Analog Haberleşme

ELM362 Final (2007), Soru 1 çözümü

Orijinal not: 22.pdf

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orj. s. 1

Örnek 1

ELM362 Analog Haberleşme Sistemleri Final Sınavı, 8.6.2007. (30p.) $x(t) = 2\,\mathrm{Sinc}(40\pi t)$ işareti, transfer fonksiyonu $H(\omega)$ olan bir kanaldan iletilmektedir. Çıkış işareti $y(t) = 20\,\mathrm{Sinc}(40\pi t - 200\pi)$ olduğuna göre,

a) $H(\omega)$'yı bulunuz ve genlik, faz spektrumlarını çiziniz.
b) $h(t)$'yi bulunuz.
c) Bu sistem bozulmasız mıdır? Sistemin girişine $x(t) = t\,\mathrm{Sinc}^2(\pi t)$ işareti uygulanırsa çıkış ne olur?
d) $x(t) = 2\,\mathrm{Sinc}(40\pi t)$ işareti için, çıkışın enerji spektral yoğunluğunu bulunuz.
e) Çıkış işaretinin enerjisini bulunuz.

a)

$$A\tau\,\mathrm{sinc}(t\tau) \overset{F}{\longleftrightarrow} A\,\Pi\left(\frac{f}{\tau}\right)$$

$$x(t) = \underbrace{2}_{A\tau}\,\mathrm{sinc}\left(\underbrace{40\pi}_{\tau}\,t\right) \longleftrightarrow \frac{1}{20\pi}\,\Pi\left(\frac{f}{40\pi}\right) = X(f)$$

$$A\tau = 2$$

$$A \cdot 40\pi = 2$$

$$A = \frac{2}{40\pi} = \frac{1}{20\pi}$$

$$\underbrace{20}_{A\tau}\,\mathrm{sinc}\left(\underbrace{40\pi}_{\tau}\,t\right) \longleftrightarrow \frac{1}{2\pi}\,\Pi\left(\frac{f}{40\pi}\right), \qquad A\tau = 20, \quad A = \frac{20}{40\pi} = \frac{1}{2\pi}$$

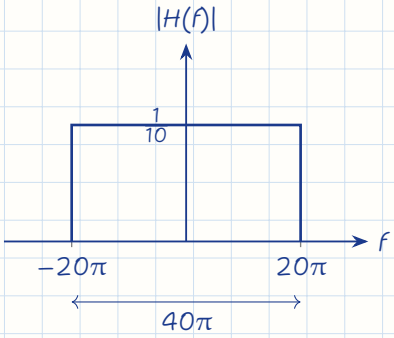
$$x(t - t_0) \overset{F}{\longleftrightarrow} X(f)\,e^{-j2\pi f t_0} \quad \leftarrow \text{Zamanda öteleme}$$

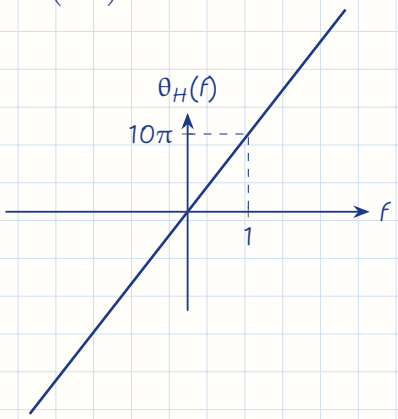
$$y(t) = 20\,\mathrm{sinc}(40\pi(t-5)) \longleftrightarrow \frac{1}{2\pi}\,\Pi\left(\frac{f}{40\pi}\right) \cdot e^{-j2\pi f 5} = Y(f)$$

$$x(t) \circledast h(t) = y(t)$$
$$X(f) \cdot H(f) = Y(f)$$

$$H(f) = \frac{X(f)}{Y(f)} = \frac{10\,\frac{1}{20\pi}\,\Pi\left(\frac{f}{40\pi}\right)}{\frac{1}{2\pi}\,\Pi\left(\frac{f}{40\pi}\right)\cdot e^{-j2\pi f5}} = \frac{1}{10}\,\Pi\left(\frac{f}{40\pi}\right)e^{j2\pi f5}$$

Claude notu: muhtemelen $H(f) = Y(f)/X(f)$ olmalı; bu durumda $H(f) = 10\,\Pi\left(\frac{f}{40\pi}\right)e^{-j2\pi f5}$ çıkardı.





orj. s. 2

b)

$$Az\,\mathrm{sinc}(tz) \overset{F}{\longleftrightarrow} A\,\Pi\left(\frac{f}{z}\right), \qquad A \to \frac{1}{10}, \quad z \to 40\pi$$

$$\frac{1}{10}\,40\pi\,\mathrm{sinc}(40\pi t) \overset{F}{\longleftrightarrow} \frac{1}{10}\,\Pi\left(\frac{f}{40\pi}\right)$$

$$\overset{h(t)}{4\pi\,\mathrm{sinc}\big(40\pi(t+5)\big)} \overset{F}{\longleftrightarrow} \frac{1}{10}\,\Pi\left(\frac{f}{40\pi}\right)\cdot e^{j2\pi f5} = H(f)$$

c) Sistem her yerde bozulmasız değildir ama -20π 20π arasında bozulmasız gibi davranır çünkü transfer fonksiyonun genliği sabit açısı lineerdir.

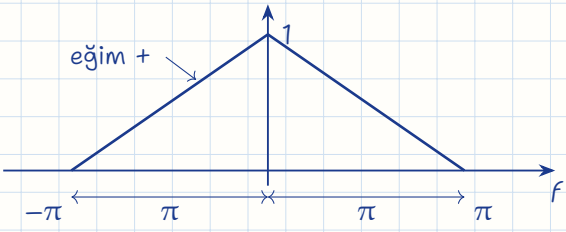
$$x(t) = t\,\mathrm{sinc}^2(\pi t)$$

$$t^n x(t) \overset{F}{\longleftrightarrow} \left(\frac{j}{2\pi}\right)^n \frac{d^n}{df^n}X(f) \qquad \leftarrow \textit{zaman ile çarpma}$$

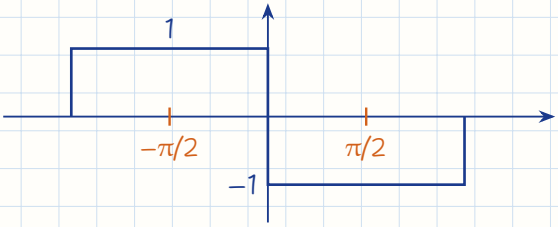
$$Az\,\mathrm{sinc}^2(tz) \overset{F}{\longleftrightarrow} A\,\Lambda\left(\frac{f}{z}\right)$$

$$\mathrm{sinc}^2(\underbrace{\pi}_{z}\,t) \longleftrightarrow \frac{1}{\pi}\,\Lambda\left(\frac{f}{\pi}\right), \qquad Az = 1, \quad A = \frac{1}{\pi}$$

$$t\,\mathrm{sinc}^2(\pi t) \longleftrightarrow \frac{1}{\pi}\left(\frac{j}{2\pi}\right)\frac{d}{df}\Lambda\left(\frac{f}{\pi}\right)$$



türevi



$$X(f) = \frac{j}{2\pi^2}\left[\Pi\left(\frac{f+\pi/2}{\pi/2}\right) - \Pi\left(\frac{f-\pi/2}{\pi/2}\right)\right]$$

orj. s. 3

$$X(f) = \frac{j}{2\pi^2}\left[\Pi\left(\frac{f+\pi/2}{\pi/2}\right) - \Pi\left(\frac{f-\pi/2}{\pi/2}\right)\right]$$

$$Y(f) = X(f)\,H(f) \qquad \left\{ \frac{1}{10}\Pi\left(\frac{f}{40\pi}\right)\cdot e^{j2\pi fS} = H(f)\right.$$

$$Y(f) = \frac{j}{20\pi^2}\left[\Pi\left(\frac{f+\pi/2}{\underbrace{\pi/2}_{z}}\right) - \Pi\left(\frac{f-\pi/2}{\pi/2}\right)\right]\cdot e^{j2\pi fS}$$

$$Az\,\mathrm{sinc}(tz) \overset{F}{\longleftrightarrow} A\Pi\left(\frac{f}{z}\right), \qquad Az \to 1, \quad A \to 1, \quad z \to \frac{\pi}{2}$$

$$\frac{\pi}{2}\mathrm{sinc}\left(\pi\frac{t}{2}\right) \longleftrightarrow \Pi\left(\frac{f}{\pi/2}\right)$$

$$\frac{\pi}{2}\mathrm{sinc}\left(\frac{\pi}{2}(t+5)\right) \longleftrightarrow \Pi\left(\frac{f}{\pi/2}\right)\cdot e^{j2\pi fS}$$

$$\frac{\pi}{2}\mathrm{sinc}\left(\frac{\pi}{2}(t+5)\right)\cdot e^{j2\pi\cdot\frac{\pi}{2}t} \longleftrightarrow \Pi\left(\frac{f+\pi/2}{\pi/2}\right)e^{j2\pi fS}$$

$$y(t) = \frac{j}{20\pi^2}\left[\frac{\pi}{2}\mathrm{sinc}\left(\frac{\pi}{2}(t+5)\right)e^{j\pi^2 t} - \frac{\pi}{2}\mathrm{sinc}\left(\frac{\pi}{2}(t+5)\right)e^{-j\pi^2 t}\right]$$

$$= \frac{j}{40\pi}\mathrm{sinc}\left(\frac{\pi}{2}(t+5)\right)\left[e^{j\pi^2 t} - e^{-j\pi^2 t}\right]$$

$$= -\frac{1}{20\pi}\mathrm{sinc}\left(\frac{\pi}{2}(t+5)\right)\cdot\underbrace{\frac{e^{j\pi^2 t} - e^{-j\pi^2 t}}{2j}}_{\sin(\pi^2 t)}$$

$$= -\frac{1}{20\pi}\mathrm{sinc}\left(\frac{\pi}{2}(t+5)\right)\sin(\pi^2 t)$$

orj. s. 4

d)

$$x(t) = 2\,\mathrm{sinc}(\underbrace{40\pi}_{z}\,t), \qquad Az = 2, \quad A = \frac{2}{40\pi}$$

$$\frac{1}{10}\,\Pi\left(\frac{f}{40\pi}\right)\cdot e^{j2\pi fS} = H(f)$$

$$Az\,\mathrm{sinc}(tz) \overset{F}{\longleftrightarrow} A\,\Pi\left(\frac{f}{z}\right)$$

$$X(f) = \frac{1}{20\pi}\,\Pi\left(\frac{f}{40\pi}\right)$$

$$Y(f) = X(f)\,H(f) = \frac{1}{200\pi}\,\Pi\left(\frac{f}{40\pi}\right)\cdot e^{j2\pi fS}$$

$$E_g(f) = |X(f)|^2 = \left(\frac{1}{200}\,\Pi\left(\frac{f}{40\pi}\right)\right)^2 = \frac{1}{40\,000}\,\Pi\left(\frac{f}{40\pi}\right)$$

e) $$E = \int_{-\infty}^{\infty} E_g(f)\,df = \int_{-20\pi}^{20\pi} \frac{1}{40000}\,df$$

$$= \frac{40\pi}{40\,000} = \frac{\pi}{1000}$$