İşaretler ve Sistemler

Ders 11 – Soru çözümleri (DTFT ile konvolüsyon ve fark denklemi)

Orijinal not: Isaret-ve-Sistemler-ders-11_Soru.pdf

UYARI Bu belge, el yazısı ders notlarından Claude (Anthropic) yardımıyla dijitale aktarılmıştır. Yardımcı kaynak olarak hazırlanmıştır; hata veya eksik içerebilir. Lütfen orijinal notları ve ders kaynaklarını esas alın.

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Örnek

orj. s. 1

$x[n] = \alpha^n u[n], \quad |\alpha| < 1 \qquad y[n] = ?$

$h[n] = \beta^n u[n], \quad |\beta| < 1$

$$y[n] = x[n] * h[n] = \mathcal{F}^{-1}\{Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega})\}$$

Reminder: $\alpha^n u[n] \overset{\mathcal{F}}{\longleftrightarrow} \dfrac{1}{1-\alpha e^{-j\omega}}$

$$Y(e^{j\omega}) = \frac{1}{(1-\alpha e^{-j\omega})(1-\beta e^{-j\omega})} = \frac{A}{1-\alpha e^{-j\omega}} + \frac{B}{1-\beta e^{-j\omega}}$$

$$\updownarrow \mathcal{F}^{-1}$$

$$y[n] = A\alpha^n u[n] - B\beta^n u[n]$$[1]

$$A = \frac{\alpha}{\alpha-\beta} \qquad B = \frac{-\beta}{\alpha-\beta}$$

$$y[n] = \frac{1}{\alpha-\beta}\left[\alpha\,\alpha^n u[n] - \beta\,\beta^n u[n]\right] = \frac{1}{\alpha-\beta}(\alpha^{n+1} - \beta^{n+1})u[n]$$

Örnek

$\alpha = \beta$ ise $y[n] = ?$

$$Y(e^{j\omega}) = \frac{1}{(1-\alpha e^{-j\omega})^2}$$

Reminder: $x[n] = \alpha^n u[n] \overset{\mathcal{F}}{\longleftrightarrow} X(e^{j\omega}) = \dfrac{1}{1-\alpha e^{-j\omega}}$

$$n\,x[n] \overset{\mathcal{F}}{\longleftrightarrow} j\frac{dX(e^{j\omega})}{d\omega}$$

$$x[n-n_0] \overset{\mathcal{F}}{\longleftrightarrow} e^{-j\omega n_0} X(e^{j\omega})$$

$$\frac{dX(e^{j\omega})}{d\omega} = -j\alpha e^{-j\omega}\frac{1}{(1-\alpha e^{-j\omega})^2} = -j\alpha e^{-j\omega}\,Y(e^{j\omega})$$

[1] Claude notu: $B = -\beta/(\alpha-\beta)$ alındığı için muhtemelen $y[n] = A\alpha^n u[n] + B\beta^n u[n]$ olmalı (sonuç doğru).

$$Y(e^{j\omega}) = \frac{1}{\alpha}\, e^{j\omega}\, \underbrace{j\frac{dX(e^{j\omega})}{d\omega}} \xleftrightarrow{\mathcal{F}^{-1}} y[n] = \frac{1}{\alpha}(n+1)\alpha^{n+1}u[n+1] = (n+1)\alpha^{n}u[n]$$

$e^{j\omega}$: Zamanda öteleme ($\downarrow$)

$j\frac{dX(e^{j\omega})}{d\omega}$: Frekansta türev ($\downarrow$ $n\alpha^n u[n]$)

$(n+1)\alpha^n u[n]$: $y[-1]=0$ ($\downarrow$)

$$(n+1)\alpha^n u[n] \xleftrightarrow{\mathcal{F}} \frac{1}{(1-\alpha e^{-j\omega})^2}$$

Örnek

orj. s. 2

$y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = 2x[n]$ ise $h[n] = ?$

$$Y(e^{j\omega}) - \frac{3}{4}e^{-j\omega}Y(e^{j\omega}) + \frac{1}{8}e^{-j2\omega}Y(e^{j\omega}) = 2X(e^{j\omega})$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{2}{1 - \frac{3}{4}e^{-j\omega} + \frac{1}{8}e^{-j2\omega}} = \frac{A}{1-\frac{1}{2}e^{-j\omega}} + \frac{B}{1-\frac{1}{4}e^{-j\omega}}, \qquad A = 4 \quad B = -2$$

$$\updownarrow \mathcal{F}^{-1}$$

$$h[n] = 4\left(\frac{1}{2}\right)^n u[n] - 2\left(\frac{1}{4}\right)^n u[n]$$