Analog Haberleşme

Uygulama 2: LTI kanal, AM ve çoklama

Orijinal not: Analog-Uygulama-2.pdf

UYARI Bu belge, el yazısı ders notlarından Claude (Anthropic) yardımıyla dijitale aktarılmıştır. Yardımcı kaynak olarak hazırlanmıştır; hata veya eksik içerebilir. Lütfen orijinal notları ve ders kaynaklarını esas alın.

NOTICE *This document was transcribed from handwritten lecture notes with the help of Claude (Anthropic). It is meant as a supplementary resource and may contain errors or omissions. Always check against the original notes and official course material.*

orj. s. 1

Örnek

$x(t) = 2\,\mathrm{sinc}(40t)$ işareti transfer fonksiyonu $H(f)$ olan bir kanaldan iletilmektedir.
$y(t) = 20\,\mathrm{sinc}(40t - 200)$ old. göre,

a) $H(f)$'yi bulun. Genlik ve faz spektrumlarını çiziniz
b) $h(t)$'yi bulun
c) Bu sistem bozulmasızmıdır?
$x(t) = t\,\mathrm{sinc}^2(t)$ ise $y(t) = ?$
d) $x(t) = 2\,\mathrm{sinc}(40t)$ için, çıkışın enerji spektral yoğunluğunu bulun
e) Çıkış işaretinin enerjisini bulun.

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = x(t) \circledast h(t)$$

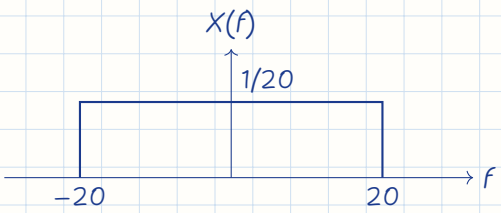
$$E_x(f) = |X(f)|^2 \qquad Y(f) = X(f)H(f)$$

$$E_y(f) = |H(f)|^2 E_x(f)$$

$$x(t - t_0) \overset{\mathcal{F}}{\longleftrightarrow} X(f)\, e^{-j2\pi f t_0}$$

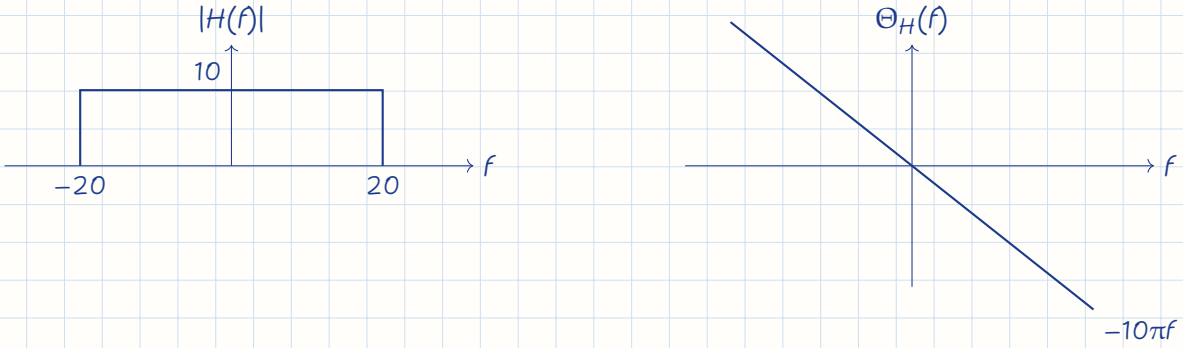
$$A\tau\,\mathrm{sinc}(\tau t) \overset{\mathcal{F}}{\longleftrightarrow} A\,\Pi\left(\frac{f}{\tau}\right)$$

a) $x(t) = 2\,\mathrm{sinc}(40t) \overset{\substack{\mathcal{F} \\ \tau=40 \\ A\tau=2 \\ A=1/20}}{\longleftrightarrow} X(f) = \frac{1}{20}\,\Pi\left(\frac{f}{40}\right)$



$$y(t) = 20\,\mathrm{sinc}(40(t-5)) \xleftrightarrow[\substack{\tau=40\\A=1/2}]{\mathcal{F}} Y(f) = \frac{1}{2}\Pi\left(\frac{f}{40}\right)e^{-j2\pi f5}$$

$$H(f) = \frac{Y(f)}{X(f)} = 10 \underbrace{\Pi\left(\frac{f}{40}\right)}_{\text{yine kare darbe gelir}} e^{-j2\pi f5}$$



b) $h(t) = 400\,\mathrm{sinc}(40(t-5))$

c) Bozulmasızdır. (Çünkü giriş işareti için) genlik spektrumu sabit, faz spektrumu lineerdir.

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = k\,x(t-t_d) \qquad k = 10,\ t_d = 5$$

$$|H(f)| = k \qquad \Theta_H(f) = -2\pi f t_d$$

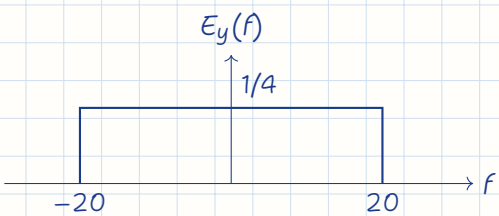
orj. s. 2

$$x(t) = t\,\mathrm{sinc}^2(t)$$

$$y(t) = 10\,x(t-5) = 10\,(t-5)\,\mathrm{sinc}^2(t-5)$$

d) $x(t) = 2\,\mathrm{sinc}(40t)$

$$E_x(f) = |X(f)|^2 = \frac{1}{400}\Pi\left(\frac{f}{40}\right) = \frac{1}{4}\Pi\left(\frac{f}{40}\right)$$ [1]



[1] Claude notu: ikinci satır muhtemelen $E_y(f) = |H(f)|^2 E_x(f) = 100 \cdot \frac{1}{400}\Pi\left(\frac{f}{40}\right)$ olmalı.

e) $E_y = \int_{-\infty}^{\infty} E_y(f)\,df$

$= \int_{-20}^{20} \frac{1}{4}\,df = \frac{1}{4} \cdot 40 = 10$ J

Örnek

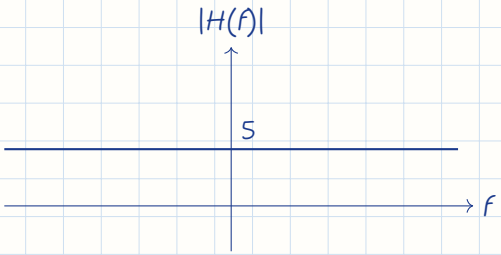
Bir LTI sistem olarak modellenen kanalın dürtü cevabı, $h(t) = 5\,\delta(t-10)$ olarak verilmektedir.

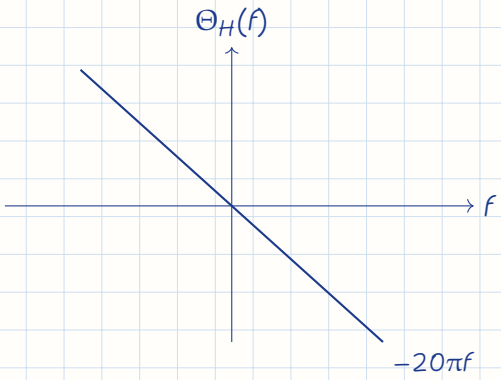
a) Bu sistemin Transfer fonksiyonunu bulun. Genlik ve faz spektrumlarını çizin. Bu sistem bozulmasız mıdır? Neden??
b) $x(t) = 10 + 10t\,\mathrm{sinc}(t)$ işareti uygulanırsa çıkışı nedir
c) $x(t) = 20\cos(2000\pi t) + 10\sin(4000\pi t)$ ise $x(t)$'nin güç yoğunluk fonksiyonunu (power spectral density) bulun
d) c şıkındaki $x(t)$ için $y(t)$'yi ve $y(t)$'nin GYF'sini bulun.
e) $y(t)$'nin özilişki fonksiyonunu bulun
f) $y(t)$'nin gücünü hem özilişki fonksiyonundan hem de GYF'den bulun.

a) $h(t) = 5\,\delta(t-10) \xrightarrow{\mathcal{F}} H(f) = 5\,e^{-j2\pi 10f}$

$H(f) = |H(f)|\,e^{j\Theta_H(f)}$

$|H(f)| = 5 \qquad \Theta_H(f) = -2\pi 10f$





Genlik spektrumu sabit, faz spektrumu lineer olduğu için sistem bozulmasızdır.

orj. s. 3

b) $x(t) = 10 + 10t\,\mathrm{sinc}(t)$

$y(t) = k\,x(t-t_d) = 5\,x(t-10)$

$= 50 + 50(t-10)\,\mathrm{sinc}(t-10)$

$x(t) = \sum_n x_n\,e^{j2\pi n f_0 t}$

$R_x(\tau) = \sum_n |x_n|^2\,e^{j2\pi n f_0 \tau}$ $\qquad \cos(a) = \frac{1}{2}\left[e^{-ja} + e^{ja}\right]$

$\overset{\mathcal{F}}{\downarrow}$

$S_x(f) = \sum_n |x_n|^2\,\delta(f - nf_0)$

c) $x(t) = 20\cos(2\pi 1000\,t) + 10\sin(2\pi 2000\,t)$

$$= \underbrace{10\,e^{-j2\pi 1000t}}_{x_{-1}} + \underbrace{10\,e^{j2\pi 1000t}}_{x_1} + \underbrace{\frac{10}{2j}\,e^{j2\pi 2000t}}_{x_2}$$

$$\underbrace{-\frac{10}{2j}\,e^{-j2\pi 2000t}}_{x_{-2}}$$

$$S_x(f) = 100\,\delta(f-1000) + 100\,\delta(f+1000) + 25\,\delta(f-2000) + 25\,\delta(f+2000)$$

d) $y(t) = 5x(t-10)$

$$= 100\cos\big(2\pi 1000\,(t-10)\big) + 50\sin\big(2\pi 2000\,(t-10)\big)$$

$$S_y(f) = |H(f)|^2 S_x(f) = 25\,S_x(f) = 2500\big[\delta(f-1000) + \delta(f+1000)\big]$$
$$+ 625\big[\delta(f-2000) + \delta(f+2000)\big]$$

e) $R_y(\tau) = 5000\cos(2\pi 1000\,\tau) + 1250\cos(2\pi 2000\,\tau)$

f) $P_y = \int_{-\infty}^{\infty} S_y(f)\,df = R_y(0)$

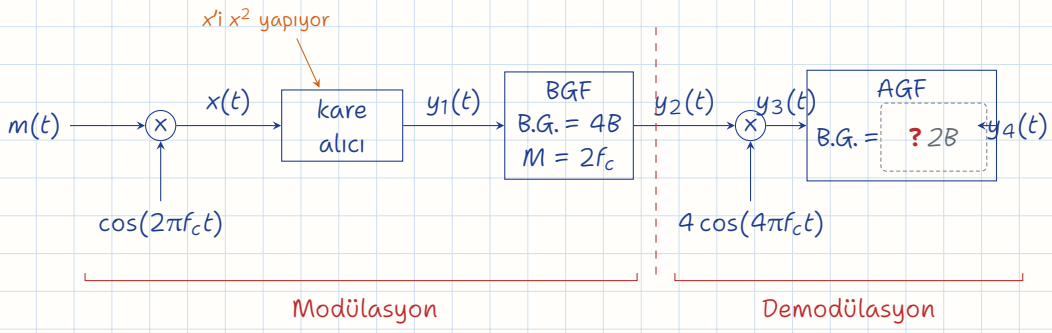
$$R_y(\tau) = \int_{-\infty}^{\infty} S_y(f)\,e^{j2\pi f\tau}\,df$$

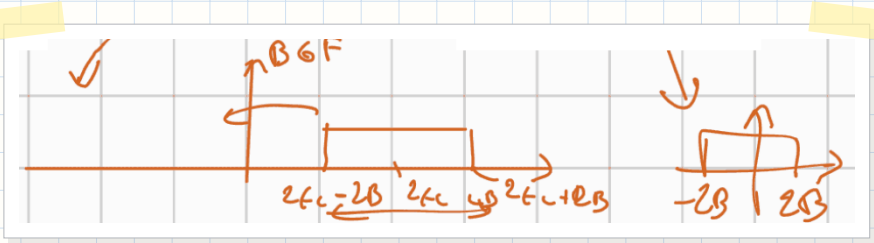
$$P_y = R_y(0) = 6250$$

Örnek

Fourier spektrumu $M(f) = \Pi(f/20)$ olan mesaj işareti aşağıda verilen sistemden geçmektedir. $x(t)$, $y_1(t)$, $y_2(t)$, $y_3(t)$ ve $y_4(t)$'ü zaman ve frekans domaininde ifade ediniz, spektrumlarını çiziniz.

$$f_c \gg B$$

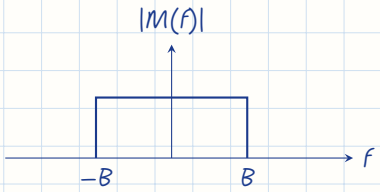




orijinal çizim, s. 3

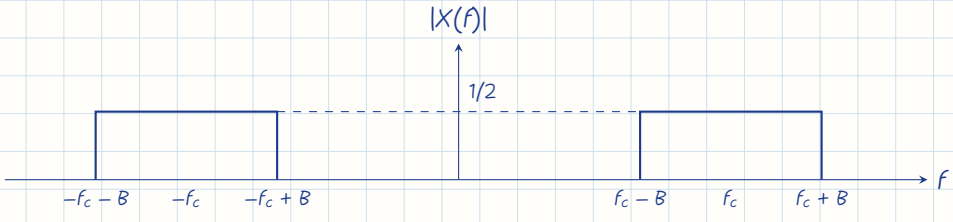
orj. s. 4

$$M(f) = \Pi\left(\frac{f}{2B}\right)$$



$$M(f) = \Pi\left(\frac{f}{2B}\right)$$
$$\hookrightarrow\ m(t) = 2B\,\mathrm{sinc}(2Bt)$$
$$= 2B\,\mathrm{sinc}(2Bt)\cos(2\pi f_c t)^2$$

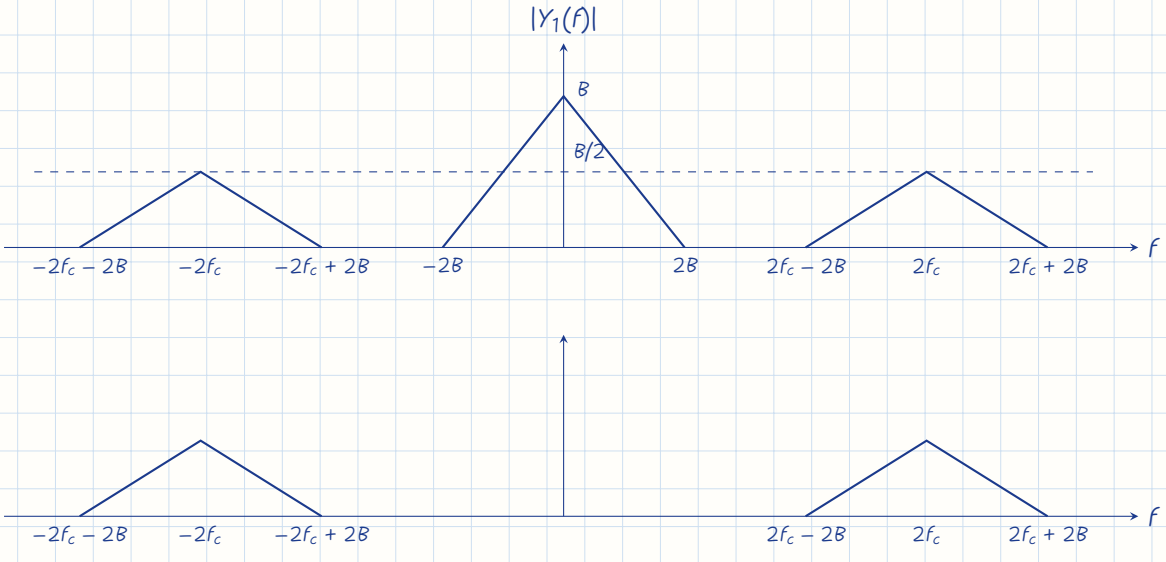
$$X(f) = \frac{1}{2}M(f-f_c) + \frac{1}{2}M(f+f_c)$$
$$X(f) = \frac{1}{2}\Pi\left(\frac{f-f_c}{2B}\right) + \frac{1}{2}\Pi\left(\frac{f+f_c}{2B}\right)$$



$$y_1(t) = x^2(t) = \left[m(t)\cos(2\pi f_c t)\right]^2$$
$$= 4B^2\,\mathrm{sinc}^2(2Bt)\underbrace{\cos^2(2\pi f_c t)}_{\frac{1+\cos(2\pi\,2f_c t)}{2}}$$
$$= \underbrace{2B^2\,\mathrm{sinc}^2(2Bt)}_{\tau=2B,\ A=B} + 2B^2\,\mathrm{sinc}^2(2Bt)\cos(2\pi\,2f_c t)$$

$$Y_1(f) = B\Lambda\left(\frac{f}{2B}\right) + \frac{B}{2}\Lambda\left(\frac{f-2f_c}{2B}\right) + \frac{B}{2}\Lambda\left(\frac{f+2f_c}{2B}\right)$$

[2] Claude notu: ikinci satır muhtemelen $x(t) = m(t)\cos(2\pi f_c t) = 2B\,\mathrm{sinc}(2Bt)\cos(2\pi f_c t)$ olmalı.



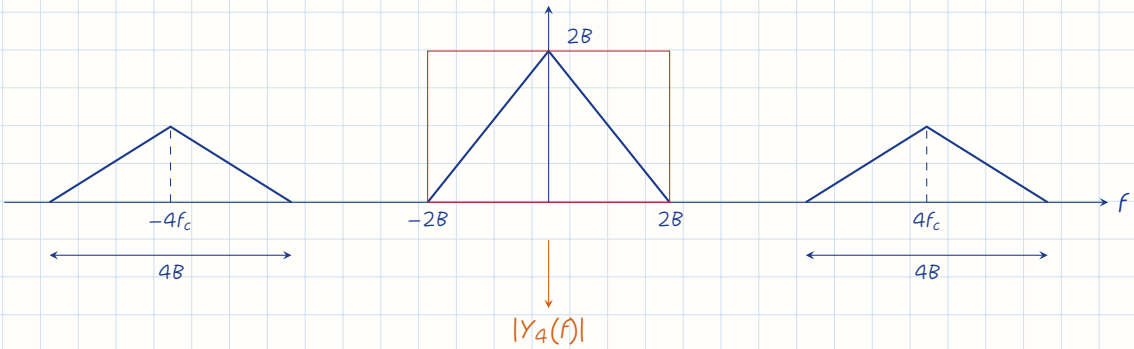
orj. s. 5

$$Y_2(f) = \frac{B}{2}\Lambda\left(\frac{f-2f_c}{2B}\right) + \frac{B}{2}\Lambda\left(\frac{f+2f_c}{2B}\right)$$

$$y_2(t) = 2B^2\,\mathrm{sinc}^2(2Bt)\cos(2\pi 2f_c t) \quad \leftarrow \text{modüle edilmiş işaret}$$

$$\begin{aligned} y_3(t) &= 4\,y_2(t)\cos(2\pi 2f_c t) \\ &= 8B^2\,\mathrm{sinc}^2(2Bt)\underbrace{\cos^2(2\pi 2f_c t)}_{\frac{1}{2}+\frac{1}{2}\cos(2\pi 4f_c t)} \\ &= \underbrace{4B^2\,\mathrm{sinc}^2(2Bt)}_{\tau=2B,\ A=2B} + 4B^2\,\mathrm{sinc}^2(2Bt)\cos(2\pi 4f_c t) \end{aligned}$$

$$Y_3(f) = 2B\Lambda\left(\frac{f}{2B}\right) + B\Lambda\left(\frac{f-4f_c}{2B}\right) + B\Lambda\left(\frac{f+4f_c}{2B}\right)$$



$$Y_4(f) = 2B\Lambda\left(\frac{f}{2B}\right)$$

$$y_4(t) = 4B^2\,\mathrm{sinc}^2(2Bt) = m^2(t)$$

Örnek

Bir büyük taşıyıcılı, çift yan bantlı genlik modülasyonu (large carrier double side band

amplitude mod.) işareti

$$y(t) = \underbrace{\left[20 + 2\cos(3000\pi t) + 10\cos(6000\pi t)\right]}_{w(t)} \cos(2\pi 10^5 t)$$

ile verilmektedir. Modülasyon indeksi $\mu = 0{,}6$'dır.

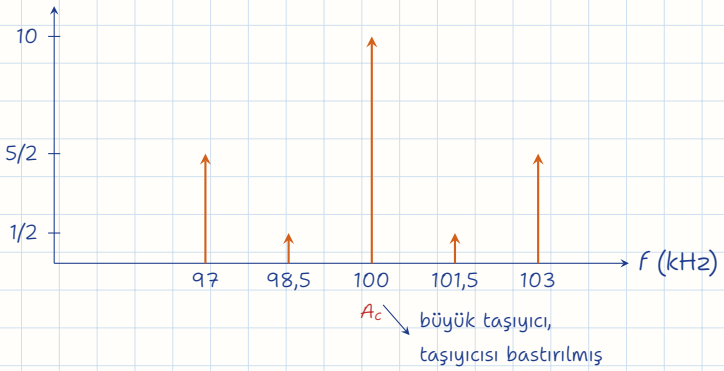
a) $y(t)$'nin spektrumunu bulun ve çizin
b) Taşıyıcı gücünü, yan bantların gücünü ve verimi bulun
c) Mesaj işareti $m(t)$'yi bulun.

$$y(t) = A_c\left[1 + \mu m_n(t)\right]\cos(2\pi f_c t)$$

orj. s. 6

$$\begin{aligned} y(t) &= w(t)\cos(2\pi 10^5 t) \\ \overset{\mathcal{F}}{\longleftrightarrow}\quad Y(f) &= \frac{1}{2}W(f-10^5) + \frac{1}{2}W(f+10^5) \\ W(f) &= 20\,\delta(f) + \delta(f-1500) + \delta(f+1500) \\ &\quad + 5\,\delta(f-3000) + 5\,\delta(f+3000) \end{aligned}$$

$$\begin{aligned} Y(f) &= 10\,\delta(f-10^5) + 10\,\delta(f+10^5) + \frac{1}{2}\delta(f-101500) \\ &\quad + \frac{1}{2}\delta(f+98500) + \frac{1}{2}\delta(f+101500) + \frac{5}{2}\delta(f-103000) \\ &\quad + \frac{5}{2}\delta(f+97000) + \frac{5}{2}\delta(f-97000) + \frac{5}{2}\delta(f+103000)^3 \end{aligned}$$



1. yol

$$\left.\begin{aligned} P_c &= 2(10)^2 = 200 \quad \text{(carrier)} \\ P_s &= 4\cdot\left(\frac{5}{2}\right)^2 + 4\cdot\left(\frac{1}{2}\right)^2 = 26 \end{aligned}\right\} \quad \text{Verim} = \frac{P_s}{P_s + P_c} = \frac{26}{226} \approx 0{,}115$$

2. yol

$$y(t) = \Big[\underbrace{20}_{\text{taşıyıcı}} + \underbrace{2\cos(2\pi 1500t) + 10\cos(2\pi 3000t)}_{\text{yan bantlar}}\Big]\cos(2\pi f_c t)$$

[3] Claude notu: $\frac{1}{2}\delta(f-98500)$ terimi eksik görünüyor.

$$P_c = \frac{20^2}{2} = 200$$
$$P_s = \frac{1}{2}\left(\frac{2^2}{2} + \frac{10^2}{2}\right) = 26$$

c) $y(t) = A_c\left[1 + \mu m_n(t)\right]\cos(2\pi f_c t)$

$$= 20\left[1 + \frac{1}{10}\cos(2\pi 1500t) + \frac{1}{2}\cos(2\pi 3000t)\right]\cos(2\pi 10^5 t)$$

$$\frac{m(t)}{m_p} = \frac{10}{6}\left[\frac{2}{20}\cos(3000\pi t) + \frac{10}{20}\cos(6000\pi t)\right]$$

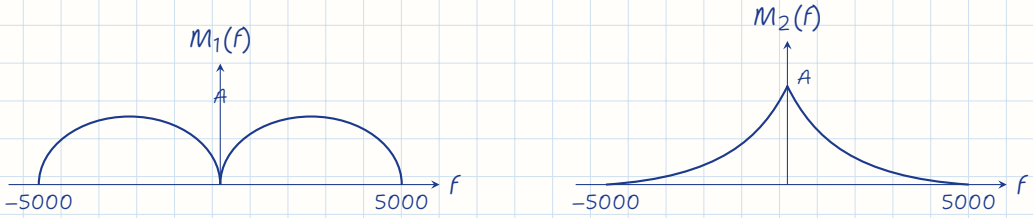
$$= \underbrace{\frac{1}{12}}_{m_p}\underbrace{\left[2\cos(3000\pi t) + 10\cos(6000\pi t)\right]}_{m(t)}$$

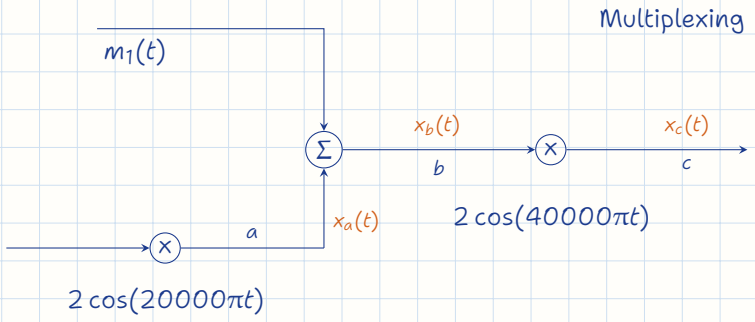
orj. s. 7

Örnek

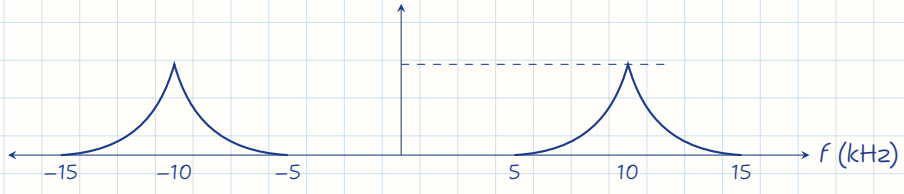
Two signals $m_1(t)$ and $m_2(t)$ both band limited to 5000 Hz, are to be transmitted simultaneously over a channel by multiplexing scheme shown in figure. The signal at point b is the multiplexing signal, which now modulates a carrier freq 20kHz. The modulated signal at point c is transmitted over a channel.

a) Sketch signal spectra (or spectrum) at points a, b, c
b) What must be the bandwidth of the channel.
c) Design a receiver to recover signals $m_1(t)$ and $m_2(t)$ from the modulated signal at point c.



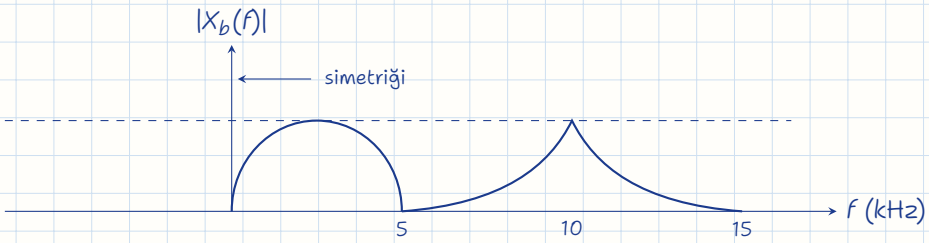


$$x_a(t) = 2m_2(t)\cos(2\pi 10k\, t)$$
$$X_a(f) = M_2(f - 10k) + M_2(f + 10k)$$

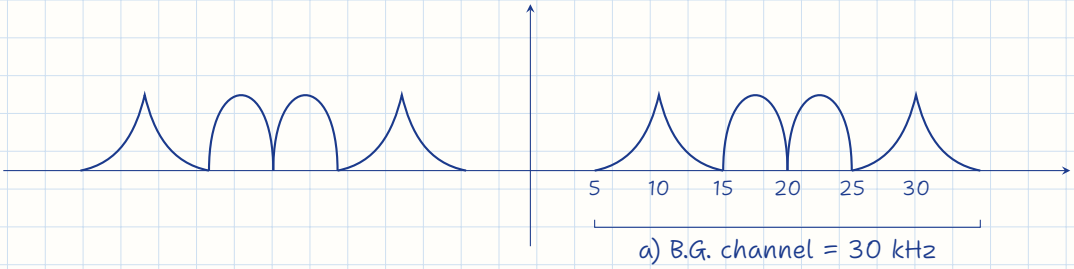


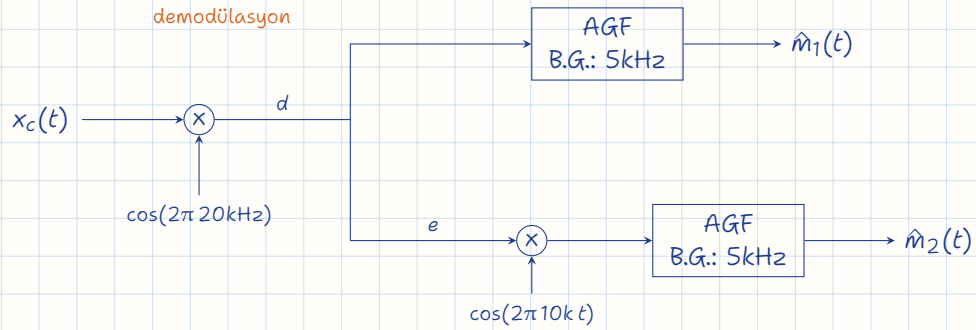
orj. s. 8

$$x_b(t) = m_1(t) + x_a(t) = m_1(t) + 2m_2(t)\cos(2\pi 10k\, t)$$
$$X_b(f) = M_1(f) + M_2(f - 10k) + M_2(f + 10k)$$

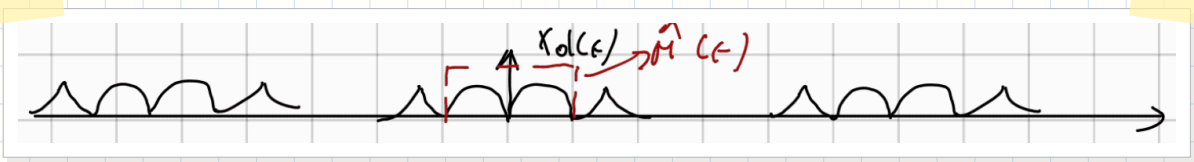


$$x_c(t) = 2x_b(t)\cos(2\pi\, 20k\, t)$$
$$X_c(f) = X_b(f - 20k) + X_b(f + 20k)$$





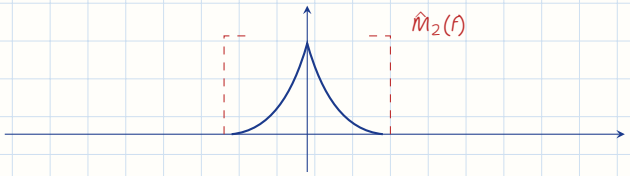
$$x_d(t) = x_c(t)\cos(2\pi\, 20k\, t)$$
$$X_d(f) = \frac{1}{2}X_c(f - 20k) + \frac{1}{2}X_c(f + 20k)$$



orijinal çizim, s. 8

orj. s. 9

$$x_e(t) = x_d(t)\cos(2\pi 10k\, t)$$
$$\longrightarrow X_e(f) = \frac{1}{2}X_d(f - 10k) + \frac{1}{2}X_d(f + 10k)$$


$\hat{M}_2(f)$