Analog Haberleşme

ELM 361 Uygulama 3 – SSB ve FM soruları

Orijinal not: ELM-361-Uygulama-3.pdf

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orj. s. 1

ELM 361 Uygulama – 3

Örnek 3.16 (SSB)

An SSB-AM signal is generated by modulating an 800 kHz carrier by the signal $m(t) = \cos(2000\pi t) + 2\sin(2000\pi t)$. The amplitude of the carrier is $A_c = 100$.

1) Determine the signal $\hat{m}(t)$.
2) Determine the (time-domain) expression for the lower sideband of the SSB-AM signal.
3) Determine the magnitude spectrum of the lower-sideband SSB signal.

1. $\hat{m}(t) = m_h(t)$ hilbert transform.

$$\hat{m}(t) = \cos\left(2000\pi t - \frac{\pi}{2}\right) + 2\sin\left(2000\pi t - \frac{\pi}{2}\right)$$

$$\hat{m}(t) = \sin(2000\pi t) - 2\cos(2000\pi t)$$

2.

$$y_{LSB}(t) = A_c\left(m(t)\cos(2\pi f_c t) + \hat{m}(t)\sin(2\pi f_c t)\right)$$

$$A_c = 100, \quad f_c = 800\,\text{kHz}, \quad m(t) = \cos(2000\pi t) + 2\sin(2000\pi t)$$

$$y_{LSB}(t) = 100 \cdot \Big(\left(\cos(2000\pi t) + 2\sin(2000\pi t)\right) \cdot \cos(2\pi\, 800k \cdot t) + \left(\sin(2000\pi t) - 2\cos(2000\pi t)\right) \cdot \sin(2\pi\, 800k \cdot t) \Big)$$

$$= 100\Big(\cos\big(\overbrace{2000\pi t}^{2\pi f_m}\big)\cos\big(2\pi\overbrace{800k\,t}^{f_c}\big) + 2\sin\big(\overbrace{2000\pi t}^{2\pi f_m}\big)\cos\big(2\pi\overbrace{800k\,t}^{f_c}\big) + \sin\big(\overbrace{2000\pi t}^{2\pi f_m}\big)\sin\big(2\pi\overbrace{800k\,t}^{f_c}\big) - 2\cos\big(\overbrace{2000\pi t}^{2\pi f_m}\big)\sin\big(2\pi\overbrace{800k\,t}^{f_c}\big)\Big)$$

$$f_c - f_m = f_1$$
$$f_c + f_m = f_2$$

$$= 100\Big(\tfrac{1}{2}\big(\cos(2\pi(f_m+f_c)t)+\cos(2\pi(f_m-f_c)t)\big)\ ①$$
$$+\tfrac{2}{2}\big(\sin(2\pi(f_m+f_c)t)+\sin(2\pi(f_m-f_c)t)\big)\ ②③$$
$$+\tfrac{1}{2}\big(\cos(2\pi(f_m-f_c)t)-\cos(2\pi(f_m+f_c)t)\big)\ ①$$
$$-\tfrac{2}{2}\big(\sin(2\pi(f_c+f_m)t)+\sin(2\pi(f_c-f_m)t)\big)\Big)\ ②③$$
$$-\sin\big(2\pi(f_m-f_c)t\big)$$

orj. s. 2

$$y_{LSB}(t) = 100\cdot\Big(\cos\big(2\pi(f_m-f_c)t\big)+2\sin\big(2\pi(f_m-f_c)t\big)\Big)$$
$$= 100\cdot\Big(\cos\big(2\pi(1000-800k)t\big)+2\sin\big(2\pi(1000-800k)t\big)\Big)$$
$$y_{LSB}(t) = 100\Big(\cos\big(2\pi(-1000+\underbrace{800k}_{f_c})t\big)-2\sin\big(2\pi(\underbrace{800k}_{f_c}-1000)t\big)\Big)$$

3. $Y_{LSB}(f) = ?$

Hint: cos

$$\cos\alpha\cdot\cos\beta = \tfrac{1}{2}\big(\cos(\alpha+\beta)+\cos(\alpha-\beta)\big)$$
$$\cos\alpha\cdot\sin\beta = \tfrac{1}{2}\big(\sin(\beta+\alpha)+\sin(\beta-\alpha)\big)$$
$$\sin\alpha\cdot\cos\beta = \tfrac{1}{2}\big(\sin(\alpha-\beta)+\sin(\alpha+\beta)\big)$$
$$\sin\alpha\cdot\sin\beta = \tfrac{1}{2}\big(\cos(\alpha-\beta)-\cos(\alpha+\beta)\big)$$

$$y_{LSB}(t) = 100\cdot\Big(\cos\big(2\pi\underbrace{(f_c-f_m)}\,t\big)$$
$$-2\sin\big(2\pi(f_c-f_m)t\big)\Big)$$

$$Y_{LSB}(f) = 100\cdot\tfrac{1}{2}\cdot\Big(\delta\big(f-(f_c-f_m)\big)+\delta\big(f+(f_c-f_m)\big)\Big)$$
$$-\frac{100}{\not 2 j}\,2\Big(\delta\big(f-(f_c-f_m)\big)-\delta\big(f+(f_c-f_m)\big)\Big)$$

$$Y_{LSB}(f) = 50\cdot\Big(\delta(f-800k+1000)+\delta(f+800k-1000)\Big)$$
$$+100j\cdot\Big(\delta(f-800k+1000)-\delta(f+800k-1000)\Big)$$
$$= (50+100j)\Big(\delta(f-800k+1000)\Big)+(50-100j)\cdot\Big(\delta(f+800k-1000)\Big)$$

↓ magnitude spectrum

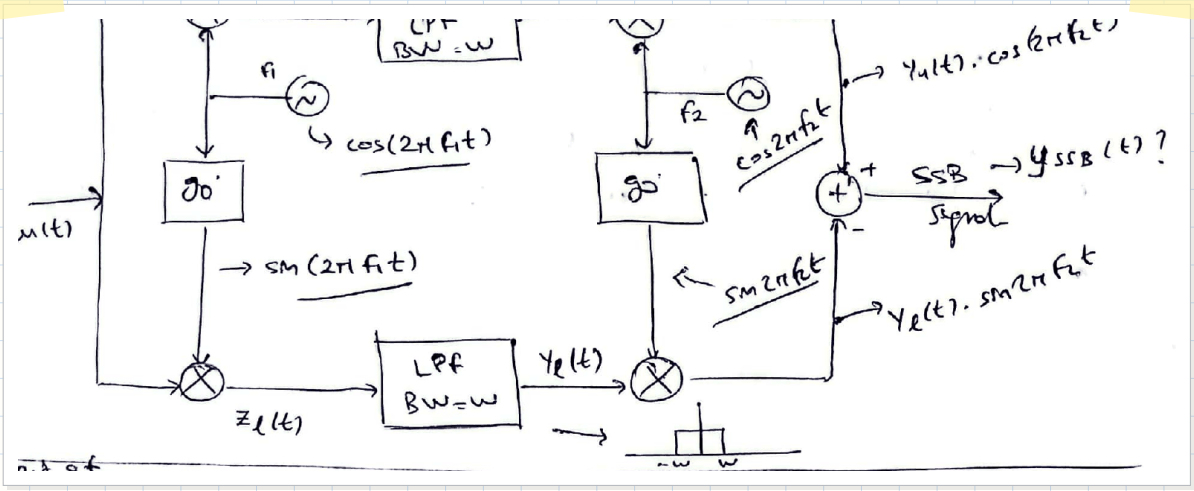
$$|Y_{LSB}(f)| = \sqrt{50^2+100^2}\cdot\Big(\delta(f-800k+1000)+\delta(f+800k-1000)\Big)$$
$$= 10\sqrt{125}\cdot\Big(\delta(f-800k+1000)+\delta(f+800k-1000)\Big)$$

orj. s. 3

Örnek 3.17

Weaver's SSB modulator is illustrated in figure P-3.17. By taking the input signal as

$m(t) = \cos 2\pi f_m t$ where $f_m < W$ demonstrate that by proper choice of f_1 and f_2 the output is an SSB signal.



orijinal çizim, s. 3

input of upper LPF

$$z_u(t) = m(t) \cdot \cos(2\pi f_1 t)$$

$$= \cos(2\pi f_m t) \cdot \cos(2\pi f_1 t) = \frac{1}{2}\left[\cos\left(2\pi \underbrace{(f_1 - f_m)}_{f_a} t\right) + \cos\left(2\pi \underbrace{(f_1 + f_m)}_{f_b} t\right)\right]$$

input of lower LPF

$$z_l(t) = \cos(2\pi f_m t) \cdot \sin(2\pi f_1 t)$$

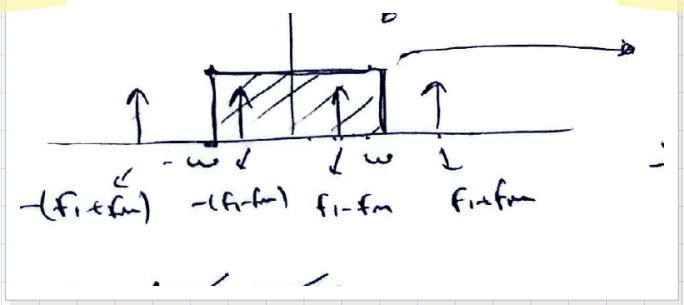
$$= \frac{1}{2}\left[\sin\left(2\pi(f_1 - f_m)t\right) + \sin\left(2\pi(f_1 + f_m)t\right)\right]$$

$f_m < W$ ⟶ we will select f_1. ? *okunamadı*

$$|f_1 - f_m| < W \quad \text{and} \quad f_1 + f_m > W$$

? *W, W/2 (kutunun altındaki küçük notlar)* → SSB olması için seçilir.

$$Z_u(f) = \frac{1}{4} \cdot \left(\delta(f - f_a) + \delta(f + f_a)\right) + \frac{1}{4} \cdot \left(\delta(f - f_b) + \delta(f + f_b)\right)$$



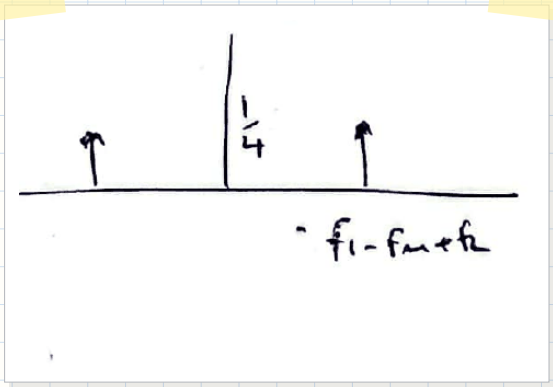
orijinal çizim, s. 3

$$y_U(t) = \frac{1}{2}\cos\left(2\pi(f_1 - f_m)t\right)$$
$$y_\ell(t) = \frac{1}{2}\sin\left(2\pi(f_1 - f_m)t\right)$$

$$y_{SSB}(t) = y_U(t)\cdot\cos(2\pi f_2 t) - y_\ell(t)\cdot\sin(2\pi f_2 t)$$
$$= \frac{1}{2}\cos\left(2\pi(f_1 - f_m)t\right)\cdot\cos(2\pi f_2 t) - \frac{1}{2}\sin\left(2\pi(f_1 - f_m)t\right)\cdot\sin(2\pi f_2 t)$$

orj. s. 4

$$y_{SSB}(t) = \frac{1}{4}\cdot\left(\cos\left(2\pi(f_1 - f_m + f_2)t\right) + \cos\left(2\pi(f_1 - f_m - f_2)t\right)\right)$$
$$-\frac{1}{4}\cdot\left(\cos\left(2\pi(f_1 - f_m - f_2)t\right) - \cos\left(2\pi(f_1 - f_m + f_2)t\right)\right)$$
$$\cos(A - B) = \cos A\cos B + \sin A\sin B$$
$$y_{SSB}(t) = \frac{1}{2}\cos\left(2\pi(f_1 - f_m + f_2)t\right)$$

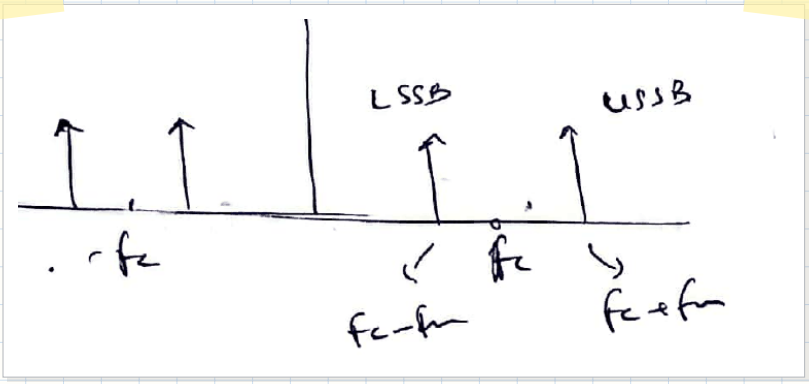


orijinal çizim, s. 4

$$f_1 + f_2 - f_m = f_c + f_m \longrightarrow \overset{\text{at } f_c}{\text{USSB (upper)}}$$
$$f_1 + f_2 - f_m = f_c - f_m \longrightarrow \text{LSSB (lower)}$$

we are choosing different f_1 and f_2 for USSB and LSSB.

$f_c = \boxed{-(f_1 + f_2)}$ $\quad$ $f_c = \boxed{f_1 + f_2}$



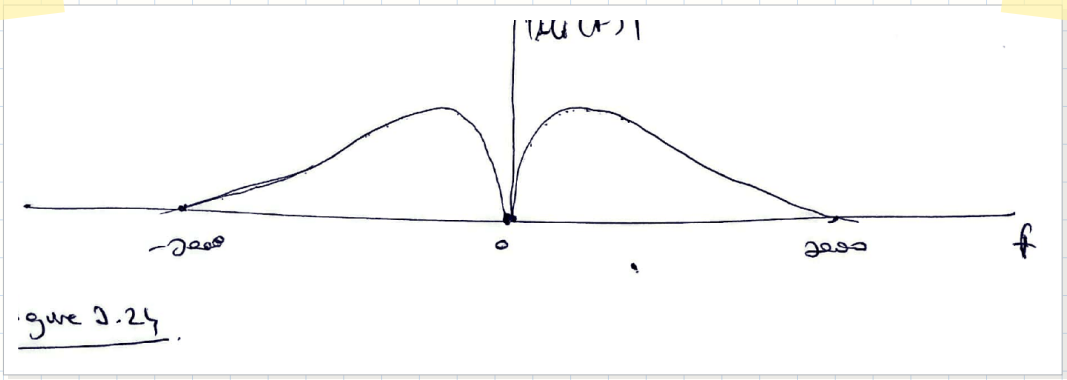
orijinal çizim, s. 4

orj. s. 5

Örnek 3.24 (FDM)

We wish to transmit 60 voice-band signals by SSB (upper-sideband) modulation and freq. division multiplexing (FDM). Each of the 60 signals has a spectrum as shown figure P-3.24. Note that the voice-band signal is band limited to 3 kHz. If each signal is frequency translated separately, we require a frequency synthesizer that produces 60 carrier frequencies to perform the FDM. On the other hand, if we subdivide the channels into L groups of K subchannels each, such that $LK = 60$; we may reduce the number of frequencies, from the synthesizer to $L + K$.

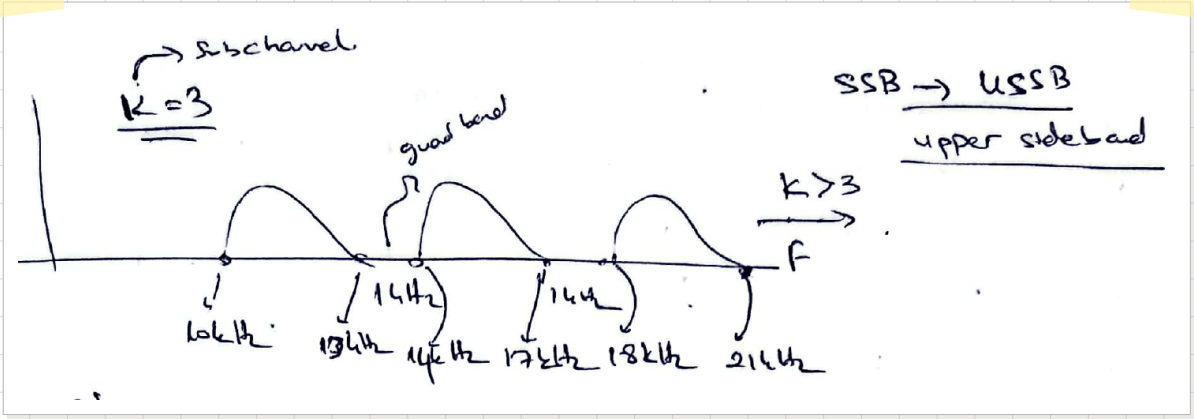
1) Illustrate the spectrum of the SSB signals in a group of K subchannels. Assume that a 1 kHz guardband separates the signals in adjacent frequency subchannels and that the carrier frequencies are $f_{c1} = 10$ kHz, $f_{c2} = 14$ kHz – etc.
2) Sketch L and K such that $LK = 60$ and $L + K$ is a minimum.
3) Determine the frequencies of the carriers if the 60 FDM signals occupy the frequency band 300–540 kHz, and each group of K signals occupies the band 10 kHz to $(10 + 4K)$ kHz.



orijinal çizim, s. 5

orj. s. 6

1)



orijinal çizim, s. 6

2) $\overset{\text{groups}}{L} \quad \underset{\text{subchannels}}{K} = 60$ $\quad (L,K)$ or $(K,L) \rightarrow (1,60),\ (2,30),\ (3,20),\ (4,15),\ (6,10).$

Minimum value $\rightarrow L + K$

$6 + 10 \rightarrow \underline{16}$

3) $L = 6,\ \underline{K = 10}$ we need 16 carriers with frequencies.

$K = 10$

- $f_{c1} = 10\ kHz$
- $f_{c2} = 14\ kHz$
- $f_{c3} = 18\ kHz$
- $f_{c4} = 22\ kHz$
- ⋮
- $f_{c10} = 46\ kHz$

$f_{c11} = 50\ kHz$ $\quad 10 + 4K$

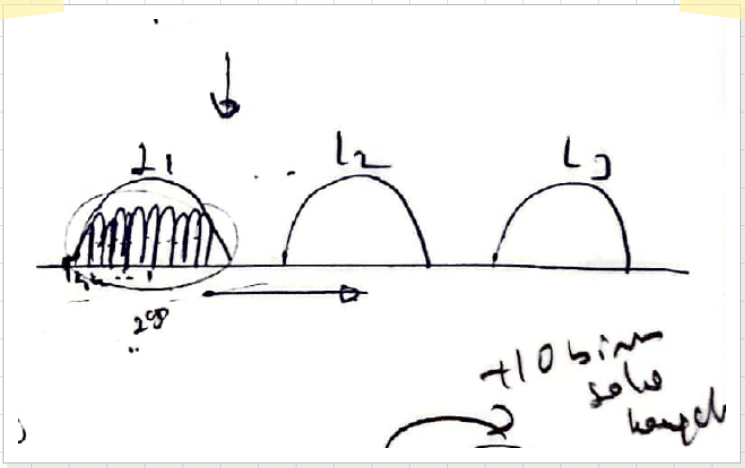
36 kHz ? +4 kHz

$L = 6$

$f_{L1} = 280\ kHz$
$f_{L2} = 320\ kHz$
360
⋮ 400
440
$f_{L6} = 480\ kHz$
~~$f_{L7} = 520\ kHz$~~

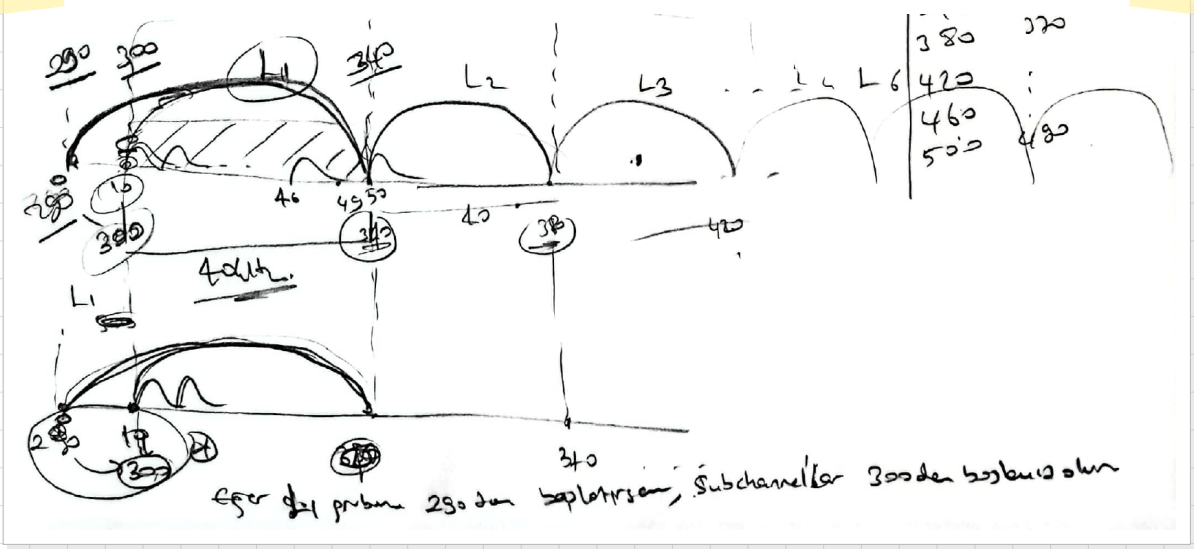
300 – 540 kHz

$L \rightarrow$ groups
$K \rightarrow$ subchannels.



orijinal çizim, s. 6

300 290
340 330
380 370
420 ⋮
460
500 490



orijinal çizim, s. 6

Eğer f_{L1} grubunu 290'dan başlatırsak, subchannel'lar 300'den başlıyor olur.

orj. s. 7

Örnek 4.3 (FM)

The carrier $c(t) = 100\cos(2\pi f_c t)$ is frequency modulated by the signal $m(t) = 5\cos(20000\pi t)$ where $f_c = 10^8$ Hz. The peak frequency deviation is 20 kHz.

1.) Determine the amplitude and frequency of all signal components that have a power-level of at least 10% of the power of the unmodulated carrier component.
2.) from Carson's rule, determine the approximate BW of FM signal.

$$y(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cdot \cos\left(2\pi(f_c + n \cdot f_m)t\right)$$

$$A_c = 100 \qquad f_m = 10000\,\text{Hz}$$
$$f_c = 10^8\,\text{Hz} \qquad = 10^4\,\text{Hz}$$

$$\beta = \frac{\Delta f}{B} = \frac{k_f \cdot \overset{\max|m(t)|}{m_p}}{f_m} = \frac{\Delta f_{max}}{f_m}$$

$$f_m = \frac{20 \cdot 10^3}{10^4} = 2$$

$$\beta = 2.$$

$$y(t) = \underline{100} \sum_{n=-\infty}^{\infty} \boxed{J_n(2)} \cdot \cos\left(2\pi(10^8 + n \cdot 10^4)t\right)$$

1) unmodulated carrier component $\rightarrow c(t)$

$$P_c = \frac{A_c^2}{2} = \frac{100^2}{2} = 5000$$

$$\text{at least } 10\% \rightarrow 5000 \cdot \frac{10}{100} = \boxed{500} \rightarrow \boxed{P_{J_n(2)} > 500} \qquad P_{J_n(2)} = \frac{(A_c J_n(2))^2}{2}$$

Index n	$J_n(2)$	freq(Hz)	Amplitude ($100\,J_n(2)$)	Power →
0	.2239	10^8	22.39	250,63
1	.5767	$10^8 + 10^4$	57.67	1663.1
2	.3528	$10^8 + 2\cdot10^4$	35.28	622.46
3	.1289	$10^8 + 3\cdot10^4$	12.89	83.13
4	.0340	$10^8 + 4\cdot10^4$	3.40	5.7785

Index → ? $n = 1,2$
It's given → x ×100 ↑

orj. s. 8

index	freq (Hz)	Amplitude
1	$10^8 + 10^4$	57.67
2	$10^8 + 2\cdot10^4$	35.28
−1	$10^8 - 10^4$	57.67
−2	$10^8 - 2\cdot10^4$	35.28

$$J_n^2(2) = J_{-n}^2(2)$$

2) $B_c = 2\,B\,(\beta + 1) = 2\cdot10^4(2+1) = \underline{6\cdot10^4\ \text{Hz}}$ ($B \downarrow f_m$)

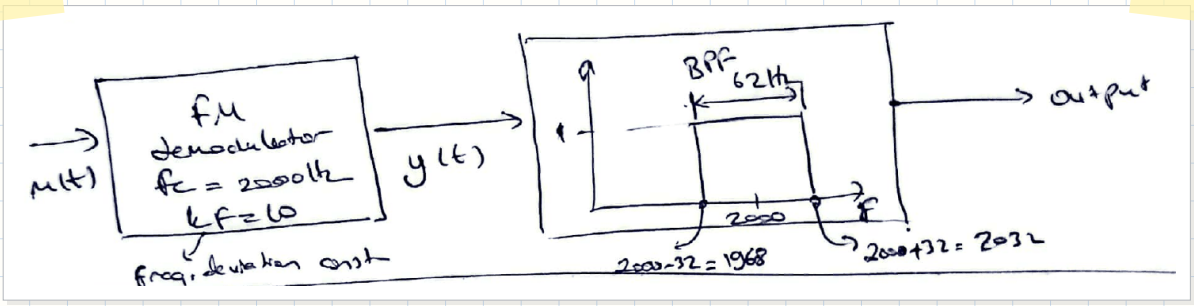
orj. s. 9

Örnek 4.18 (FM)

The modulation signal that is the input into an FM modulator is $m(t) = 10\cos(16\pi t)$. The output of the FM modulator is

$$y(t) = \underset{A_c}{10}\cos\Big[\overbrace{\underset{f_c}{4000\pi t} + 2\pi k_f\int_{-\infty}^{t} m(\tau)\,d\tau}^{\theta(t)}\Big]$$

where $k_f = 10$ (See figure P-4.18). If the output of the FM modulator is passed through an ideal BPF centered at $f_c = 2000$ with a bandwidth of 62 Hz, determine the power of the frequency components at the output of the filter. What percentage of the transmitter power appears at the output of the BPF?



orijinal çizim, s. 9

$$y(t) = A_c\sum_{n=-\infty}^{\infty} J_n(\beta)\cdot\cos\left(2\pi(f_c + n\cdot f_m)t\right)$$

$$A_c = 10, \quad f_c = 2000, \quad f_m = 8, \quad k_f = 10$$

$$\underset{B\to}{\beta} = \frac{\Delta f}{f_m} = \frac{k_f \cdot m_p}{f_m} = \frac{k_f \cdot \max|m(t)|}{f_m} = \frac{10 \cdot 10}{8} = 12.5$$

$$y(t) = 10 \cdot \sum_{n=-\infty}^{\infty} J_n(12.5) \cdot \cos\left(2\pi(2000 + n \cdot 8)t\right) \quad \text{at the output of the BPF.}$$

	freq		freq
J_0	2000	J_{-1}	1992
J_1	2008	J_{-2}	1984
J_2	2016	J_{-3}	1976
J_3	2024	J_{-4}	1968
J_4	2032		

$$\cdots -4 \cdots \overset{n}{0} \cdots 4 \cdots$$

→ Power at the output of the BPF

$$P_y = \frac{10^2}{2} \cdot J_0^2(12.5) + 2\sum_{n=1}^{4} J_n^2(12.5) \cdot \frac{10^2}{2} = 50 \cdot 0.2630$$

$$= \underline{13.15}$$

β	$J_0(\beta)$	$J_1(\beta)$	$J_2(\beta)$	$\cdots$
1				
2				
3				
⋮	**Hint**			

orj. s. 10

$$\underset{\substack{\uparrow \\ \text{total} \\ \text{transmitted} \\ \text{power}}}{P_{tot}} = \frac{A_c^2}{2} = \frac{10^2}{2} = 50.$$

$$\frac{P_y}{P_{tot}} = \frac{13.15}{50} = \frac{26.30}{100} \to \underline{\%26.30}$$

Power at the output of BPF is only %26.30 of transmitted power.

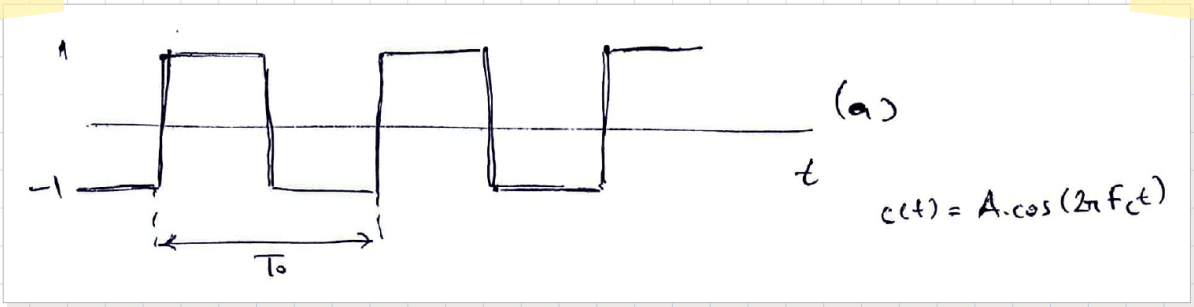
- Effective BW → Carson Rule %99

%26.30 → %99 olurdu

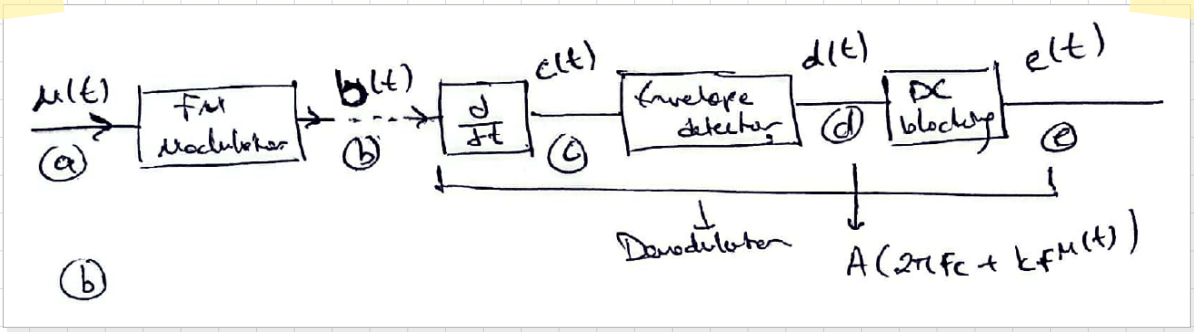
orj. s. 11

Örnek 5.4.2

A periodic square wave $m(t)$ (Fig. P.5.4-2a) frequency-modulates a carrier of frequency $f_c = \underline{10\ kHz}$ with $\underline{\Delta f = 1\ kHz}$. The carrier amplitude is A. The resulting FM signal is demodulated, as shown in Fig. P.5.4-2.b. by the method discussed in sec. 5.4 (Fig. 5.11) sketch wave forms. b, c, d, e.



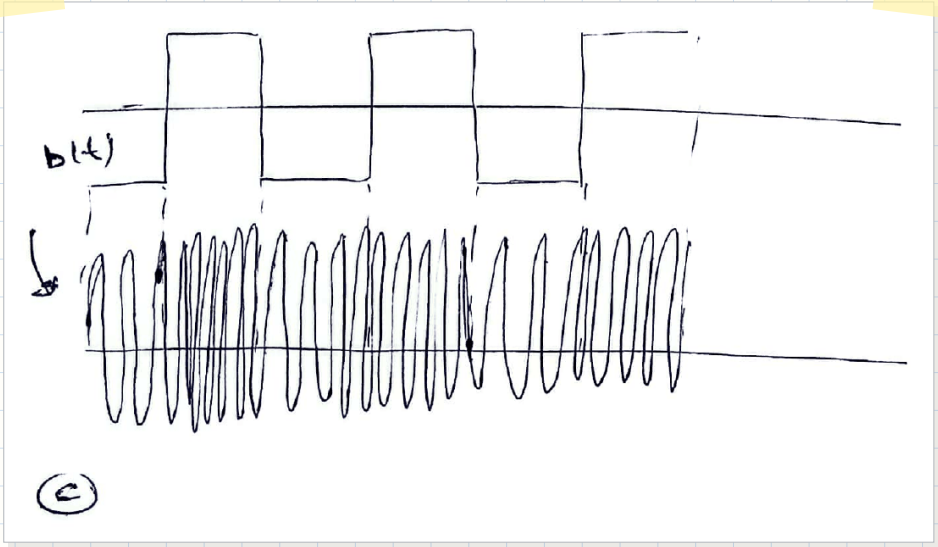
orijinal çizim, s. 11



orijinal çizim, s. 11

$$\Delta f = k_f \cdot \overset{\max|m(t)|}{m_p} = k_f$$
$$= 1k$$

$$f_i = f_c + k_f \cdot m(t)$$
$$f_i = 10k + 1k \cdot m(t)$$
$$f_{i,min} = 10k + 1k\,(m(t)_{min})$$
$$f_{i,min} = 9\ kHz$$
$$f_{i,max} = 10k + 1k\,(m(t)_{max})$$
$$f_{i,max} = \underline{11\ kHz}$$



orijinal çizim, s. 11

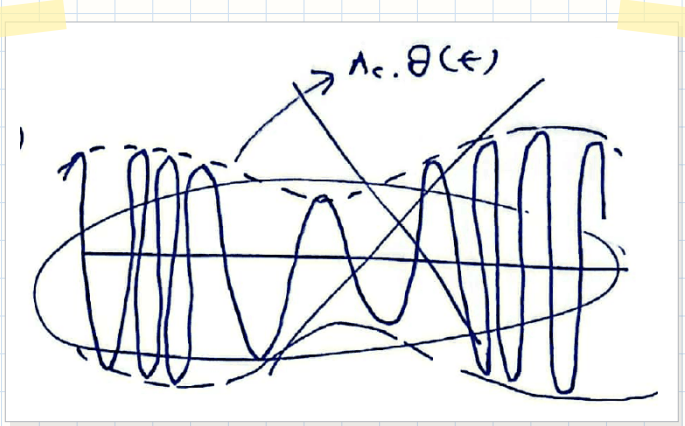
orj. s. 12

b(t): $y(t) = A_c \cdot \cos\left(\underbrace{2\pi f_c t + 2\pi k_f \int m(\tau)\,d\tau}_{\theta(t)}\right)$

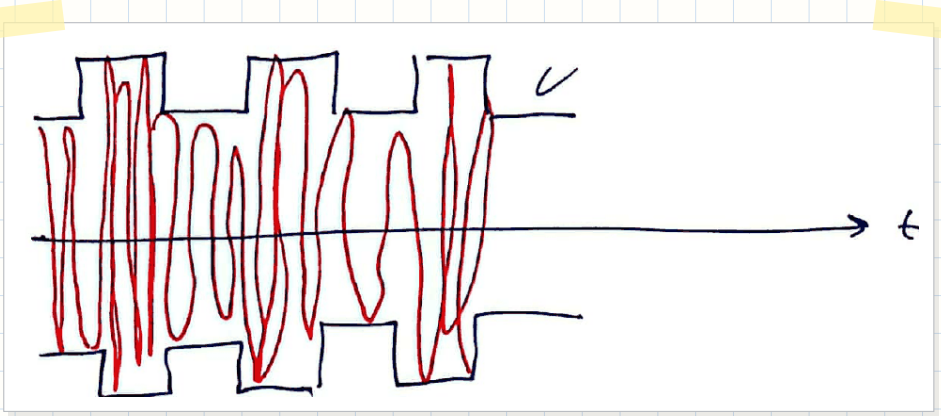
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$$\frac{d}{dt}y(t) = A_c \cdot \theta'(t) \sin\left(\theta(t)\right)$$

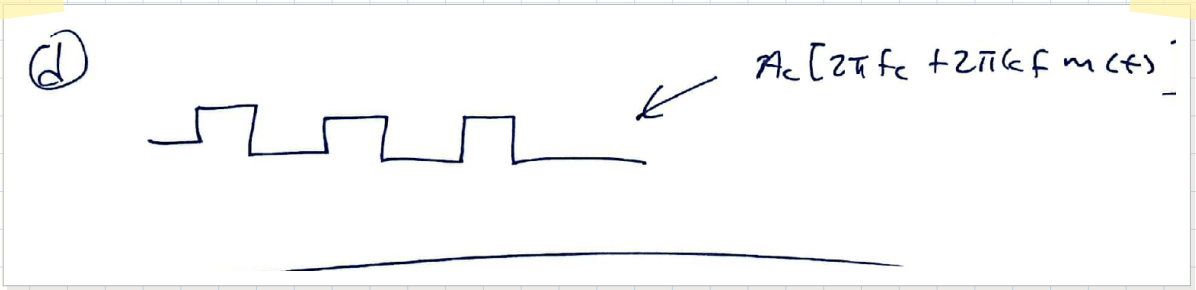
$$A_c\left[2\pi f_c + 2\pi k_f m(t)\right] \cdot \sin\theta(t)$$



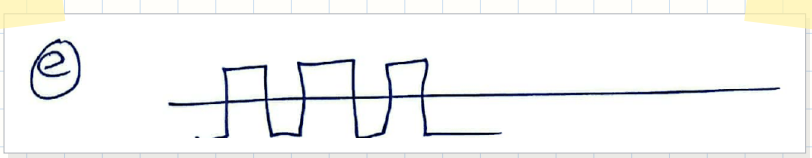
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orijinal çizim, s. 12



orijinal çizim, s. 12



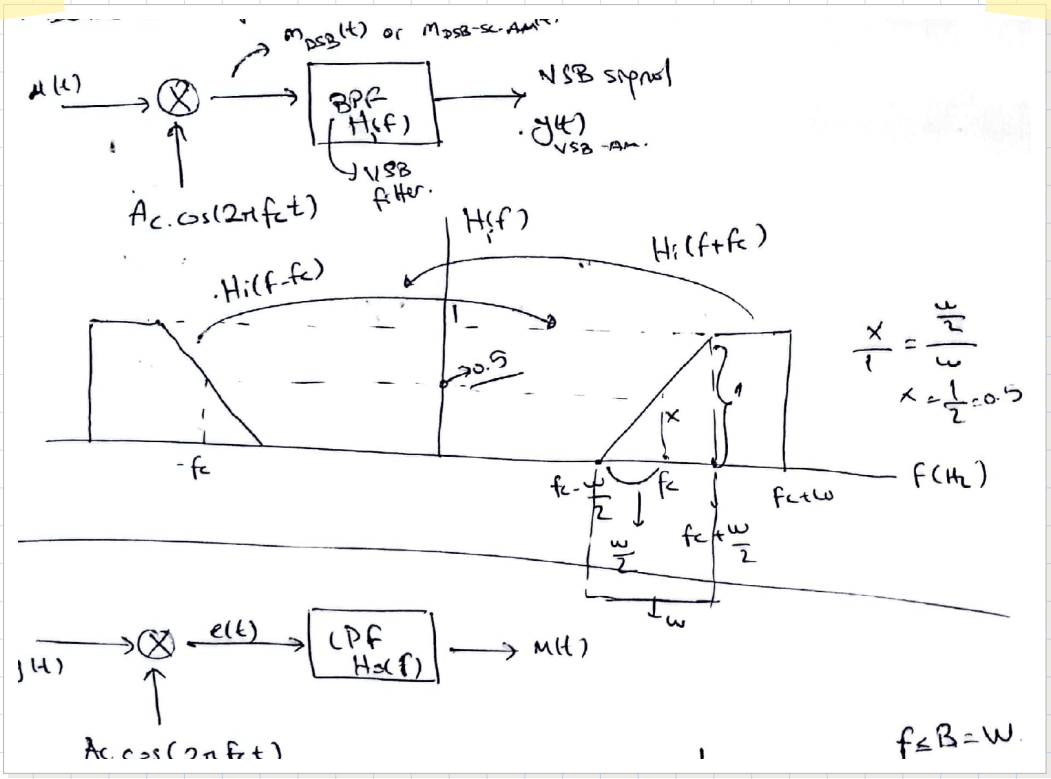
orijinal çizim, s. 12

orj. s. 13

Örnek 3.21

A vestigial sideband modulation system is shown in figure P-3.21. The bandwidth of the message signal $m(t)$ is W, and the transfer func. of the band pass filter is shown in the figure.

1) Determine $h_\ell(t)$, which is the low pass equivalent of $h(t)$, where represents the impulse response of the band pass filter.
2) Derive an expression for the modulated signal $y(t)$.



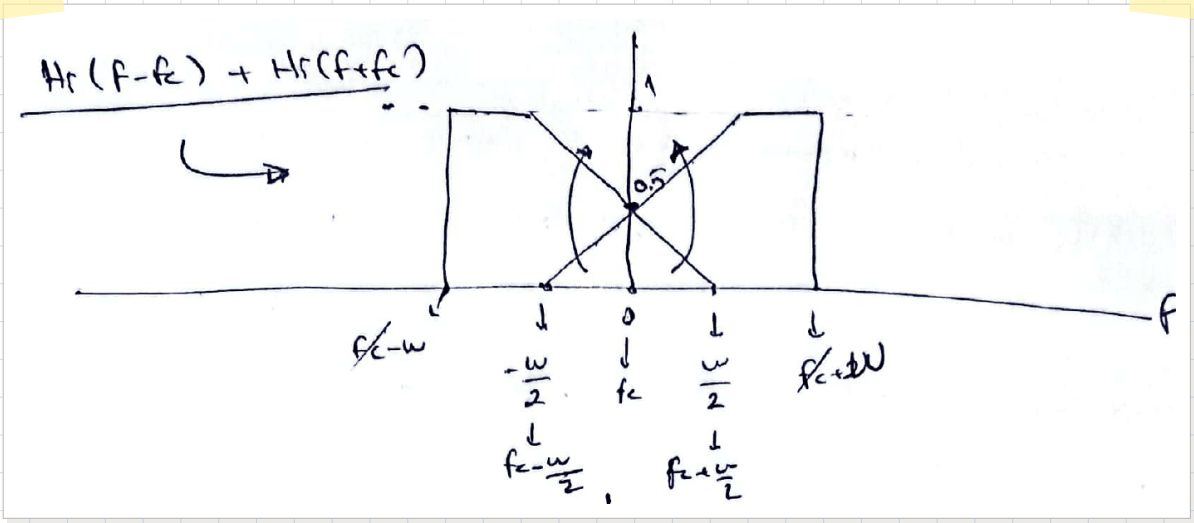
orijinal çizim, s. 13

$$M(f) = E(f) \cdot H_o(f)$$

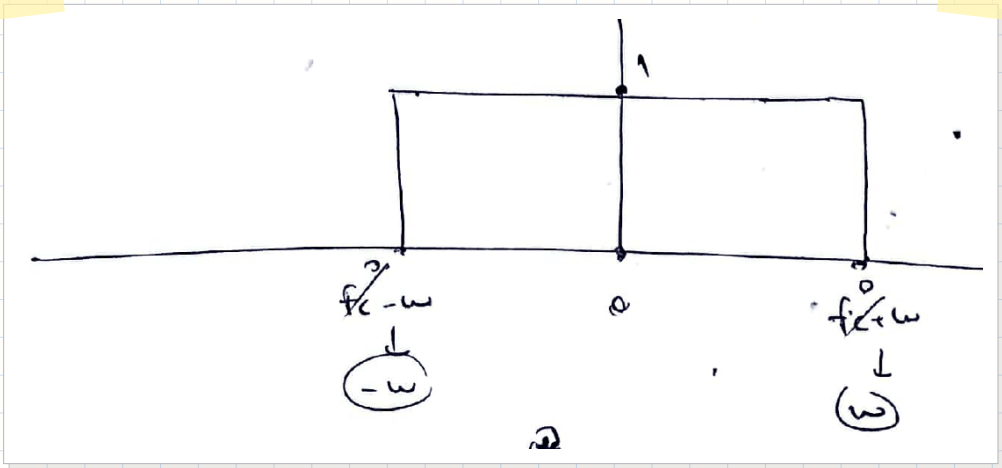
$$M(f) = M(f) \cdot \left(H_i(f - f_c) + H_i(f + f_c)\right) \cdot H_o(f)$$

$$H_o(f) = \frac{1}{H_i(f - f_c) + H_i(f + f_c)}, \qquad f \le B = W$$

orj. s. 14

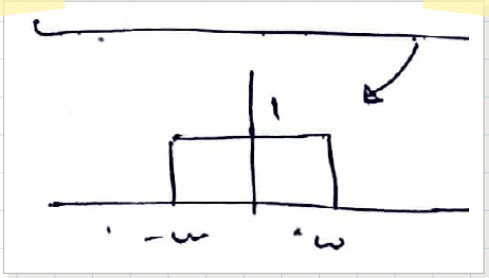


orijinal çizim, s. 14

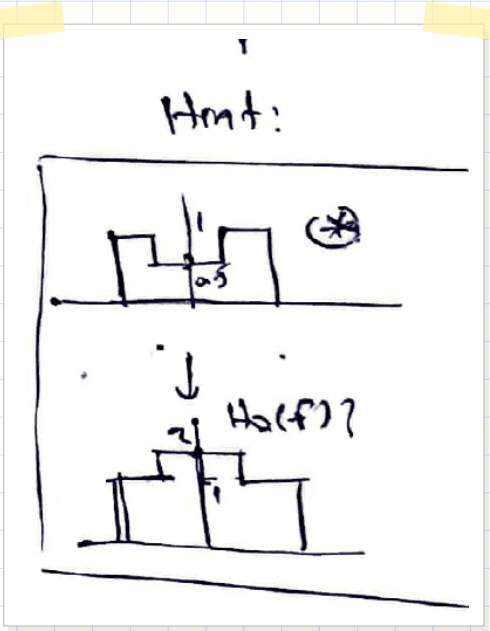


orijinal çizim, s. 14

$$M(f) = M(f) \cdot \left(H_i(f - f_c) + H_i(f + f_c)\right) \cdot H_o(f)$$



orijinal çizim, s. 14



orijinal çizim, s. 14