Görüntü İşleme

Filtering in the Frequency Domain (Ders 5)

Orijinal not: Ders-5.pdf

UYARI Bu belge, el yazısı ders notlarından Claude (Anthropic) yardımıyla dijitale aktarılmıştır. Yardımcı kaynak olarak hazırlanmıştır; hata veya eksik içerebilir. Lütfen orijinal notları ve ders kaynaklarını esas alın.

NOTICE *This document was transcribed from handwritten lecture notes with the help of Claude (Anthropic). It is meant as a supplementary resource and may contain errors or omissions. Always check against the original notes and official course material.*

Not: Orijinal dosyadaki basılı sayfalar (s. 5–6) dijitalleştirilmedi; yalnızca el yazısı sayfalar aktarıldı.

orj. s. 1

Filtring in the frequency domain

Fourier serisi :

Fourier dönüşümü

$$x(t) \longleftrightarrow X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi ft}\, dt$$

DSP ↘

$$x[n] \longleftrightarrow \underset{k=0,\ldots,N-1}{X[k]} = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi}{N}kn}$$

yeni notasyon

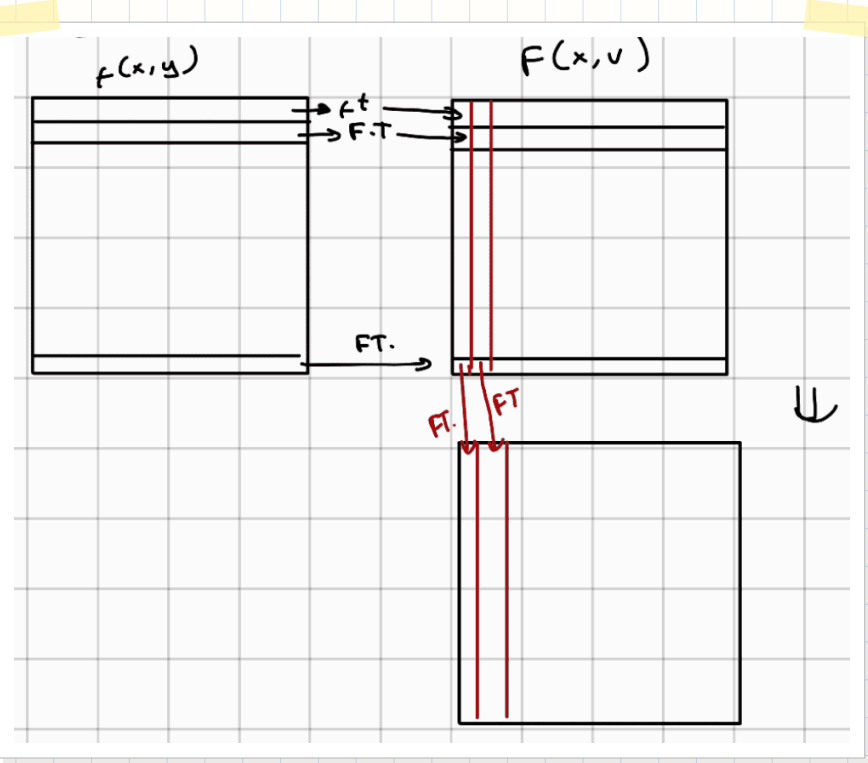
$$\underset{u=0,\ldots,N-1}{F[u]} = \sum_{x=0}^{N-1} f[x]\, e^{-j\frac{2\pi}{N}ux}$$

$$F[u,v] = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\, \underbrace{e^{-j\frac{2\pi}{M}ux}\, e^{-j\frac{2\pi}{N}vy}}_{e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}}$$

$$u = 0,\ldots,M-1$$
$$v = 0,\ldots,N-1$$

$$f(x,y) = \frac{?}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)\, e^{j\frac{2\pi}{M}ux}\, e^{j\frac{2\pi}{N}vy}$$

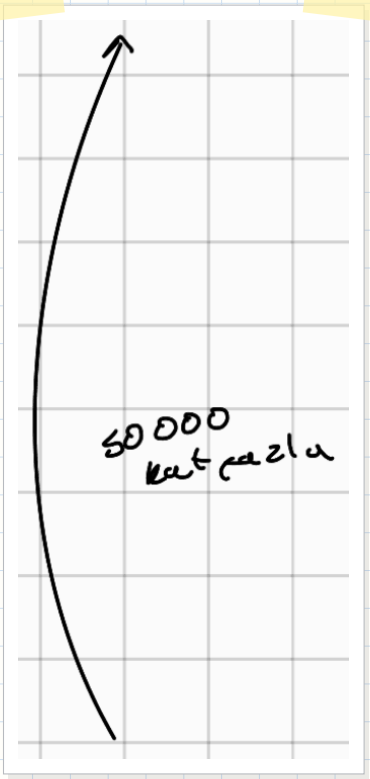
çok yavaş olduğu için aşağıdakini yapıyoruz



orijinal çizim, s. 1

Bir fourier dönüşümü $N \log_2(N)$

$M^2 \cdot N^2$ işlem yapılmalı



orijinal çizim, s. 1

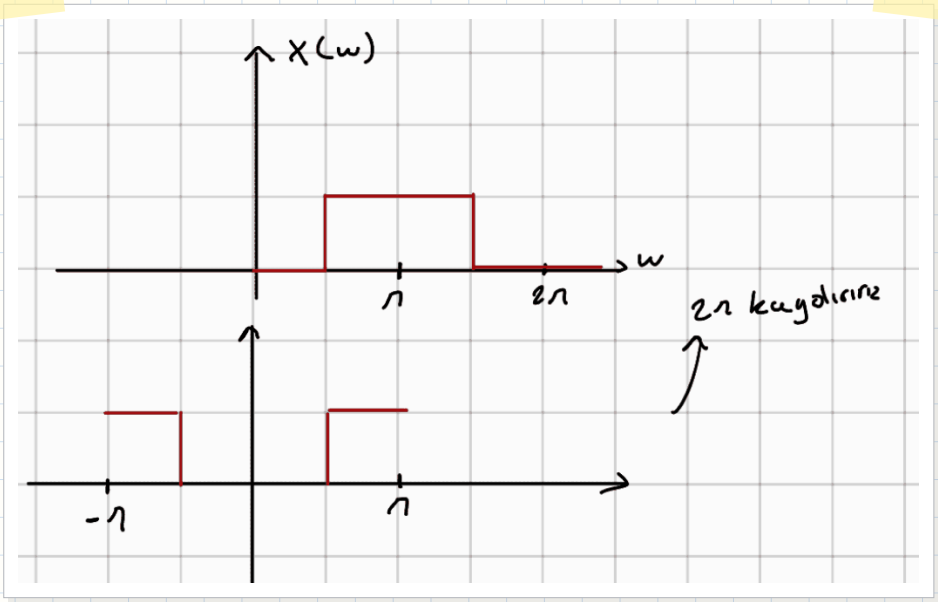
Toplam $2N^2 \log 2N$

$N = 1000 \Rightarrow$ 20 milyon

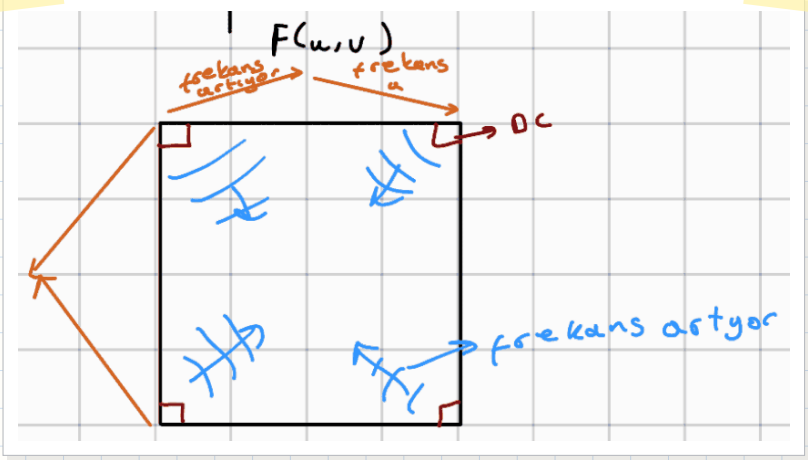
orj. s. 2

$$\underbrace{\sum_{x=0}^{M-1}\left[\underbrace{\left[\sum_{y=0}^{N-1} f(x,y)\, e^{-j\frac{2\pi}{N}vy}\right]}_{F(x,v)} e^{-j\frac{2\pi}{M}ux}\right]}_{F(u,v)}$$

$$f(x,y)\exp\left(2\pi j\left(\frac{u_0 x}{M}+\frac{v_0 y}{N}\right)\right) \longleftrightarrow F(u-u_0,\, v-v_0)$$

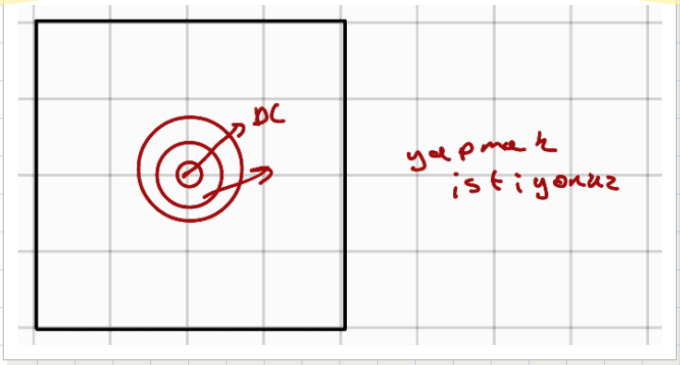


orijinal çizim, s. 2

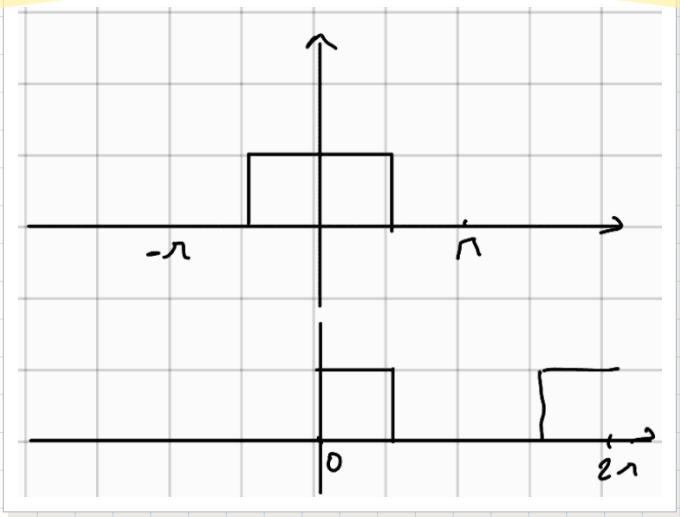


orijinal çizim, s. 2

↓ böyle yapsak



orijinal çizim, s. 2



orijinal çizim, s. 2

orj. s. 3

Merkezi nasıl taşıyacağız ?

$$f(x,y)\exp\left(2\pi j\left(\frac{u_0 x}{M}+\frac{v_0 y}{N}\right)\right) \longleftrightarrow F(u-u_0, v-v_0)$$

$$u_0 = \frac{M}{2}, \quad v_0 = \frac{N}{2}$$

$$e^{j2\pi\frac{M}{2}\frac{x}{M}} f(x,y) \longleftrightarrow \sum_{x=0}^{M-1}\left[e^{j2\pi\frac{M}{2}\frac{x}{M}} f(x)\right] e^{-j\frac{2\pi}{M}ux}$$

$$= \sum_{x=0}^{M-1} f(x)\, e^{-j\frac{2\pi}{M}\left(\frac{u}{M}-\underbrace{\left(\frac{M}{2}\right)}_{u_0}\right)x}$$

$$\sum_{x=0}^{M-1}\left[f(x)\, e^{j2\pi\frac{u_0 x}{M}}\right] e^{-j\frac{2\pi}{M}ux} = \sum_{x=0}^{M-1} f(x)\, e^{-j\frac{2\pi}{M}(u-u_0)x}$$

$$= F(u-u_0)$$

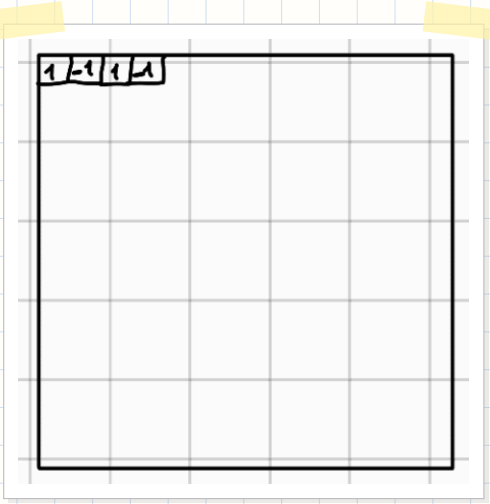
$$u_0 = \frac{M}{2}$$

$$f(x) = e^{j2\pi \frac{\frac{M}{2}x}{M}}$$

$$f(x)\left(\underbrace{e^{j\pi}}_{-1}\right)^x$$

$$f(x)(-1)^x \longleftrightarrow F\left(u - \frac{M}{2}\right)$$

$$f(x,y)(-1)^{x+y} \longleftrightarrow F\left(u - \frac{M}{2}, v - \frac{N}{2}\right)$$



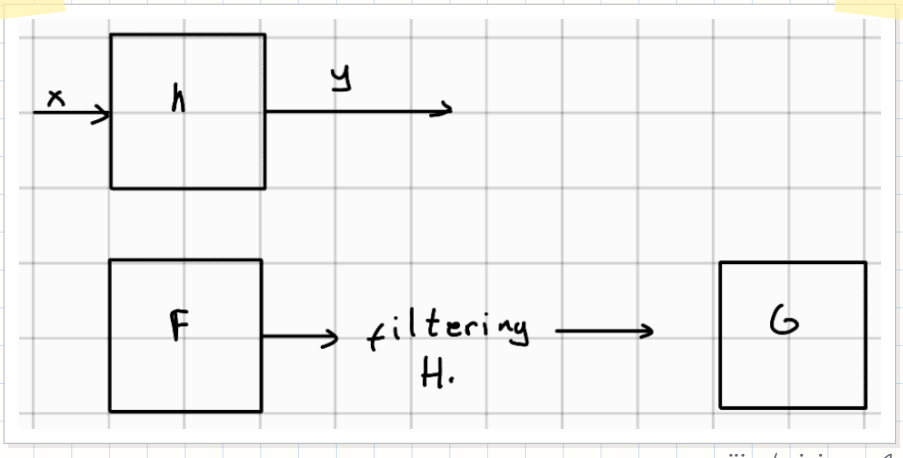
orijinal çizim, s. 3

giriş realse fourier konjüge simetriktir

if $f(x,y)$ is real , $F(u,v) = F^*(-u,-v)$

$$|F(u,v)| = |F^*(-u,-v)|$$

orj. s. 4

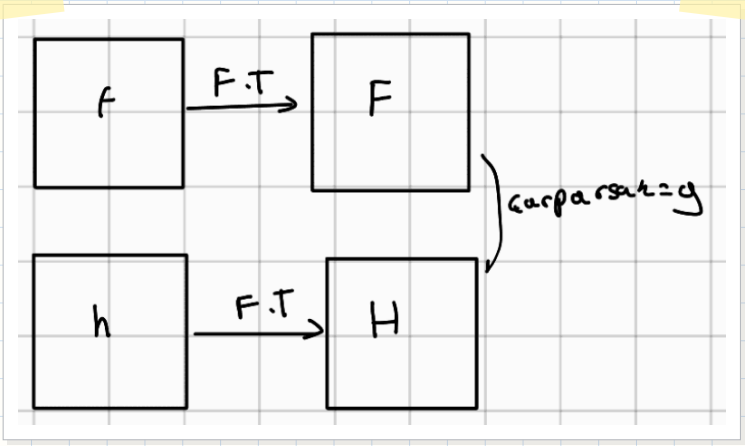


orijinal çizim, s. 4

$f * h = g$ $f \circ h = g$

frekans domaini ↘

$$F \cdot H = G$$

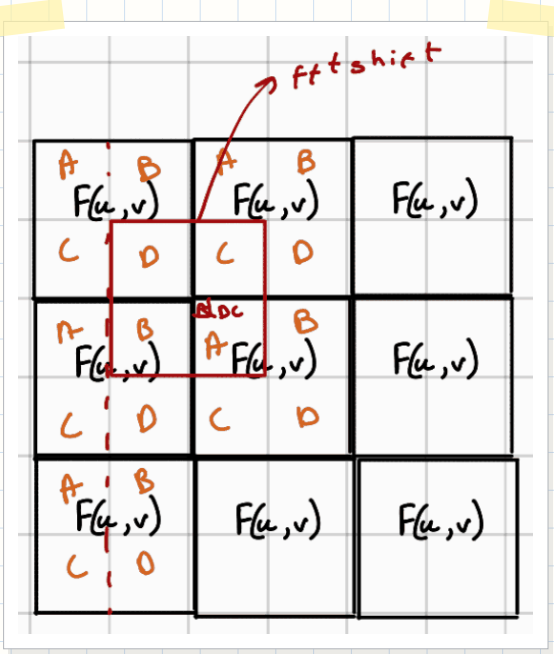


orijinal çizim, s. 4

Power spectrum $P(u,v) = |F(u,v)|^2$

Average value $\bar{f}(x,y) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y) = \frac{1}{MN}F(0,0)$

⋆ Periodicity $F(u,v) = F(u + k_1M,\ v + k_2N)$



orijinal çizim, s. 4