Devre Teorisi II

Final çalışması – Laplace, s-domeni analizi, Fourier serileri, frekans yanıtı ve pasif filtreler

Orijinal not: Devre-2-final-not.pdf

UYARI Bu belge, el yazısı ders notlarından Claude (Anthropic) yardımıyla dijitale aktarılmıştır. Yardımcı kaynak olarak hazırlanmıştır; hata veya eksik içerebilir. Lütfen orijinal notları ve ders kaynaklarını esas alın.

NOTICE *This document was transcribed from handwritten lecture notes with the help of Claude (Anthropic). It is meant as a supplementary resource and may contain errors or omissions. Always check against the original notes and official course material.*

Laplace transform

orj. s. 1

$$F(s) = \mathcal{L}\left[f(t)\right] = \int_{0^-}^{\infty} f(t)\, e^{-st}\, dt \qquad \text{(one sided, for } t > 0\text{)}$$

$$e^{-at} \longleftrightarrow \frac{1}{s+a} \qquad \sin \omega t \longleftrightarrow \frac{\omega}{s^2+\omega^2}$$

$$\frac{df}{dt} \longleftrightarrow sF(s) - f(0^-) \qquad \cos \omega t \longleftrightarrow \frac{s}{s^2+\omega^2}$$

> **Şekil:** Basılı materyal (ders kitabından ekran görüntüsü): *Table 14.2-1 Laplace Transform Pairs* ve *Table 14.2-2 Laplace Transform Properties* tabloları; dijitalleştirilmedi.
> *(orijinal notta, s. 1)*

★ $A\cos(\omega t + \theta) = (A\cos\theta)\cos(\omega t) - (A\sin\theta)\sin(\omega t)$

Inverse Laplace Transform: Simple, Real poles

> **Şekil:** Basılı formül (kitaptan): basit reel kutuplar için $F(s) = N(s)/D(s)$'nin kısmi kesirlere açılımı ve $\mathcal{R}_i$ katsayılarının formülü; dijitalleştirilmedi.
> *(orijinal notta, s. 1)*

Repeated poles

> **Şekil:** Basılı formül (kitaptan): p_1'de q katlı kutup için kısmi kesir açılımı ve $\mathcal{R}_{q-k}$, $\mathcal{R}_1$, $\mathcal{R}_2$, $\mathcal{R}_q$ katsayılarının türevli formülleri; dijitalleştirilmedi. El yazısı ekler: $\frac{d^k}{ds^k}$ terimine ok ile "k'ıncı dereceden türev alacağız" notu ve formüllerin yanında kırmızı parantez ile $\star$ işareti.
> *(orijinal notta, s. 1)*

Improper Rational function

$\star$ payın derecesi büyükse polinom bölmesi yapıl? ır

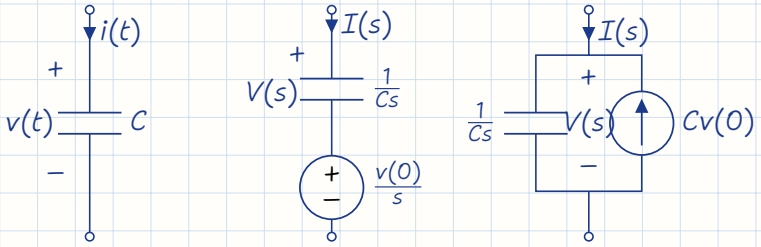
Circuit Analysis Using Impedance and Initial Conditions

$$v(t) = i(t)\mathcal{R} \longleftrightarrow V(s) = I(s)\mathcal{R} \quad \Rightarrow \quad Z(s) = \frac{V(s)}{I(s)}$$

For capacitor

$$v(t) = \frac{1}{C}\int_0^t i(\tau)\,d\tau + v(0) \longleftrightarrow V(s) = \frac{1}{Cs}I(s) + \frac{v(0)}{s}$$

$$\Rightarrow Z_C(s) = \frac{1}{Cs} \quad \text{and} \quad I(s) = CsV(s) - Cv(0)$$

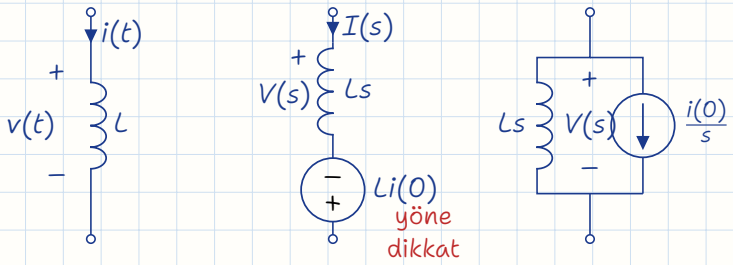


> **Şekil:** Yanda kitaptan küçük basılı bir şekil (kondansatör ve bobin s-domeni modelleri) yapıştırılmış; dijitalleştirilmedi.
> *(orijinal notta, s. 1)*

For Inductor

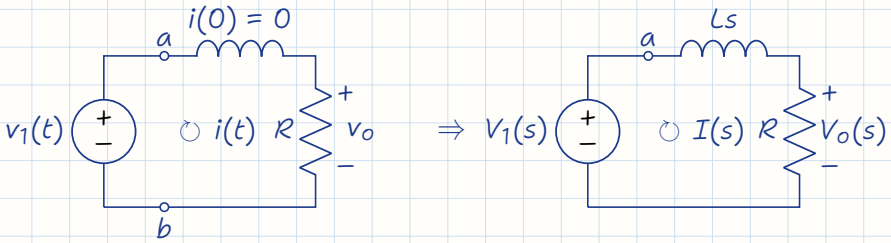
$$v(t) = L\frac{d}{dt}i(t) \longleftrightarrow V(s) = LsI(s) - Li(0)$$

$$Z_L(s) = Ls \quad \text{and} \quad I(s) = \frac{1}{Ls}V(s) + \frac{i(0)}{s}$$



Transfer function and impedance

$$H(s) = \frac{V_o(s)}{V_1(s)}$$



$$V_o(s) = H(s)\,V_1(s)$$

when the input is a unit step function

$$V_1(s) = \mathcal{L}\left[u(t)\right] = \frac{1}{s} \qquad V_o(s) = \frac{H(s)}{s}$$

$$\Rightarrow \text{ Step response} = \mathcal{L}^{-1}\left[\frac{H(s)}{s}\right]$$

when the input is an impulse function

$$v_1(t) = \delta(t) \longleftrightarrow V_1(s) = \mathcal{L}\left[\delta(t)\right] = 1 \qquad V_o(s) = H(s)$$

$$\text{Impulse response} = \mathcal{L}^{-1}\left[H(s)\right]$$

$$H(s) = \text{Voltage gain} = \frac{V_o(s)}{V_i(s)} \qquad H(s) = \text{current gain} = \frac{I_o(s)}{I_i(s)}$$

$$H(s) = \text{Impedance} = \frac{V(s)}{I(s)} \qquad H(s) = \text{Admittance} = \frac{I(s)}{V(s)}$$

The Fourier Series

orj. s. 2

Trigonometric Fourier series

$$f(t) = f(t + nT)$$

$$f(t) = \underbrace{a_0}_{dc} + \underbrace{\sum_{n=1}^{\infty}\left(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t\right)}_{ac}$$

$$a_0 = \frac{1}{T}\int_0^T f(t)\,dt \qquad a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_0 t\,dt \qquad b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega_0 t\,dt$$

As alternative

$$f(t) = a_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t + \phi_n), \qquad A_n = \sqrt{a_n^2 + b_n^2}, \quad \phi_n = -\tan^{-1}\frac{b_n}{a_n}$$

important integrals

$$\int \cos at\,dt = \frac{1}{a}\sin at \qquad \int t\cos at\,dt = \frac{1}{a^2}\cos at + \frac{1}{a}t\sin at$$

$$\int \sin at\,dt = -\frac{1}{a}\cos at \qquad \int t\sin at\,dt = \frac{1}{a^2}\sin at - \frac{1}{a}t\cos at$$

Symmetry Considerations

Even Symmetry (çift)

$$f(t) = f(-t) \qquad a_0 = \frac{2}{T}\int_0^{T/2} f(t)\,dt \qquad a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos n\omega_0 t\,dt \qquad b_n = 0$$

Odd Symmetry (tek simetri)

$$f(-t) = -f(t) \qquad \int_{-T/2}^{T/2} f_o(t)\,dt = 0$$

$$a_0 = 0 \quad a_n = 0 \qquad b_n = \frac{4}{T}\int_0^{T/2} f(t)\sin n\omega_0 t\,dt$$

Halfwave Symmetry

$$f\left(t - \frac{T}{2}\right) = -f(t) \qquad a_0 = 0$$

$$a_n = \begin{cases} \frac{4}{T}\int_0^{T/2} f(t)\cos n\omega_0 t\,dt, & \text{for } n \text{ odd} \\ 0, & \text{for } n \text{ even} \end{cases} \qquad b_n = \begin{cases} \frac{4}{T}\int_0^{T/2} f(t)\sin n\omega_0 t\,dt, & \text{for } n \text{ odd} \\ 0, & \text{for } n \text{ even} \end{cases}$$

Circuit Applications

$$v(t) = V_0 + \sum_{n=1}^{\infty} V_n \cos(n\omega_0 t + \theta_n)$$

$v(t)$ consists of two parts: the dc component V_0 and the ac component $\mathbf{V}_n = |V_n|\angle\theta_n$ with several harmonics.

> *Şekil:* Basılı şekil (kitaptan): seri bağlı V_0, $V_1\cos(\omega_0 t + \theta_1)$, $V_2\cos(2\omega_0 t + \theta_2)$, …, $V_n\cos(n\omega_0 t + \theta_n)$ kaynakları bir "Linear network"ü besliyor, akım $i(t)$; dijitalleştirilmedi.
> *(orijinal notta, s. 2)*

$$\text{so} \quad i(t) = I_0 + \sum_{n=1}^{\infty} |I_n|\cos(n\omega_0 t + \psi_n)$$

Average Power and RMS Values

$$v(t) = V_{dc} + \sum_{n=1}^{\infty} V_n \cos(n\omega_0 t - \theta_n) \qquad i(t) = I_{dc} + \sum_{n=1}^{\infty} I_n \cos(n\omega_0 t - \phi_n)$$

$$P = V_{dc}I_{dc} + \frac{1}{2}\sum_{n=1}^{\infty} V_n I_n \cos(\theta_n - \phi_n)$$

$$F_{rms} = \sqrt{a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty}\left(a_n^2 + b_n^2\right)}$$

Frequency Response

orj. s. 3

$$H(\omega) = \frac{Y(\omega)}{X(\omega)}$$

The decibel Scale

1. $\log P_1P_2 = \log P_1 + \log P_2$
2. $\log P_1/P_2 = \log P_1 - \log P_2$
3. $\log P^n = n \log P$
4. $\log 1 = 0$

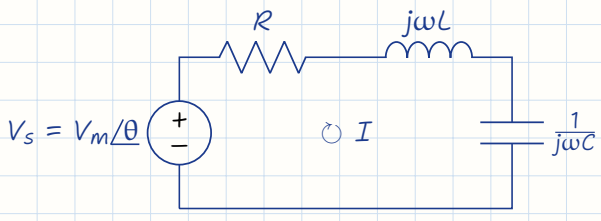
$$G = \text{Number of bels} = \log_{10}\frac{P_2}{P_1}$$

$$G_{dB} = 10\log_{10}\frac{P_2}{P_1} \qquad G_{dB} = 20\log_{10}\frac{V_2}{V_1} \qquad G_{dB} = 20\log_{10}\frac{I_2}{I_1}$$

$$H = |H|\angle\phi = He^{j\phi} \qquad \ln H = \ln|H| + \ln e^{j\phi} = \ln|H| + j\phi \qquad H_{dB} = 20\log_{10} H$$

> **Şekil:** Basılı materyal (kitaptan): Bode diyagramı tablosu – Factor / Magnitude / Phase sütunları; K, $(j\omega)^N$, $1/(j\omega)^N$, $(1 + j\omega/z)^N$, $1/(1 + j\omega/p)^N$ ve ikinci dereceden terimler için asimptotik genlik ($\pm 20N$, $\pm 40N$ dB/decade) ve faz ($0°$, $\pm 90N°$, $\pm 180N°$) çizimleri; dijitalleştirilmedi. *(orijinal notta, s. 3)*

Series Resonance



$$Z = H(\omega) = \frac{V_s}{I} = R + j\omega L + \frac{1}{j\omega C}$$

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$\mathrm{Im}(Z) = \omega L - \frac{1}{\omega C} = 0$$

$$\omega_0 L = \frac{1}{\omega_0 C} \qquad \boxed{\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s}}$$

since $\omega_0 = 2\pi f_0$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

The frequency response of the circuit's current magnitude

$$I = |I| = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

$$P(\omega) = \frac{1}{2} I^2 R \qquad P(\omega_0) = \frac{1}{2}\frac{V_m^2}{R}$$

$$P(\omega_1) = P(\omega_2) = \frac{(V_m/\sqrt{2})^2}{2R} = \frac{V_m^2}{4R} \quad \leftarrow \text{half power}$$

$$\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{2}R$$

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \qquad \omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

$$Q = 2\pi \frac{\text{Peak energy stored in the circuit}}{\text{Energy dissipated by the circuit in one period at resonance}}$$

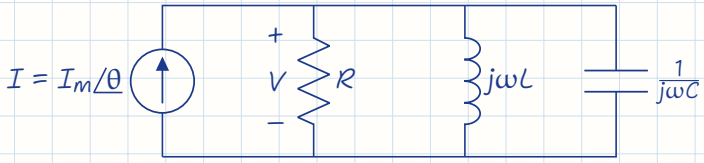
$$Q = 2\pi \frac{\frac{1}{2}LI^2}{\frac{1}{2}I^2 R\,(1/f_0)} = \frac{2\pi f_0 L}{R}$$

$$\boxed{Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R}}$$

$$\boxed{\text{Bandwidth} = \frac{R}{L} = \frac{\omega_0}{Q}}$$

$$\omega_1 \simeq \omega_0 - \frac{B}{2}, \qquad \omega_2 \simeq \omega_0 + \frac{B}{2}$$

Parallel Resonance



$$Y = H(\omega) = \frac{I}{V} = \frac{1}{R} + j\omega C + \frac{1}{j\omega L} \qquad Y = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

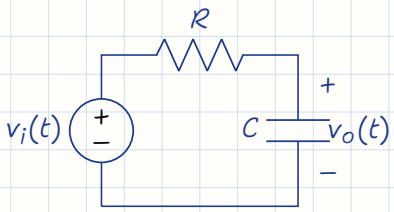
$$\omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} \qquad \omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$\boxed{B = \omega_2 - \omega_1 = \frac{1}{RC}} \qquad \boxed{Q = \frac{\omega_0}{B} = \omega_0 RC = \frac{R}{\omega_0 L}}$$

Passive Filters

orj. s. 4

Low pass filter

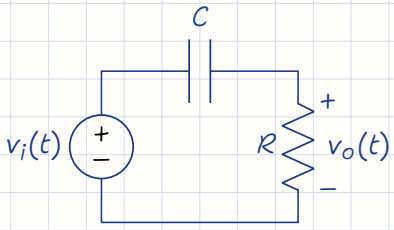


$$H(\omega) = \frac{V_o}{V_i} = \frac{1/j\omega C}{R + 1/j\omega C} \qquad H(\omega) = \frac{1}{1 + j\omega RC}$$

★ $H(0) = 1, \quad H(\infty) = 0$

$$\underset{\text{cutoff frequency}}{H(\omega_c)} = \frac{1}{\sqrt{1 + \omega_c^2 R^2 C^2}} = \frac{1}{\sqrt{2}} \qquad \boxed{\omega_c = \frac{1}{RC}}$$

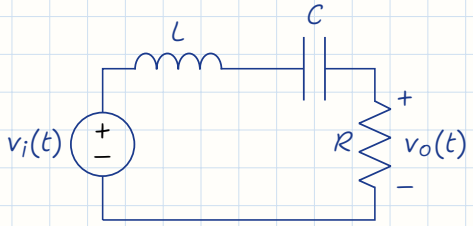
High pass filter



$$H(\omega) = \frac{V_o}{V_i} = \frac{R}{R + 1/j\omega C} \qquad H(\omega) = \frac{j\omega RC}{1 + j\omega RC}$$

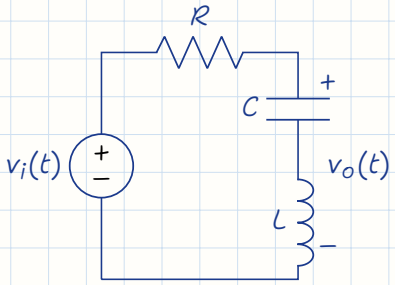
★ $H(0) = 0, \quad H(\infty) = 1 \Rightarrow \quad \omega_c = \frac{1}{RC}$

Band pass filter



$$H(\omega) = \frac{V_o}{V_i} = \frac{R}{R + j(\omega L - 1/\omega C)} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

Bandstop filter



$$H(\omega) = \frac{V_o}{V_i} = \frac{j(\omega L - 1/\omega C)}{R + j(\omega L - 1/\omega C)}$$

Notice that $H(0) = 1$, $H(\infty) = 1$

and $\omega_0 = \frac{1}{\sqrt{LC}}$ //