Elektromekanik Enerji Dönüşümü

Final notu – elektromekanik dönüşüm, DC ve asenkron makineler

Original notes: Final-notu.pdf

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orig. p. 1

Chapter 3

externally force move the conductor opposite to the magnetic force, mech. energy converted into electric energy

Energy

$$W_i = \underbrace{W_o}_{\text{output}} + \underbrace{W_e}_{\text{loss}} + \underbrace{W_f}_{\text{change in stored energy}}$$

Figure: Printed figure: N-turn coil on a magnetic core, source $e(t)$, current i, flux ϕ.
(in the original notes, p. 1)

In a conservative system:

$$dW_i = v\,i\,dt \qquad \text{(input energy)}$$
$$= -e\,i\,dt$$
$$e = -\frac{d\lambda}{dt} = -\frac{dN\Phi}{dt}$$

The energy supplied by the source stored in the system as mag. energy ($W_i = W_m$)

$$L = \frac{N\Phi}{i} \qquad \text{(inductance)}$$

λ = total flux linkages $\rightarrow$ $dW_i = i\,d\lambda = N\,i\,d\Phi$

$$\Rightarrow W_i = \int i\,d\lambda = N\int i\,d\Phi$$

In a linear system the energy stored in the system

$$W_m = \tfrac{1}{2}N\Phi i = \tfrac{1}{2}Li^2$$

$$Ni = Hl \quad \text{and} \quad \Phi(t) = A\,dB \Rightarrow W_i = lA\int H\,dB$$

3.3 Magnetic Field as a Medium

Case 1. Constant Flux

A magnetic circuit is moved by a distance dx in time dt in a region where the flux is held constant. The energy supplied by the source must be zero because the initial and final values of the flux are the same.

For a conservative system

$$F_m\,dx + dW_m = 0 \qquad F_m = -\frac{dW_m}{dx}$$

The mechanical force acting on the magnetic circuit.

Case 2. Constant Current

For a change in flux $d\phi$ when I is held constant. the energy input:

$$dW_i = I\,d\phi \quad \text{and} \quad dW_m = \tfrac{1}{2} I\,d\phi \;\Rightarrow\; dW_i = 2\,dW_m$$

For conservative system $F_m\,dx + dW_m = 2\,dW_m$ $F_m = \dfrac{dW_m}{dx}$

When the flux is constant $F_m = -\dfrac{\partial W_m(\phi, x)}{\partial x}$

When the current is constant $F_m = \dfrac{\partial W_m(I, x)}{\partial x}$

The rate of change of stored magnetic energy (constant flux)

$$\frac{dW_m}{dx} = \frac{1}{2} N\Phi \frac{\partial i}{\partial x} = -\frac{1}{2} i^2 \frac{\partial \mathcal{L}}{\partial x} \qquad F_m = \tfrac{1}{2} i^2 \frac{\partial \mathcal{L}}{\partial x}$$

total airgap inductance

Magnetic Circuit with Air Gap

> *Figure:* Printed figure: magnetic circuit with air gap (fixed member, movable member, gap x, winding N, force F).
> *(in the original notes, p. 1)*

$$W_i = \ell A \int H\,dB$$

so the energy in each air gap

$$W_g = \tfrac{1}{2}\mu_0 H_0^2 A x = \frac{1}{2\mu_0} B_0^2 A x$$

$$B_0 = \mu_0 H_0$$

When the Flux constant $\Phi = BA$

$$F_g = -\tfrac{1}{2}\mu_0 H_0^2 A = -\frac{1}{2\mu_0} B_0^2 A$$

μ relative permeability

$L = \frac{N^2}{\mathcal{R}}$ $\mathcal{R} = \frac{\ell}{\mu A}$ $F_{gravity} = mg$

$$F_{spring} = \frac{tork}{çap}$$

The rotor of dc machine called armature.

Wound machine (mach. with electromagnets)

Generator Action

Figure: Printed figure: cross-section of a 2-pole machine (yoke, stator, magnets, rotor, air gap); student's labels: YOKE, stator, Rotor, Air.
(in the original notes, p. 1)

2 pole machine.
mechanical Angle $\theta_m = \theta$ (Electrical Angle)
The arc by the coil = 180° ⇒ full pitch coil
less than 180° fractional pitch coil

Figure: Printed plots: (a) N and S magnet poles over the yoke, (b) flux per pole set by the magnets, (c) flux passing through a coil rotating at a constant speed, induced emf in the coil. Student's labels: "Flux per pole set by the magnets", "Flux passing through a coil rotating at a constant speed", "Induced emf in the coil".
(in the original notes, p. 1)

Figure: Printed figure: "2 pole ac current generator" (rotating coil between N and S poles, forces F, speed ω_m, torque T_s).
(in the original notes, p. 1)

If ϕ_p is the flux per pole: $\phi = \phi_p \cos\theta$

θ: the angular position of the coil in electrical degrees

$$e = -\frac{d\phi}{dt} = \phi_p \sin\theta \frac{d\theta}{dt} \qquad \text{where } \frac{d\theta}{dt} \text{ is the angular freq.}$$

$$e = \phi_p \omega \sin\omega t \quad \} \quad \theta = \theta_m \text{ in 2pole machine.}$$

Multiple Machines

Ex: 4 pole machine ⇒ θ is 720° while θ_m still 360°

If there are P poles in a machine $\theta = \frac{P}{2}\theta_m$

angular f. of the induced emf $\omega = \frac{P}{2}\omega_m$ → angular f. of the rotor

$$\omega = 2\pi f \Rightarrow f = \frac{P}{4\pi}\omega_m \quad \Big\} \quad \underset{\text{rad/s}}{\omega_m} = \frac{2\pi}{60}{}^{\text{rpm}}N_m$$

$$f = \frac{P}{120}N_m, \qquad N_m = \frac{120f}{P}$$

The max value of the induced emf in a single turn coil

$$E_m = \phi_p\omega = \frac{P}{2}\phi_p\omega_m = \frac{2\pi P}{120}\phi_p N_m$$

In a dc machine, the average value of the induced voltage in a single turn coil is

$$E_c = \frac{2}{\pi}E_m = \frac{P}{\pi}\phi_p\omega_m = \frac{P}{30}\phi_p N_m \qquad E_{eff} = \frac{E_m}{\sqrt{2}}$$

Force on a Conductor

Lorentz equation:

$$\vec{F} = \int \underset{\text{lenght of the current carrying element}}{i\,d\vec{\ell}} \times \underset{\text{magnetic flux density}}{\vec{B}}$$

For linear current-carrying conductor:

$$\vec{F} = i\vec{L} \times \vec{B}$$

Motor action: the force on the upper conductors of the coil

$$\vec{F} = iLB\vec{a}_\phi \quad \text{(the same down)}$$

$F_c = 2iBL$ total force experienced.

Torque: $T_e = F_e \cdot r = 2Bilr$ for each conductor

Revolving Field of a Three Phase Motor:

> *Figure:* Printed figure: "2 pole, three phase motor" (three-phase stator winding with coil sides 1, 1', 2, 2', 3, 3', closed-loop rotor).
> *(in the original notes, p. 1)*

$$i_1 = I_m \sin \omega t$$
$$i_2 = I_m \sin(\omega t - 120^\circ)$$
$$i_3 = I_m \sin(\omega t + 120^\circ)$$

Instant1 $(\omega t = 0) \Rightarrow \Phi_{r1} = \sqrt{\Phi_2^2 + \Phi_3^2 + 2\Phi_2\Phi_3\cos(60)}$

$$= 1{,}5\,\Phi_m$$

Instant2 $(\omega t = 60^\circ) \Rightarrow \Phi_{r2} = 1{,}5\,\Phi_m$

Instant3 ($\omega t = 120°$) $\Rightarrow \Phi_{r3} = \Phi_{r1}$ and 120° phase shift

In a 2 pole motor, the time taken by the flux to complete one revolution along the periphery of the air gap is the same as input. T

For P pole: $T_s = \frac{P}{2}T$

the speed of the revolving field in revolutions per second $n_s = \frac{1}{T_s} = \frac{2f}{P}$ (dönme frekansı)

$$N_s = \frac{120f}{P}\,[\text{rpm}], \quad \omega_s = \frac{4\pi f}{P} = \frac{2}{P}\omega$$

Sabit frekanslı bir kaynak için hız sabit.

Slip speed: (the difference between synchronous and rotor speed)

$$N_r = N_s - N_m, \quad \omega_r = \omega_s - \omega_m \rightarrow \text{rotor speed}$$

Per unit slip: $s = \frac{\omega_r}{\omega_s} = \frac{\omega_s - \omega_m}{\omega_s}$

$$\Rightarrow \omega_m = (1-s)\omega_s, \quad N_m = (1-s)N_s$$

Revolving Field of two Phase Motor

$$i_1 = I_m \sin\omega t \,/\, i_2 = I_m \sin(\omega t - 90°) = -I_m \cos\omega t$$

$$\Phi_1 = \Phi_m \sin\omega t \,/\, \Phi_2 = -\Phi_m \cos\omega t$$

Synchronous Motor

power angle δ: rotor lags, revolving field

The power is proportional $\sin\delta$

Under no load, δ is almost zero

The torque developed max @ $\delta = 90°$

Reluctance motor:

$$\mathcal{R}(\theta) = \frac{2g}{\mu A} \rightarrow \text{effective airgap}$$

$\theta = 0 \Rightarrow$ air gap min

$$L(\theta) = 0{,}5(L_d + L_q) + 0{,}5(L_d - L_q)\cos 2\theta$$

Torque developed by rotating the system $T_e = \frac{1}{2}i^2\frac{dL}{d\theta}$

$$\underset{\text{rotor speed}}{\omega_m} = \frac{2}{P}\omega$$

Torque exerted on the rotor $T_e = -\frac{1}{2}i^2(L_d - L_q)\sin 2\theta$

$$\theta = \omega_m t + \delta \Rightarrow T_e = -\frac{1}{2}i^2(L_d - L_q)\sin[2\omega_m t + 2\delta]$$

$$\Rightarrow T_e(\text{avg}) = -0{,}125\,(L_d - L_q)I_m^2 \sin 2\delta$$

which max when $\delta = 45°$

Direct current Generators

Induced emf Equation:

Average value of the induced emf $E_c = \frac{P}{\pi}\phi_p\omega_m$ } $f = \frac{P}{4\pi}\omega_m$ } $N_m = \frac{120f}{P}$ (of induced v.)

Let N_c be the turns per coil, C total number of coil (slots for a 2 layer winding) and a the number of parallel paths ($a = 2$ for wave, $a = P$ for lap winding)

The total turns per parallel path: $N_a = \frac{C}{a}$ ($\frac{\text{turns per coil}}{N_c}$) turns between negative and positive brushes

$$\underset{\text{between two terminal nodes}}{E_a} = \frac{PC}{\pi a}N_c\omega_m\phi_p \} \; Z = 2CN_c \quad \text{total number of conductors}$$

$$E_a = k_a\phi_p\omega_m \Rightarrow k_a = \frac{2P}{2\pi a} \quad \text{(machine constant)}$$

The electrical power developed by the generator:

$$P_d = E_aI_a = k_a\phi_p\omega_mI_a = T_d\omega_m$$

Since in mech system: $P_d = T_d\omega_m \Rightarrow T_d = K_a\phi_pI_a$

The actual pole area: $A_p = \frac{2\pi rl}{P}$ → length of the machine

Effective: $A_e = A_p \times$ armature% } $\phi_p = BA_e$

$$E_{coil} = \frac{C}{a}E_a \} \; E_{turn} = E_{coil}/\text{turns}$$

$E_{conductor} = \frac{E_{coil}}{P}$ } Torque experienced by a single turn, current-carrying coil in a uniform magnetic field: T

$$T_c = 2BlrI_c \quad \text{(Average torque)}$$

The average torque developed by a dc machine:

$$T_d = \left[\frac{2BlrI_a}{a}\right][CN_c] = \frac{BlrZ}{a}I_a$$

The torque developed by the dc machine:

$$T_d = \frac{PZ}{2\pi a}BA_pI_a = k_a\phi_pI_a$$

$$\phi_p = k_fI_f, \quad E_a = k_1k_fI_f$$

$$E_{a2} = \frac{\omega_{m2}}{\omega_{m1}}E_{a1}$$

> *Figure:* Printed figure: magnetization curve (induced emf E_a at no load versus field current I_f or field mmf; air-gap line and magnetic circuit curve).
> *(in the original notes, p. 1)*

Voltage regulation

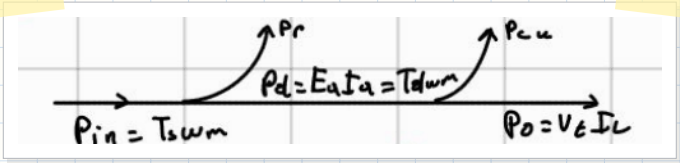
$$VR\% = \frac{V_{nL} - V_{FL}}{V_{FL}} \times 100$$

$$\underset{\text{rotational loss}}{P_r} = \underset{\text{friction windage}}{P_{fw}} + \underset{\text{magnetic loss}}{P_m}$$

For generator

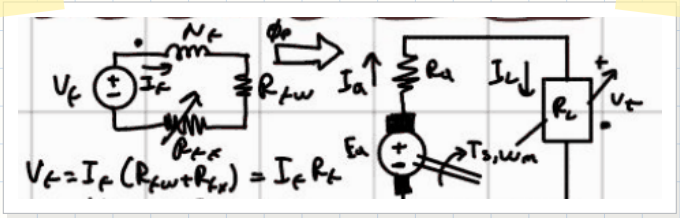
Power Flow diagram: $V_a = V_t + I_a R_a$

$$P_o = T_s \omega_m - P_r - P_{cu}$$



original drawing, p. 1

A seperately Exited DC Generator



original drawing, p. 1

$$V_t = I_f(R_{fw} + R_{fx}) = I_f R_f$$
$$E_a = V_t + I_a R_a$$
$$I_L = I_a$$

$$P_{inm} = P_d + P_r$$
$$P_{in} = P_{inm} + V_f I_f$$

$$\eta = \frac{P_{out}}{P_{in}}$$

A Shunt generator

Figure: Printed circuit: shunt generator (field winding N_f, R_{fw}, R_{fx}, armature E_a, R_a, load R_L, V_t).
(in the original notes, p. 1)

$$I_a = I_L + I_f$$
$$V_t = I_f R_f$$
$$V_t = I_L R_L = E_a - I_a R_a$$

A series Generator

> *Figure:* Printed circuit: series generator (series field R_s, N_s, diverter R_d, armature E_a, R_a, load R_L).
> *(in the original notes, p. 1)*

$$V_t = E_a - I_a R_a - I_s R_s$$
$$I_s R_s = I_d R_d$$
$$I_a = I_L = I_s + I_d$$

Compound Generators

> *Figure:* Printed circuit: compound generator (series field R_s, N_s with diverter R_d, shunt field R_{fw}, N_f, R_{fx}, armature E_a, R_a, load R_L).
> *(in the original notes, p. 1)*

orig. p. 2

Short shunt

$$I_a = I_L + I_f \} I_s = \frac{R_d}{R_d + R_s} I_L$$
$$V_t = E_a - I_a R_a - I_s R_s \} V_f = E_a - I_a R_a$$
$$\text{mmf} = I_f N_f \pm I_s N_s - \text{mmf}_{fd}$$

Maximum Efficiency Criterion

For a PM dc generator, the power output is

$$P_o = V_t I_L \} P_{in} = V_t I_L + I_L^2 R_a + P_r$$

$$\eta = \frac{V_t I_L}{V_t I_L + I^2 R_a + P_r} \} I_{LM}^2 R_a = P_r \quad I_{LM} = \sqrt{\frac{P_r}{R_a}}$$

Max efficiency

Seperately excited: $I_{LM}^2 R_a = P_r + I_f^2 R_f$

Shunt : $I_{LM}^2 R_a = P_r + I_f^2 (R_a + R_f)$

Series: $I_{LM}^2 (R_a + R_s) = P_r$

Short-Shunt: $I_{LM}^2 (R_a + R_s) = P_r + I_f^2 (R_a + R_f)$

Long Shunt: $I_{LM}^2 (R_a + R_s) = P_r + I_f^2 (R_a + R_s + R_f)$

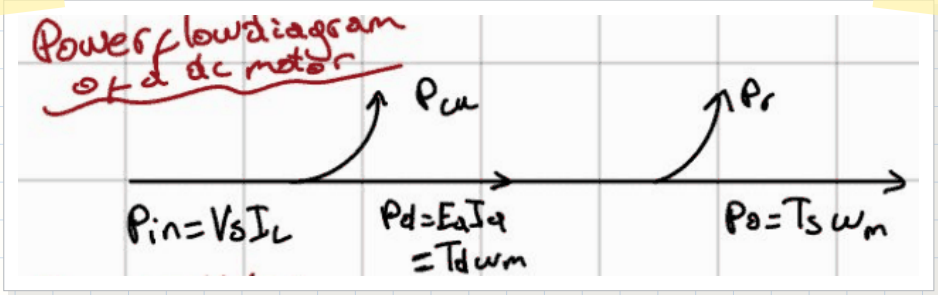
DC motor

$$T_d = k_a \phi_p I_a \} E_a = k_a \phi_p \omega_m$$

$$I_a = \frac{\overbrace{V_s - E_a}^{\text{Applied voltage across armature}}}{\underbrace{R}_{\text{total resistance at armature}}} \} V_s = E_a + I_a R$$

Speed Regulation

$$SR\% = \frac{N_{mnL} - N_{mFL}}{N_{mFL}} \times 100 = \frac{\omega_{mnL} - \omega_{mFL}}{\omega_{mFL}} \times 100$$



original drawing, p. 2

A series Motor

> *Figure:* Printed circuit: series motor (field R_s, N_s, rheostat R_{ax}, armature R_a, currents I_a, I_s, I_L, source V_s).
> *(in the original notes, p. 2)*

$$\phi_p = k_f I_a \} E_a = k_a k_f I_a \omega_m$$

$$T_d = K_a \phi_p I_a = k_a k_f I_a^2 \} \begin{matrix} E_a = V_s - I_a R \\ R = R_a + R_s + R_{ax} \end{matrix}$$

$$\omega_m = \frac{V_s - I_a R}{K_a k_f I_a} \Rightarrow I_a = \frac{V_s}{K_a k_f \omega_m + R}$$

$$P_d = E_a I_a = [V_s - I_a R_a] I_a$$

The max power developed when

$$P_{dm} = \frac{V_s^2}{4R} \quad \text{when} \quad I_{am} = \frac{V_s}{2R}$$

A shunt motor

$$T_d = K_a \Phi_p I_a = K I_a$$

> *Figure:* Printed circuit: shunt motor (starter rheostat R_{ax}, armature E_a, R_a, I_a, shunt field R_{fw}, N_f, R_{fx}, I_f, source V_s, I_L). *(in the original notes, p. 2)*

$$k_a \Phi_p = \frac{E_a}{\omega_m}$$

$$E_a = V_s - I_a R_a \;\} \; E_a = k_a \Phi_p \omega_m \;\} \; \omega_m = \frac{V_s - I_a R_a}{k_a \Phi_p}$$

$$\omega_m = \omega_{mnL} - \frac{R_a}{k_a \Phi_p} I_a \;\} \Rightarrow \omega_{mnL} = V_s / k_a \Phi_p$$

As $R_a \to 0$, the speed of shunt motor $\omega_m \to \omega_{mnL}$. In other words, a shunt motor is basically a constant speed motor.

$$T_d = \frac{k_a \Phi_p V_s}{R_a} - \frac{k_a^2 \Phi_p^2}{R_a} \omega_m \;\} \; P_{d\,max} = \frac{V_s^2}{4R_a}$$

$$I_{fnL} = \frac{V_s}{R_f} \qquad P_{cunL} = I_a^2 R_a + I_f^2 R_f$$

$$E_a = V_s - I_a R_a \;\} \; P_{in} = V_s I_L \;\} \; P_d = E_a \cdot I_a \;\} \; \eta = \frac{P_o}{P_{in}}$$

> *Figure:* Printed circuit: compound motor (starter resistor R_{ax}, series field winding R_s, N_s, shunt field winding R_{fw}, N_f, R_{fx}, armature E_a, R_a, T_s, ω_m, source V_s); the student marked the short shunt and long shunt connections in red. *(in the original notes, p. 2)*

$$\Phi_p = \Phi_{sh} \;\overset{?\ cum/diff}{\pm}\; k_f I_a$$

$$\Rightarrow T_d = k_a I_a \Phi_{sh} \pm k_a k_f I_a^2$$

$$\omega_m = \frac{V_s - I_a (R_a + R_s)}{k_a (\Phi_{sh} \pm k_f I_a)}$$

Methods of Speed Control

Armature Resistance Control

For series or a compound motor:

$$\omega_m = \frac{V_s - I_a R_a}{k_a \phi_p} \qquad \text{where } R = R_a + R_s + R_c$$

for a shunt motor

$$R = R_a + R_c$$

Field Control method

Inserting an external resistor in series with the field winding.

$$\underset{\text{new}}{I_{an}} \overset{\text{For same torque}}{=} \frac{\phi_p}{\phi_{pn}} I_a \;\} N_{mn} = \frac{\phi_p E_{an}}{\phi_{pn} E_a} N_m$$

$$P_{in} = V_s(I_{an} + I_{fn}) \qquad P_{dn} = E_{an} I_{an}$$

Braking or Reversing DC motors

Plugging: Reverse Supply connections to the armature terminals. At any time during the plugging,

$$I_a = \frac{V_s + E_a}{R + R_a} = \frac{V_s}{R + R_a} + \overset{E_a = k_a \phi_p \omega_m}{\frac{E_a}{R + R_a}} \leftarrow \text{external Res.}$$

So the braking torque:

$$T_b = K_a I_a \phi_p = k_1 \phi_p + k_2 \phi_p^2 \omega_m$$

$$k_1 = \frac{k_a V_s}{R + R_a} \qquad k_2 = \frac{K_a^2}{R + R_a} \qquad \text{for series motor}$$

For shunt: $T_b = k_3 + k_4 \omega_m$

$$k_3 = k_1 \phi_p \quad \text{and} \quad k_4 = k_2 \phi_p^2$$

When the speed reach 0, $T_b = k_3$.

Dynamic Braking

The armature disconnected from S. connected across a variable resistor R. The back emf produce a reverse current.

At any time in $t \geq 0$:

$$I_a = \frac{E_a}{R + R_a} = \frac{k_a \phi_p \omega_m}{R + R_a}$$

Braking torque:

$$T_b = k_a \phi_p I_a = \frac{k_a^2 \phi_p^2 \omega_m}{R + R_a} = k_2 \phi_p^2 \omega_m$$

$$\text{for series motor: } T_b = K_2 k_f^2 I_a^2 \omega_m \;\} \text{ for shunt: } T_b = k_4 \omega_m$$

Polyphase Induction Motors

The Synchronous Speed of the revolving field is

$$N_s = \frac{120f}{P} \Rightarrow \omega_s = \frac{4\pi f}{P} \qquad \text{(f of the current in the stator.)}$$

(slip) revolving field relative rotor speed

$$\left.\begin{aligned} N_r &= N_s - N_m \quad \omega_r = \omega_s - \omega_m \\ N_m &= (1-s)N_s \quad \omega_m = (1-s)\omega_s \end{aligned}\right\} s = \frac{N_r}{N_s} = \frac{\omega_r}{\omega_s} \quad \text{(per unit)}$$

$$\overset{\text{rotor emf}}{f_r} = \frac{PN_r}{120} = \frac{P(N_s - N_m)}{120} = \frac{PN_s}{120}\left[\frac{N_s - N_m}{N_s}\right]$$

$f_r = sf$ | At stand still, the slip is 1 and $f_r = f$

> *Figure:* Printed per-phase equivalent circuit of the induction motor with ideal transformer $a:1$ (R_1, jX_1, I_1, V_1, I_ϕ, R_c, jX_m, I_c, I_m, E_1; rotor side R_r, jX_r, I_r, E_r); labels re-written by hand, rotor branch marked R_r/s, jX_b ("de olabilir").
> *(in the original notes, p. 2)*

Per phase eq. circuit

$$\tilde{I}_r = \frac{\tilde{E}_r}{R_r + jX_r} = \frac{s\tilde{E}_b}{R_r + jsX_b} = \frac{\tilde{E}_b}{(R_r/s) + jX_b} \qquad a = \frac{N_1 k_{w1}}{N_2 k_{w2}}$$

per phase emf in rotor cct.

$$\tilde{E}_1 = \overset{4.44}{\frac{2\pi}{\sqrt{2}}} f N_1 \underset{\text{stator w. fac.}}{k_{w1}} \Phi_m$$

per phase induced emf in the stator winding.

$$\tilde{E}_b = \frac{2\pi}{\sqrt{2}} f N_2 k_{w2} \Phi_m$$

per phase induced emf in the rotor winding under blocked-rotor condition.

> *Figure:* Printed equivalent circuit referred to the stator (R_1, jX_1, R_2/s, jX_2, I_1, I_ϕ, I_2, R_c, jX_m, I_c, I_m, E_1); student's note: "E_1 reflected to stator".
> *(in the original notes, p. 2)*

$$R_2 = a^2 R_r \} X_2 = a^2 X_b \} \tilde{I}_2 = \frac{\tilde{I}_r}{a} \} \tilde{E}_1 = \frac{I_2 R_2}{s} + jI_2 X_2$$

$$I_\phi = I_c + I_m \} I_c = \frac{E_1}{R_c} \} I_m = \frac{E_1}{jX_m} \} I_1 = I_\phi + I_2$$

$$V_1 = E_1 + I_1(R_1 + jX_1) \} P_{dr} = 3I_2^2 R_2/s$$

Figure: Printed equivalent circuit with load resistance $\mathcal{R}_2(1-s)/s$ ($\mathcal{R}_1$, jX_1, $\mathcal{R}_2$, jX_2, I_1, I_ϕ (excitation), I_2, $\mathcal{R}_c$, jX_m, I_c, I_m, E_1).
(in the original notes, p. 2)

$$P_{in} = 3V_1 I_1 \cos\theta \;\}\; \underbrace{P_{scl}}_{\text{total stator cct.}} = 3I_1^2\mathcal{R}_1 \;\}\; \underbrace{P_m}_{\substack{\text{total core loss}\\ \text{(magnetic loss)}}} = 3I_c^2\mathcal{R}_c$$

$$\underbrace{P_{ag}}_{\text{air gap}} = P_{in} - P_{scl} - P_m = \frac{3I_2^2\mathcal{R}_2}{s} \;\}\; P_{rcl} = 3I_2^2\mathcal{R}_2 = sP_{ag}$$

$$P_d = P_{ag} - P_{rcl} = \frac{3I_2^2(1-s)\mathcal{R}_2}{s} = (1-s)P_{ag} \;\boxed{?\ \text{illegible}}\; sP_{ag}$$

$$S = 1 - s = \frac{N_m}{N_s} = \frac{\omega_m}{\omega_s}$$

Figure: Printed power-flow diagrams of the induction motor ($P_{in} = 3V_1I_1\cos\theta$, $P_{scl} = 3I_1^2\mathcal{R}_1$, $P_m = 3I_c^2\mathcal{R}_c$, $P_{ag} = 3I_2^2\mathcal{R}_2/s$, $P_{rcl} = 3I_2^2\mathcal{R}_2$, $P_d = (1-s)P_{ag}$, $P_o = T_s\omega_m$, $P_r = P_{fw} + P_{st}$ in the first diagram and $P_r = P_{fw} + P_{st} + P_m$ in the second), labels re-written by hand; notes: "when core loss sim. by $\mathcal{R}_c$" (first diagram) and "treated as a part of rot loss" (second diagram).
(in the original notes, p. 2)

Torque developed by the motor

$$T_d = \frac{P_d}{\omega_m} = \frac{P_{ag}}{\omega_s} = \frac{3I_2^2\mathcal{R}_2}{s\omega_s} \qquad \boxed{P_o = P_d - P_r}$$

Speed Torque Characteristic

When $s = 1$

$$I_2 = \frac{E_1}{\sqrt{\mathcal{R}_1^2 + X_2^2}}^{1}, \quad T_{ds} = \frac{3I_2^2\mathcal{R}_2}{\omega_s} \leftarrow \text{starting torque}$$

because $(\mathcal{R}_2 \ll X_2)$

$$I_2 \approx \frac{E_1}{X_2}$$

When the slip falls (below breakdown slip s_b), the hypothetical resistance becomes the dominating factor.

$\mathcal{R}_2/s \gg X_2$

$$I_2 = \frac{sE_1}{\mathcal{R}_2}$$ The torque developed by the motor is now proportional to the slip s

⋆ Y bağlantıdaki hat gerilimine V_L dersek faz gerilim $V_p = \frac{V_L}{\sqrt{3}}$ olur.

[1] Claude notu: muhtemelen $\mathcal{R}_2^2 + X_2^2$ olmalı.

The stator current

$$\tilde{I}_1 = \frac{\tilde{V}_1}{\tilde{Z}_{in}} = a\angle\theta \;\}\; \text{p.f} = \cos(\theta)$$

An approximate Eq. circuit

Figure: Printed approximate equivalent circuit of the induction motor (R_1, jX_1, R_2, jX_2, I_1, I_ϕ, I_2, R_c, jX_m, I_c, I_m, load $R_2(1-s)/s$); labels also written by hand.
(in the original notes, p. 2)

Maximum Power Criterion

$$R_e = R_1 + R_2 \;\}\; X_e = X_1 + X_2$$

$$Z_e = |R_e + jX_e| = \sqrt{R_e^2 + X_e^2} = \frac{R_2}{s_p}(1 - s_p)$$

P_d is max when the equivalent s_p load resistance is equal to the magnitude of the standstill impedance of the motor.

$$\Rightarrow s_p = \frac{R_2}{R_2 + Z_e} \Rightarrow P_{dm} = \frac{3}{2}\left[\frac{V_1^2}{R_e + Z_e}\right]$$

Maximum Torque Criterion

$$T_d = \frac{3V_1^2 R_2/s}{\left[R_e^2 + X_e^2 + \left[\frac{R_2(1-s)}{s}\right]^2 + \frac{2R_eR_2(1-s)}{s}\right]\omega_s}$$

$$\underset{\text{breakdown slip}}{s_b} = \frac{R_2}{\sqrt{R_1^2 + (X_1 + X_2)^2}}$$

$$T_{dm} = \frac{3V_1^2}{2\omega_s}\left[\frac{1}{R_1 + \sqrt{R_1^2 + (X_1 + X_2)^2}}\right] \;\}\; P_d = T_{dm}(1 - s_b)\omega_s$$

⋆ If the stator impedance is so small

When the stator $s_b = \frac{R_2}{X_2}$, $T_{dm} = \frac{3V_1^2}{2\omega_s}\left[\frac{1}{X_2}\right] = \frac{3V_1^2}{2\omega_s}\left[\frac{s_b}{R_2}\right]$

$$\tilde{I}_2 = \frac{\tilde{V}_1}{R_2/s + jX_2}, \quad T_d = \frac{3I_2^2R_2}{s\omega_s} = \left[\frac{3V_1^2}{(R_2/s)^2 + X_2^2}\right]\frac{R_2}{s\omega_s}$$

$$\frac{T_d}{T_{dm}} = \frac{2ss_b}{s^2 + s_b^2}$$

When the core loss is considered a part of the rotational loss the power input to the motor using the approximate circuit.

$$P_{in} = 3V_1I_2\cos\theta \;\leftarrow \text{pf angle} \;\}\; P_o = 3V_1I_2\cos\theta - 3I_2^2(R_1 + R_2) - P_r$$

The motor efficiency is

$$\eta = \frac{3V_1 I_2 \cos\theta - 3I_2^2(R_1 + R_2) - P_r}{\text{?illegible}}$$

$3I_2^2(R_1 + R_2) = P_r$ as the criterion for the maximum efficiency of an induction motor. (stator and rotor copper losses.)

Equivalent Circuit Parameters

The Stator resistance test:

$$R_1 = 0{,}5R_{\text{?dc}}\ Y \} R_1 = 1{,}5R\,\Delta$$

The blocked-Rotor Test:

$$\hat{Z}_e = R_1 + R_2 + j(X_1 + X_2) = R_e + jX_e$$

> *Figure:* Printed circuit for the blocked-rotor test (source $\tilde{V}_{br}$, current $\tilde{I}_{br}$, R_1, jX_1, R_2, jX_2); labels written by hand.
> *(in the original notes, p. 2)*

$$X_1 = X_2 = 0{,}5X_e$$

$$R_e = \frac{P_{br}}{I_{br}^2} \} R_2 = R_e - R_1$$

$$Z_e = \frac{V_{br}}{I_{br}} \} X_e = \sqrt{Z_e^2 - R_e^2}$$

The No-load test

W_{oc} = power input } I_{oc} input Current } V_{oc} rated applied voltage

Let P_{fw} be the friction and windage loss on a per-phase basis.

Then, the power loss in R_c is $P_{oc} = W_{oc} - P_{fw\phi}$

$$R_c = \frac{V_{oc}^2}{P_{oc}} \} \cos\theta_{oc} = \frac{W_{oc}}{V_{oc}I_{oc}} \} X_m = \frac{V_{oc}}{I_{oc}\sin\theta_{oc}}$$

$$S_{oc} = V_{oc}I_{oc} \} Q_{oc} = \sqrt{S_{oc}^2 - W_{oc}^2} \} X_m = \frac{V_{oc}^2}{Q_{oc}}$$

> *Figure:* Printed circuit for the no-load test (source $\tilde{V}_{oc}$, current $\tilde{I}_{oc}$, R_c, jX_m).
> *(in the original notes, p. 2)*

The load test