Analog Haberleşme

ELEC 361 Midterm Exam (Fall 2023) çözümleri

Orijinal not: ELM361_Fall2023_Midterm_Solutions.pdf

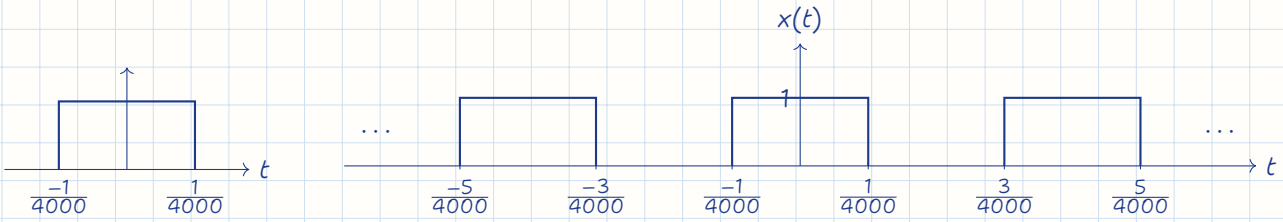
UYARI Bu belge, el yazısı ders notlarından Claude (Anthropic) yardımıyla dijitale aktarılmıştır. Yardımcı kaynak olarak hazırlanmıştır; hata veya eksik içerebilir. Lütfen orijinal notları ve ders kaynaklarını esas alın.

NOTICE *This document was transcribed from handwritten lecture notes with the help of Claude (Anthropic). It is meant as a supplementary resource and may contain errors or omissions. Always check against the original notes and official course material.*

orj. s. 1

Şekil: Bu sayfa basılı materyal içeriyor (ör. ders slaytı); dijitalleştirilmedi.
(orijinal notta, s. 1)

$$x(t) = \sum_{k=-\infty}^{\infty} \Pi\left(\frac{t - \frac{k}{1000}}{\frac{1}{2000}}\right) \qquad \Pi\left(\frac{t}{1/2000}\right) \Rightarrow$$



$$T_0 = \frac{4}{4000} = 10^{-3}\ \text{sn} \qquad f_0 = 1\ \text{kHz} \qquad \omega_0 = 2000\pi$$

a)

$$x(t) = \sum_{n=-\infty}^{\infty} x_n e^{jn\omega_0 t}$$

$$x_0 = \frac{1}{T_0}\int_{T_0} x(t)\,dt = 1000\int_{-\frac{1}{4000}}^{\frac{1}{4000}} 1\,dt = 1000 \cdot \frac{2}{4000} = \frac{1}{2}$$

$$x_n = \frac{1}{T_0}\int_{T_0} x(t)\,e^{-jn\omega_0 t}\,dt = 10^3 \int_{-\frac{1}{4}10^{-3}}^{\frac{1}{4}10^{-3}} 1 \cdot e^{-jn\,2000\pi t}\,dt$$

$$= 10^3 \cdot \frac{-1}{jn \cdot 2 \cdot 10^3 \pi}\left\{e^{-jn\cdot 2\cdot 10^3 \cdot \frac{1}{4}10^{-3}\pi} - e^{jn\cdot 2\cdot 10^3 \cdot \frac{1}{4}10^{-3}\pi}\right\}$$

$$= \frac{1}{\pi n} \cdot \frac{1}{2j}\left\{e^{j\frac{n}{2}\pi} - e^{-j\frac{n}{2}\pi}\right\}$$

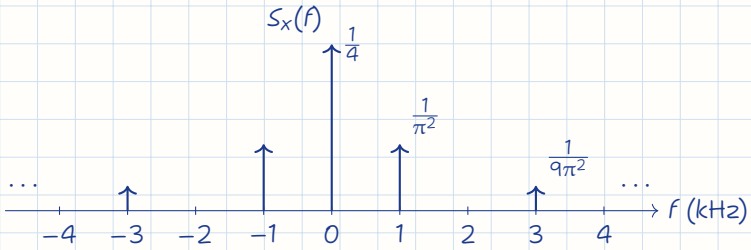
$$= \frac{1}{\pi n} \cdot \sin\left(\frac{\pi n}{2}\right), \qquad n \neq 0$$

$$x(t) = \frac{1}{2} + \sum_{\substack{n\neq 0 \\ n \text{ tek}}} \frac{1}{\pi n} \sin\left(\frac{\pi n}{2}\right) e^{jn\,2000\pi t}$$

$$\hookrightarrow \sin\left(\frac{\pi n}{2}\right) = \begin{cases} 1, & n = 1,5,9,13,\dots \\ -1, & n = 3,7,11,\dots \end{cases}$$

$$S_x(f) = \sum_n |x_n|^2\, \delta(f - nf_0)$$

$$= \frac{1}{4}\delta(f) + \sum_{\substack{n\neq 0 \\ n \text{ tek}}} \frac{1}{\pi^2 n^2}\delta(f - n1000)$$

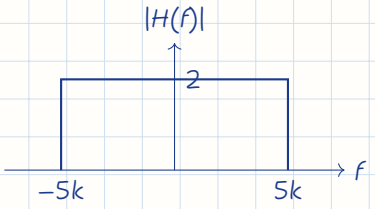


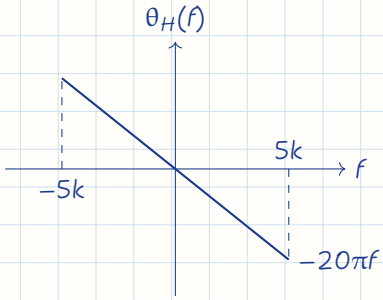
b) $x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$

$$h(t) = 20000\,\text{sinc}\underbrace{(10000t - 100000)}_{10000(t-10)} \implies H(f) = 2\,\Pi\left(\frac{f}{10000}\right) e^{-j2\pi f10}$$

$\tau = 10000,\ A = 2,\ t_0 = 10$

$$|H(f)| = 2\,\Pi\left(\frac{f}{10000}\right), \qquad \theta_H(f) = -2\pi 10 f$$





$\Rightarrow |H(f)|$, $x(t)$'nin spektrumunun tamamında $|K| \neq 0$ olmadığından bu işaret için bozulmasız değildir.

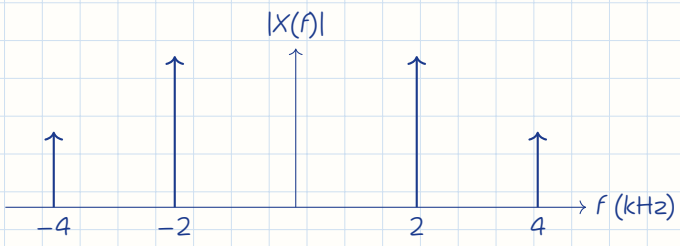
orj. s. 2

c)

$$x(t) = 20\cos(2\pi\,2000t) + 10\cos(2\pi\,4000t)$$

$$\downarrow$$

$$X(f) = 10\delta(f - 2000) + 10\delta(f + 2000) + 5\delta(f - 4000) + 5\delta(f + 4000)$$



$|H(f)|$, $X(f)$'in BG'si içinde sabit, $\theta_H(f)$'de lineer olduğu için sistem bu işaret için bozulmasızdır.

$K = 2, \quad t_d = 10$

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = 2x(t-10)$$

$$S_y(f) = |H(f)|^2 \cdot S_x(f)$$
$$S_x(f) = 100\delta(f-2000) + 100\delta(f+2000) + 25\delta(f-4000) + 25\delta(f+4000)$$
$$S_y(f) = 400\delta(f-2000) + 400\delta(f+2000) + 100\delta(f-4000) + 100\delta(f+4000)$$

$$\mathcal{F}^{-1} \downarrow$$

$$R_y(\tau) = 800\cos(2\pi 2000\tau) + 200\cos(2\pi 4000\tau)$$

1. yol: $P_y = R_y(0) = 800 + 200 = 1000$ W

2. yol: $P_y = \int_{-\infty}^{\infty} S_y(f)\,df = 2\cdot 400 + 2\cdot 100 = 1000$ W

orj. s. 3

> **Şekil:** Bu sayfa basılı materyal içeriyor (ör. ders slaytı); dijitalleştirilmedi.
> *(orijinal notta, s. 3)*

a)

$$x(t) = m(t) + \cos(2\pi f_c t) \xrightarrow{\mathcal{F}} X(f) = M(f) + \tfrac{1}{2}\delta(f-f_c) + \tfrac{1}{2}\delta(f+f_c)$$

$$z(t) = 5x^2(t) = 5\left[m(t) + \cos(2\pi f_c t)\right]^2$$
$$= 5\Big[m^2(t) + 2m(t)\cos(2\pi f_c t) + \underbrace{\cos^2(2\pi f_c t)}_{\frac{1}{2}+\frac{1}{2}\cos(2\pi 2f_c t)}\Big]$$

$$z(t) = 5m^2(t) + 10\,m(t)\cos(2\pi f_c t) + \tfrac{5}{2} + \tfrac{5}{2}\cos(2\pi 2f_c t)$$

$$\mathcal{F} \downarrow$$

$$Z(f) = \underbrace{5M(f) * M(f)}_{\text{BG'si } 2B} + 5M(f-f_c) + 5M(f+f_c)$$
$$+ \tfrac{5}{2}\delta(f) + \tfrac{5}{4}\delta(f-2f_c) + \tfrac{5}{4}\delta(f+2f_c)$$

i) LPF, Gain= 5, BW= $2B$

$$Y(f) = 25M(f) * M(f) + \tfrac{25}{2}\delta(f) \longrightarrow y(t) = 25m^2(t) + \tfrac{25}{2}$$

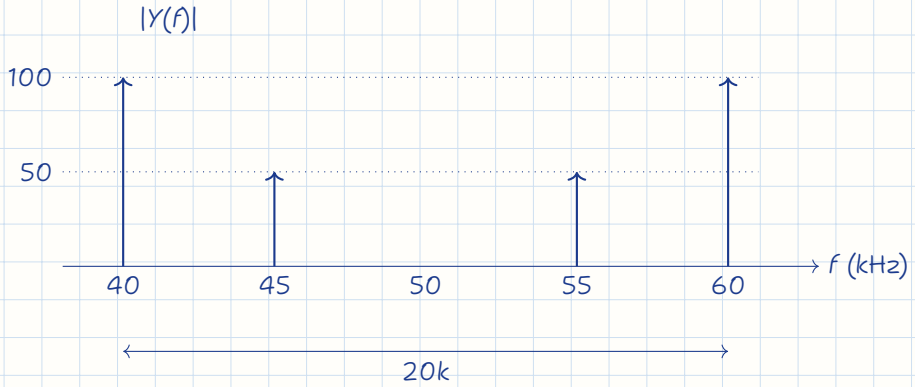
ii) BPF, Gain= 2, BW= $2B$, MF= f_c

$$Y(f) = 10M(f-f_c) + 10M(f+f_c) \longrightarrow y(t) = 20\,m(t)\cos(2\pi f_c t)$$

b) → DSB-SC-AM, LC yok.

$$m(t) = 10\cos(2\pi 5k\,t) + 20\cos(2\pi 10k\,t), \qquad f_c = 50\text{ kHz}$$
$$y(t) = 200\cos(2\pi 5k\,t)\cos(2\pi 50k\,t) + 400\cos(2\pi 10k\,t)\cos(2\pi 50k\,t)$$
$$= 100\cos(2\pi 55k\,t) + 100\cos(2\pi 45k\,t)$$
$$+ 200\cos(2\pi 60k\,t) + 200\cos(2\pi 40k\,t)$$

$$Y(f) = 50\delta(f-55k) + 50\delta(f+55k) + 50\delta(f-45k) + 50\delta(f+45k)$$
$$+ 100\delta(f-60k) + 100\delta(f+60k) + 100\delta(f+40k) + 100\delta(f-40k)$$



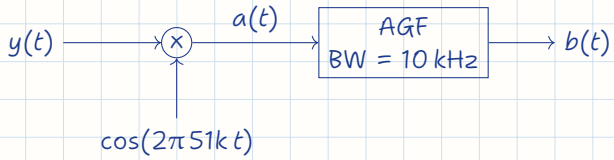
$BG\{y(t)\} = 20\ kHz$

$$P_y = 4 \cdot 100^2 + 4 \cdot 50^2 = 50k\ W$$

$$\text{verim} = \frac{P_m}{P_y} = \frac{\frac{10^2}{2} + \frac{20^2}{2}}{50k} = 0{,}005$$

orj. s. 4

c)



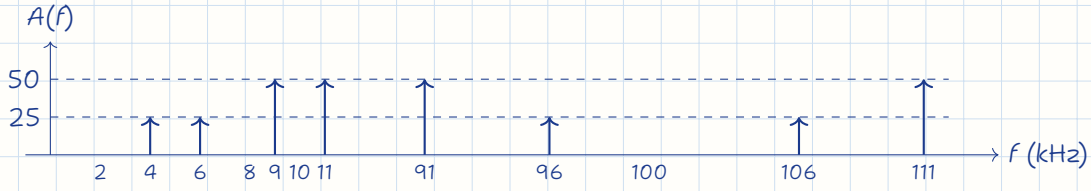
$$a(t) = y(t)\cos(2\pi 51k\,t) = 20\,m(t)\underbrace{\cos(2\pi 50k\,t)\cdot\cos(2\pi 51k\,t)}_{\frac{1}{2}\cos(2\pi 1k\,t)+\frac{1}{2}\cos(2\pi 101k\,t)}$$

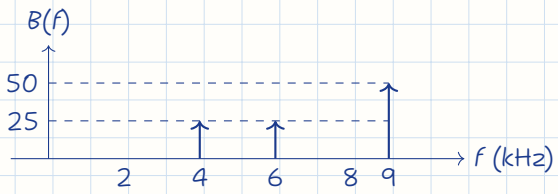
$$= 10\,m(t)\cos(2\pi 1k\,t) + 10\,m(t)\cos(2\pi 101k\,t),$$

$$m(t) = 10\cos(2\pi 5k\,t) + 20\cos(2\pi 10k\,t)$$

$$= 50\cos(2\pi 6k\,t) + 50\cos(2\pi 4k\,t) + 100\cos(2\pi 9k\,t) + 100\cos(2\pi 11k\,t)$$
$$+ 50\cos(2\pi 96k\,t) + 50\cos(2\pi 106k\,t) + 100\cos(2\pi 91k\,t) + 100\cos(2\pi 111k\,t)$$

$$A(f) = 25\delta(f-4k) + 25\delta(f+4k) + 25\delta(f-6k) + 25\delta(f+6k)$$
$$+ 50\delta(f-91k) + 50\delta(f+91k) + 50\delta(f-111k) + 50\delta(f+111k)$$
$$+ 50\delta(f-9k) + 50\delta(f+9k) + 25\delta(f-96k) + 25\delta(f+96k)$$
$$+ 25\delta(f-106k) + 25\delta(f+106k) + 50\delta(f-11k) + 50\delta(f+11k)$$





$$B(f) = 50\delta(f-9k) + 50\delta(f+9k) + 25\delta(f-4k) + 25\delta(f+4k)$$
$$+ 25\delta(f-6k) + 25\delta(f+6k)$$

$$b(t) = 100\cos(2\pi 9k\,t) + 50\cos(2\pi 4k\,t) + 50\cos(2\pi 6k\,t)$$

Claude ile dijitalleştirildi · hata içerebilir / Digitised with Claude · may contain errors

* taşıyıcı frekans doğru şekilde tahmin edilmediğinden mesaj işareti geri elde edilememiştir.

orj. s. 5

> **Şekil:** Bu sayfa basılı materyal içeriyor (ör. ders slaytı); dijitalleştirilmedi.
> *(orijinal notta, s. 5)*

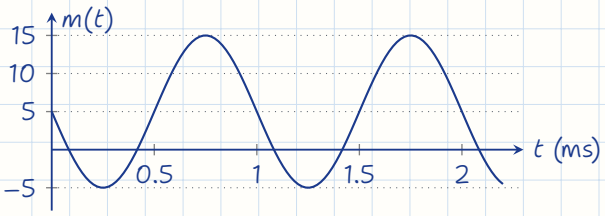
a) verim $= \dfrac{\mu^2 P_{m_n}}{1+\mu^2 P_{m_n}} = 0{,}15$.

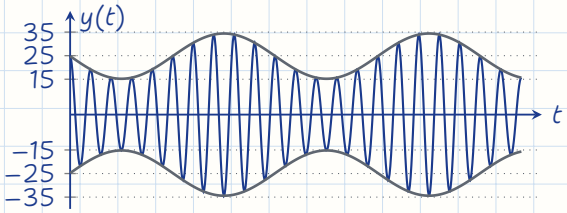
$$m(t) = 5 - 10\sin(2000\pi t) \longrightarrow m_p = 15$$
$$\Rightarrow m_n(t) = \frac{1}{3} - \frac{2}{3}\sin(2000\pi t)$$
$$P_{m_n} = \left(\frac{1}{3}\right)^2 + \left(-\frac{2}{3}\right)^2 \cdot \frac{1}{2} = \frac{3}{9} = \frac{1}{3}\ \text{W}$$
$$\mu^2 \cdot \frac{1}{3} = \frac{15}{100} + \mu^2 \cdot \frac{1}{3} \cdot \frac{15}{100}$$
$$\longrightarrow \mu^2 = 0{,}53 \rightarrow \mu = 0{,}7276 \cong 0{,}728$$

b) $c(t) = 20\cos(2\pi 10k\,t) \longrightarrow A_c = 20,\ f_c = 10\ \text{kHz}$

$$y(t) = A_c\left[1 + \mu m_n(t)\right] \cdot \cos(2\pi f_c t)$$
$$= 20\left[1 + 0{,}24 - 0{,}48\sin(2\pi 1000t)\right]\cos(2\pi 10k\,t)$$
$$P_c = \frac{A_c^2}{2} = \frac{20^2}{2} = 200\ \text{W}$$
$$P_s = \mu^2 \frac{A_c^2}{2} P_{m_n} = (0{,}728)^2 \cdot \frac{20^2}{2} \cdot \frac{1}{3} = 35{,}33\ \text{W}$$
$$P_y = P_c + P_s = 235{,}33\ \text{W}$$

c)

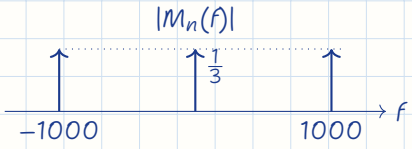


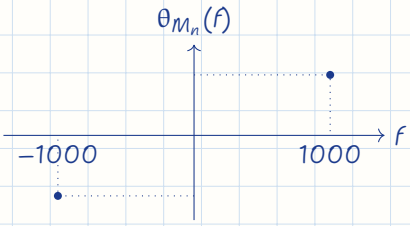


$$\left.\begin{aligned} 20\left[1 + 0{,}728 \cdot \tfrac{1}{3}\right] &= 24{,}85 \cong 25 \\ 20\left[1 + 0{,}728 \cdot \left(-\tfrac{1}{3}\right)\right] &= 15{,}14 \cong 15 \end{aligned}\right\} \qquad 20\left[1 + 0{,}728 \cdot 1\right] \cong 34{,}56 \cong 35$$

d)

$$m_n(t) = \frac{1}{3} - \frac{2}{3}\sin(2\pi 1000t)$$
$$\longrightarrow M_n(f) = \frac{1}{3}\delta(f) - \frac{2}{3} \cdot \frac{1}{2j}\left[\delta(f-1000) - \delta(f+1000)\right]$$
$$= \frac{1}{3}\delta(f) + \frac{1}{3}\delta(f-1000)\,e^{j\pi/2} + \frac{1}{3}\delta(f+1000)\,e^{-j\pi/2}$$





$$y(t) = 24{,}8\cos(2\pi 10k\,t) - 9{,}6\underbrace{\sin(2\pi 1000t)\cdot\cos(2\pi 10k\,t)}_{\frac{1}{2}\sin(-2\pi 9k\,t)+\frac{1}{2}\sin(2\pi 11k\,t)}$$

$$= 24{,}8\cos(2\pi 10k\,t) + 4{,}8\sin(2\pi 9k\,t) - 4{,}8\sin(2\pi 11k\,t)$$

$$Y(f) = 12{,}4\delta(f-10k) + 12{,}4\delta(f+10k) + 2{,}4\delta(f-9k)\,e^{j\pi/2}$$
$$+ 2{,}4\delta(f+9k)\,e^{-j\pi/2} + 2{,}4\delta(f-11k)\,e^{-j\pi/2} + 2{,}4\delta(f+11k)\,e^{j\pi/2}$$

orj. s. 6

