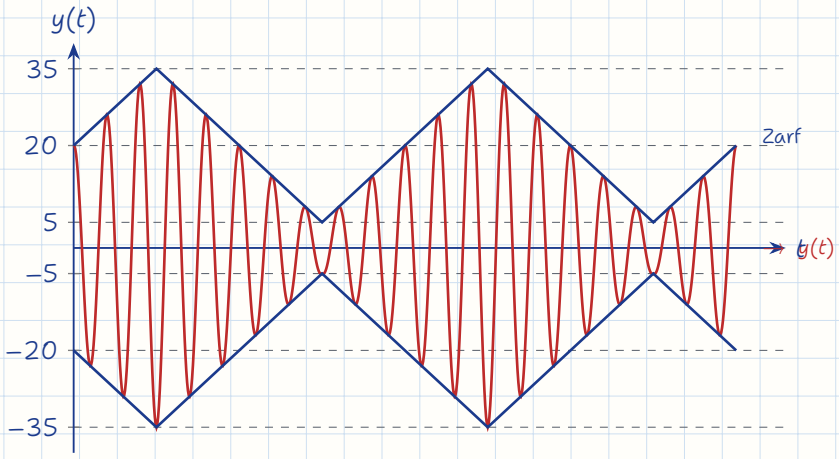
⋆ yaklaşık olarak maksimum bulunabilir böyle cos 1 olur veya sin 1 seçilebilir

Örnek Lathi, Pr. 4.3-3



$A_c = 20,\ \mu = 0{,}75$
a) Sketch $y(t)$ =?
b) $P_c,\ P_s,\ P_y,\ V$ = ?

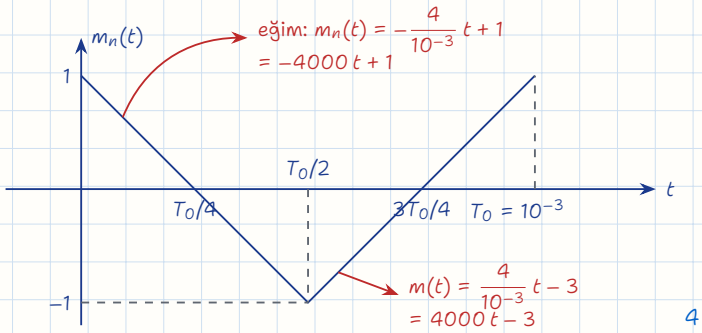
$$y(t) = A_c\left[1 + \mu m_n(t)\right]\cos(2\pi f_c t)$$
$$= 20\left[1 + 0{,}75\, m_n(t)\right]\cos(2\pi f_c t)$$



orj. s. 12

b) $P_c = \frac{A_c^2}{2} = \frac{20^2}{2} = 200$ W

$$P_s = \frac{\mu^2 A_c^2}{2} P_{m_n}$$



[4]

$$P_{m_n} = \frac{1}{T_0}\int_{T_0} |m_n(t)|^2\,dt$$

$$= \frac{1}{10^{-3}}\left[\int_0^{10^{-3}/2} (-4000\,t + 1)^2\,dt + \int_{10^{-3}/2}^{10^{-3}} (4000\,t - 3)^2\,dt\right]$$

$$= 0{,}3334 \text{ W}$$

$$P_s = \frac{\mu^2 A_c^2}{2} P_{m_n} = \frac{(0{,}75)^2 \times 20^2}{2} \times 0{,}3334 = 37{,}5 \text{ W}$$

$$P_y = P_c + P_s = 200 + 37{,}5 = 337{,}5 \text{ W}^{5}$$

$$V = \frac{P_s}{P_y} = \frac{37{,}5}{237{,}5} = 0{,}1578 \simeq 0{,}16 = \%16$$

[4] Şekilde ikinci formül el yazısında $m(t)$ olarak yazılmış; Claude notu: muhtemelen $m_n(t)$ olmalı.
[5] Claude notu: 200 + 37,5 = 237,5 olmalı (bir sonraki satırda da 237,5 kullanılıyor).