İşaretler ve Sistemler

12. hafta – DTFT, Laplace dönüşümü ve ROC

Orijinal not: Sinyaller-ve-sistemler-12hafta.pdf

UYARI Bu belge, el yazısı ders notlarından Claude (Anthropic) yardımıyla dijitale aktarılmıştır. Yardımcı kaynak olarak hazırlanmıştır; hata veya eksik içerebilir. Lütfen orijinal notları ve ders kaynaklarını esas alın.

NOTICE *This document was transcribed from handwritten lecture notes with the help of Claude (Anthropic). It is meant as a supplementary resource and may contain errors or omissions. Always check against the original notes and official course material.*

orj. s. 1

Tekrar

s Laplace

$s = j\omega$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

z-Transform

$z = e^{j\omega}$

$X(e^{j\omega})$

Ayrık zamanlı Fourier transformu (Dönüşümü)

(Discrete-time Fourier Transform)

FT, periyodik olmayan işaretler için tanımlıdır.

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n} \qquad (n: \text{ tamsayı})$$

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$x[n] \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftarrows}} X(e^{j\omega})$$

$\Rightarrow$ Ayrık zamanlı FT, $X(e^{j\omega})$, 2π ile periyodiktir.

$$X(e^{j\omega}) = X(e^{j(\omega + 2\pi)})$$

Örnek

$x[n] = a^n u[n]$, $|a| < 1$ ise $X(e^{j\omega}) = ?$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n} \underset{\uparrow}{=} \sum_{n=0}^{\infty} a^n e^{-j\omega n} = \sum_{n=0}^{\infty} (ae^{-j\omega})^n \underset{\uparrow}{=} \frac{1}{1 - ae^{-j\omega}}$$

Örnek

$x[n] = a^{|n|}$, $|a| < 1$ ise $X(e^{j\omega}) = ?$

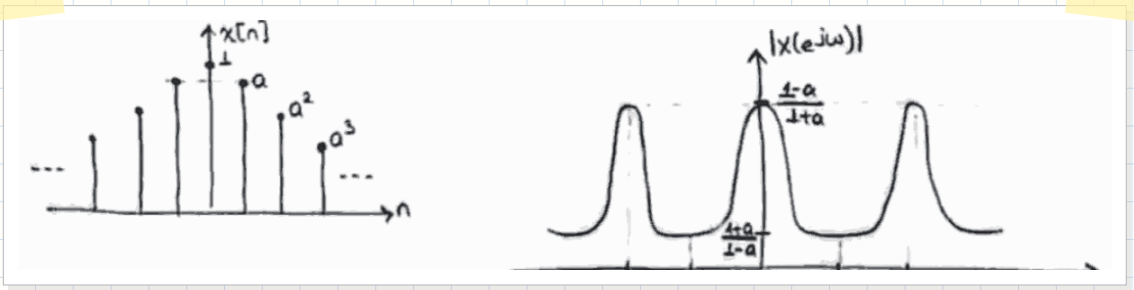
$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\,e^{-j\omega n} = \sum_{n=-\infty}^{\infty} a^{|n|}e^{-j\omega n} = \sum_{n=-\infty}^{-1} a^{-n}e^{-j\omega n} + \sum_{n=0}^{\infty} a^{n}e^{-j\omega n}$$

$$\sum_{n=-\infty}^{-1} (ae^{j\omega})^{-n} \underset{\uparrow\, k=-n}{=} \sum_{k=1}^{\infty} (ae^{j\omega})^{k} \underset{\uparrow\, (ae^{j\omega})^0=1}{=} \sum_{k=0}^{\infty} (ae^{j\omega})^{k} - 1$$

$$= \frac{1}{1-ae^{-j\omega}} - 1 = \frac{ae^{j\omega}}{1-ae^{j\omega}} \quad [1]$$

$$X(e^{j\omega}) = \frac{ae^{j\omega}}{1-ae^{j\omega}} + \frac{1}{1-ae^{-j\omega}} = \frac{1-a^2}{1+a^2-2a\cos\omega} \quad : \text{Reel}$$

$$X(e^{j\omega}) = |X(e^{j\omega})|, \qquad \angle X(e^{j\omega}) = 0$$



orijinal çizim, s. 1

orj. s. 2

Örnek

$x[n] = \begin{cases} 1, & |n| \le N_1 \\ 0, & |n| > N_1 \end{cases}$ şeklinde verilen ayrık zamanlı dikdörtgen darbe işaretinin FT bulunuz.

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\,e^{-j\omega n} = \sum_{n=-N_1}^{N_1} e^{-j\omega n}$$

$$= \sum_{n=0}^{2N_1} e^{-j\omega(n-N_1)} = e^{j\omega N_1} \sum_{n=0}^{2N_1} e^{-j\omega n}$$

$$= e^{j\omega N_1}\left[\frac{1-e^{-j\omega(2N_1+1)}}{1-e^{-j\omega}}\right]$$

$$= e^{j\omega N_1}\,\frac{e^{-j\omega(N_1+1/2)}\left[e^{j\omega(N_1+1/2)} - e^{-j\omega(N_1+1/2)}\right]}{e^{-j\omega/2}\left[e^{j\omega/2} - e^{-j\omega/2}\right]}$$

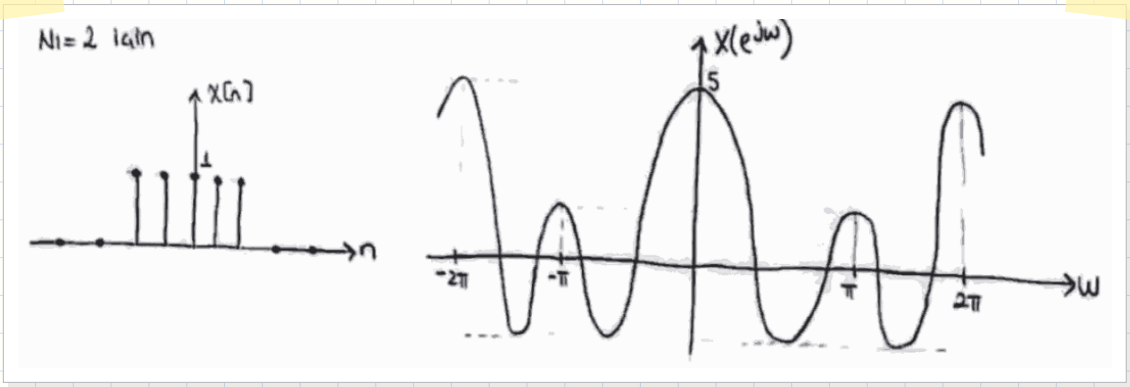
$$= \frac{\sin\left(\omega(N_1+1/2)\right)}{\sin(\omega/2)}$$

[1] Claude notu: muhtemelen $\frac{1}{1-ae^{j\omega}} - 1$ olmalı.

Reminder:

$$\sum_{k=0}^{M} \alpha^k = \frac{1-\alpha^{M+1}}{1-\alpha}, \qquad 1 - e^{j\theta} = e^{-j\theta/2}\left[e^{j\theta/2} - e^{-j\theta/2}\right] \quad 2$$

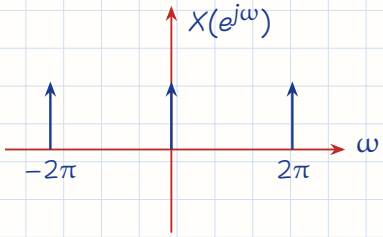
$N_1 = 2$ için



orijinal çizim, s. 2

★ $\delta[n] \overset{\mathcal{F}}{\longleftrightarrow} 1$

$1 \longleftrightarrow 2\pi\delta(e^{j\omega})$



$$\delta[n-n_0] \longleftrightarrow e^{-j\omega n_0}$$

$$e^{j\omega_0 n} \overset{\mathcal{F}}{\longleftrightarrow} 2\pi\delta\left(e^{j(\omega-\omega_0)}\right)$$

$$e^{-j\omega_0 n} \overset{\mathcal{F}}{\longleftrightarrow} 2\pi\delta\left(e^{j(\omega+\omega_0)}\right)$$

orj. s. 3

Periyodik işaretlerin Fourier Transformu

$$x[n] = \sum_{k=\langle N\rangle} x_k e^{jk\omega_0 n} = x_0 + x_1 e^{j\omega_0 n} + x_2 e^{j2\omega_0 n} + \cdots +$$

$$x_k = \frac{1}{N}\sum_{n=\langle N\rangle} x[n] e^{-jk\omega_0 n}, \qquad x_k = x_{k+N}$$

$$x[n] = \sum_{k=\langle N\rangle} x_k e^{jk\omega_0 n} \overset{\mathcal{F}}{\longleftrightarrow} X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} 2\pi x_k \delta(\omega - k\omega_0)$$

orj. s. 4

[2] Claude notu: muhtemelen $1 - e^{-j\theta}$ olmalı.

Ayrık Zamanlı Fourier Transformunun Özellikleri

$$x[n] \overset{\mathcal{F}}{\longleftrightarrow} X(e^{j\omega})$$
$$X(e^{j\omega}) = \mathcal{F}\{x[n]\}$$
$$x[n] = \mathcal{F}^{-1}\{X(e^{j\omega})\}$$

1) Periyodiklik: $X(e^{j\omega}) = X(e^{j(\omega+2\pi)})$, 2π ile periyodiktir.
2) Lineerlik: $\alpha x_1[n] + \beta x_2[n] \overset{\mathcal{F}}{\longleftrightarrow} \alpha X_1(e^{j\omega}) + \beta X_2(e^{j\omega})$
3) Zamanda öteleme: $x[n-n_0] \overset{\mathcal{F}}{\longleftrightarrow} e^{-j\omega n_0} X(e^{j\omega})$
   Genlik spektrumu değişmez, faz spektrumu $-\omega n_0$ kadar ötelenir.
4) Frekansta öteleme: $e^{j\omega_0 n} x[n] \overset{\mathcal{F}}{\longleftrightarrow} X(e^{j(\omega-\omega_0)})$
5) Zamanda katlama: $x[-n] \overset{\mathcal{F}}{\longleftrightarrow} X(e^{-j\omega})$
6) Eşlenik alma: $x^*[n] \overset{\mathcal{F}}{\longleftrightarrow} X^*(e^{-j\omega})$
7) Fark alma (differencing): $x[n] - x[n-1] \overset{\mathcal{F}}{\longleftrightarrow} (1-e^{-j\omega})X(e^{j\omega})$
8) Birikim (Accumulation):

$$\sum_{m=-\infty}^{n} x[m] \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{1-e^{-j\omega}} X(e^{j\omega}) + \pi X(e^{j0}) \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$$

9) Frekansta türev: $n\,x[n] \overset{\mathcal{F}}{\longleftrightarrow} j\frac{dX(e^{j\omega})}{d\omega}$

   İspat: $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\,e^{-j\omega n}$

$$\frac{d}{d\omega}X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\frac{d}{d\omega}(e^{-j\omega n}) = -jn\sum_{n=-\infty}^{\infty} x[n]\,e^{-j\omega n}$$ [3]

10) Parseval teoremi: $E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{2\pi} |X(e^{j\omega})|^2\,d\omega$
11) Çarpma:

$$x_1[n]\cdot x_2[n] \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{2\pi}\int_{2\pi} X_1(e^{j\alpha})X_2(e^{j(\omega-\alpha)})\,d\alpha$$

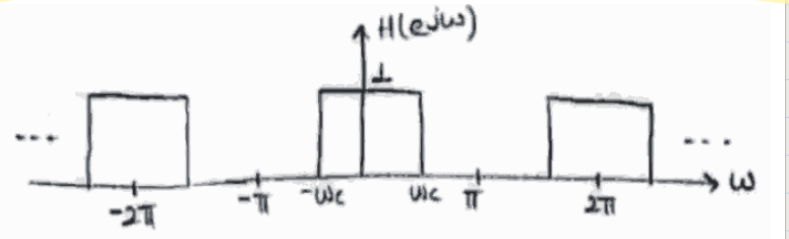
orj. s. 5

Konvülasyon

Örnek

$H(e^{j\omega}) = \begin{cases} 1, & 0 \le |\omega| \le \omega_c \\ 0, & \omega_c < |\omega| < \pi \end{cases}$ ayrık zamanlı alçak geçiren filtrenin dürtü cevabını bulunuz.

[3] Claude notu: $-jn$ çarpanı toplamın içinde olmalı: $\sum_n (-jn)\,x[n]\,e^{-j\omega n}$.



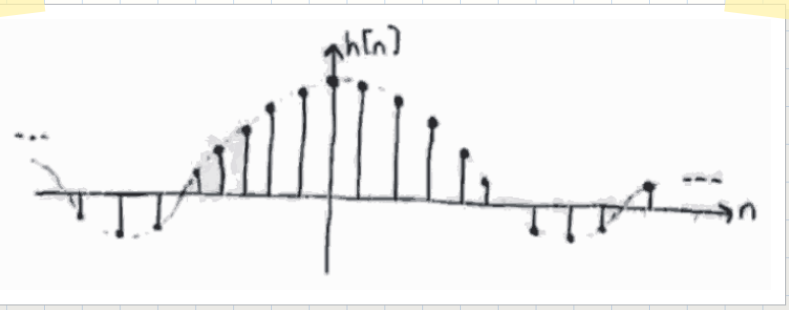
orijinal çizim, s. 5

$$h[n] = \frac{1}{2\pi}\int_{2\pi} H(e^{j\omega})\, e^{+j\omega n}\, d\omega = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\,\frac{e^{j\omega n}}{jn}\Big|_{-\omega_c}^{\omega_c} = \frac{1}{2j\pi n}\left[e^{j\omega_c n} - e^{-j\omega_c n}\right],$$

$$h[n] = \frac{1}{\pi n}\sin(\omega_c n) = \frac{\sin\left(\pi\frac{\omega_c}{\pi}n\right)}{\pi\frac{\omega_c}{\pi}n}\cdot\frac{\omega_c}{\pi} = \frac{\omega_c}{\pi}\,\mathrm{sinc}\left(\frac{\omega_c}{\pi}n\right)$$

↖ tam sayı olunca 0'ı yani n eksenini keser



orijinal çizim, s. 5

Örnek

$x[n] = \alpha^n u[n],\ |\alpha| < 1,\ y[n] = ?$
$h[n] = \beta^n u[n],\ |\beta| < 1$

Reminder: $\alpha^n u[n] \overset{\mathcal{F}}{\longleftrightarrow} \dfrac{1}{1-\alpha e^{-j\omega}}$

$$Y(e^{j\omega}) \underset{\mathcal{F}^{-1}}{\updownarrow} \frac{1}{(1-\alpha e^{-j\omega})(1-\beta e^{-j\omega})} = \frac{A}{1-\alpha e^{-j\omega}} + \frac{B}{1-\beta e^{-j\omega}}$$

$$y[n] = A\alpha^n u[n] + B\beta^n u[n]$$

orj. s. 6

$$A = \frac{\alpha}{\alpha-\beta} \qquad B = \frac{-\beta}{\alpha-\beta}$$

$$y[n] = \frac{1}{\alpha-\beta}\left[\alpha\,\alpha^n u[n] - \beta\,\beta^n u[n]\right]$$

$$= \frac{1}{\alpha-\beta}\left(\alpha^{n+1} - \beta^{n+1}\right) u[n]$$

Örnek

$\alpha = \beta$ ise $y[n] = ??$

$$Y(e^{j\omega}) = \frac{1}{(1-\alpha e^{-j\omega})^2}$$

Reminder:

$$x[n] = \alpha^n u[n] \overset{\mathcal{F}}{\longleftrightarrow} X(e^{j\omega}) = \frac{1}{1-\alpha e^{-j\omega}}$$

$$n x[n] \overset{\mathcal{F}}{\longleftrightarrow} j\frac{dX(e^{j\omega})}{d\omega}$$

$$x[n-n_0] \overset{\mathcal{F}}{\longleftrightarrow} e^{-j\omega n_0} X(e^{j\omega})$$

$$\frac{dX(e^{j\omega})}{d\omega} = -j\alpha e^{-j\omega}\frac{1}{(1-\alpha e^{-j\omega})^2} = -j\alpha e^{-j\omega} Y(e^{j\omega})$$

$$Y(e^{j\omega}) = \frac{1}{\alpha} e^{j\omega} j\frac{dX(e^{j\omega})}{d\omega} \overset{\mathcal{F}^{-1}}{\longleftrightarrow} \begin{aligned} y[n] &= \frac{1}{\alpha}(n+1)\alpha^{n+1}u[n+1] \\ &= (n+1)\alpha^n u[n] \\ &\uparrow \\ &y[-1]=0 \end{aligned}$$

$$(n+1)\alpha^n u[n] \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{(1-\alpha e^{-j\omega})^2}$$

Örnek

$y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = 2x[n]$ ise $h[n] = ?$

$$Y(e^{j\omega}) - \frac{3}{4}e^{-j\omega}Y(e^{j\omega}) + \frac{1}{8}e^{-j2\omega}Y(e^{j\omega}) = 2X(e^{j\omega})$$

$$\underset{\updownarrow \mathcal{F}^{-1}}{H(e^{j\omega})} = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{2}{1-\frac{3}{4}e^{-j\omega}+\frac{1}{8}e^{-j2\omega}} = \frac{A}{1-\frac{1}{2}e^{-j\omega}} + \frac{B}{1-\frac{1}{4}e^{-j\omega}}$$

$$A = 4 \qquad B = -2$$

$$h[n] = 4\left(\frac{1}{2}\right)^n u[n] - 2\left(\frac{1}{4}\right)^n u[n]$$

orj. s. ?

Laplace Transformu

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

$$x(t) \overset{\mathcal{L}}{\longleftrightarrow} X(s)$$

$$X(s) = \mathcal{L}\{x(t)\}$$

$$\Rightarrow s = \sigma + j\omega \quad \begin{cases} \rightarrow & \sigma : \text{Reel kısmı} \\ \hookrightarrow & \omega : \text{Sanal kısmı} \end{cases}$$

$$X(\sigma + j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-(\sigma+j\omega)t}\, dt$$

$$= \int_{-\infty}^{\infty} x(t)\, e^{-\sigma t}\, e^{-j\omega t}\, dt = \mathcal{F}\left\{x(t)\, e^{-\sigma t}\right\}$$

$\Rightarrow s = j\omega$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt = \mathcal{F}\{x(t)\} = X(s)\Big|_{s=j\omega}$$

s herhangi bir complex sayı olabilir.

Laplace transformu, $x(t)e^{-\sigma t}$ şeklinde $x(t)$ işaretinin bir reel üstelle çarpımının F. Transformudur.

$$e^{-\sigma t} \rightarrow \begin{array}{ll} \sigma > 0 & \text{zamanda azalan fonk.} \\ \sigma < 0 & \text{"} \quad \text{artan fonk.} \end{array}$$

Örnek

$x(t) = e^{-at}u(t)$

$$\begin{aligned} X(s) &= \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt = \int_{-\infty}^{\infty} e^{-at}u(t)\, e^{-st}\, dt \\ &= \int_{0}^{\infty} e^{-(a+s)t}\, dt = \frac{e^{-(a+s)t}}{-(a+s)}\Big|_0^{\infty} \\ &= \frac{1}{s+a}, \quad \mathrm{Re}\{s+a\} > 0 \end{aligned}$$

$$s = \sigma + j\omega \rightarrow X(\sigma + j\omega) = \frac{1}{(\sigma + a) + j\omega}, \quad \sigma + a > 0$$

$$s = j\omega,\ \sigma = 0 \rightarrow X(j\omega) = \frac{1}{a + j\omega}, \quad a > 0$$

$$e^{-at}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+a}, \quad \mathrm{Re}\{s\} > -a \ \triangle$$

$$u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s}, \quad \mathrm{Re}\{s\} > 0 \qquad (a = 0)$$

orj. s. 8

Örnek

$x(t) = -e^{-at}u(-t)$

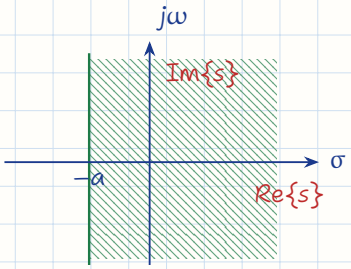
$$\begin{aligned} X(s) &= \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt = -\int_{-\infty}^{\infty} e^{-at}u(-t)\, e^{-st}\, dt \\ &= \int_{-\infty}^{0} e^{-(a+s)t}\, dt = \frac{1}{s+a}, \quad \mathrm{Re}\{s+a\} < 0 \end{aligned}$$

[4]

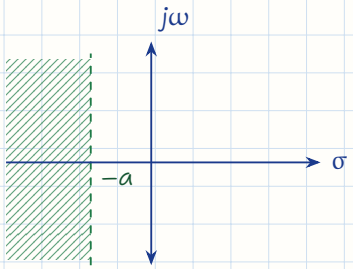
$$-e^{-at}u(-t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+a}, \quad \mathrm{Re}\{s\} < -a \ \star$$

$\Rightarrow$ Yakınsaklık bölgesi (Region of converge, ROC): Laplace transform integralinin yakınsaklığı değer aralığına yakınsaklık bölgesi denir.

[4] Claude notu: ikinci satırda integralin önünde – işareti olmalı: $-\int_{-\infty}^{0} e^{-(a+s)t}\, dt = \frac{1}{s+a}$.



$$\mathcal{L}\{e^{-at}u(t)\} = \frac{1}{s+a}, \quad \mathrm{Re}\{s\} > -a$$



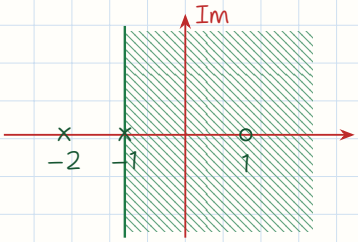
$$\mathcal{L}\{-e^{-at}u(-t)\} = \frac{1}{s+a}, \quad \mathrm{Re}\{s\} < -a$$

Örnek

$x(t) = 3e^{-2t}u(t) - 2e^{-t}u(t)$

$$e^{-2t}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+2}, \quad \mathrm{Re}\{s\} > -2$$
$$e^{-t}u(t) \longleftrightarrow \frac{1}{s+1}, \quad \mathrm{Re}\{s\} > -1$$

$$X(s) = \frac{3}{s+2} - \frac{2}{s+1} = \frac{s-1}{s^2+3s+2}, \quad \mathrm{Re}\{s\} > -1$$



× : pole (kutup)

○ : zero (sıfır)

s düzlemi kutup sıfır çizimi

orj. s. 9

Örnek

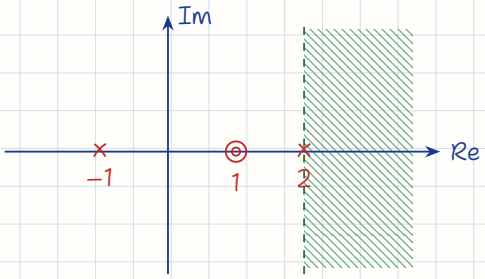
$\delta(t) - \frac{4}{3}e^{-t}u(t) + \frac{1}{2}e^{2t}u(t)$

$$\mathcal{L}\{\delta(t)\} = \int_{-\infty}^{\infty} \delta(t)\,e^{-st}\,dt = \int_{-\infty}^{\infty} \delta(t)\,e^{-s0}\,dt$$
$$= \int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

$$\delta(t) \overset{\mathcal{L}}{\longleftrightarrow} 1, \quad \forall s \ \text{(ROC, bütün s-düzlemi)}$$
$$e^{-t}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+1}, \quad \mathrm{Re}\{s\} > -1$$
$$e^{2t}u(t) \longleftrightarrow \frac{1}{s-2}, \quad \mathrm{Re}\{s\} > 2$$

$$X(s) = 1 - \frac{4}{3}\frac{1}{s+1} + \frac{1}{2}\frac{1}{s-2} = \frac{(s-1)^2}{(s+1)(s-2)}, \quad \mathrm{Re}\{s\} > 2$$ [5]

[5] Claude notu: eşitliğin sağ tarafı için $e^{2t}u(t)$ katsayısı muhtemelen $\frac{1}{3}$ olmalı (notun devamında da $\frac{1}{3}e^{2t}u(t)$ yazılmış).



⇒ Aynı noktada tekrar eden s sayısına mertebe denir. Paydada, $s = -1$ ve $s = 2$ olmak üzere birinci mertebeden iki adet kutup ve payda[6] $s = 1$'de ikinci mertebeden bir adet sıfır vardır.

Not: Laplace transformun ROC'u $j\omega$ eksenini kesmiyorsa Fourier transformu yakınsamaz. $\frac{1}{3}e^{2t}u(t)$ ifadesinin Fourier transformu yoktur.

Yakınsama bölgesi (ROC) özellikleri:

1) $X(s)$'in ROC'u s-düzlemindeki $j\omega$ eksenine paralel şeritlerden oluşur.

$$X(s) = X(\sigma + j\omega)$$

orj. s. 10

2) Rasyonel Laplace transform için, ROC herhangi bir kutup içermez.

3) $x(t)$ sonlu süreli ve mutlak entegre edilebilir ise, ROC bütün s düzlemidir.

$$\int_{T_1}^{T_2} |x(t)|\,dt < \infty \qquad \int_{T_1}^{T_2} |x(t)|\,e^{-\sigma t}\,dt < \infty \quad \leftarrow \text{1) özellik}$$

→ $\mathrm{Re}\{s\} = \sigma = 0$ iken $s = j\omega$ olup ROC içinde kalır.

→ $\mathrm{Re}\{s\} = \sigma > 0$ iken

$$\int_{T_1}^{T_2} |x(t)|\,e^{-\sigma t}\,dt < e^{-\sigma T_1} \int_{T_1}^{T_2} |x(t)|\,dt < \infty$$

olur ve $\mathrm{Re}\{s\} > 0$ s düzlemi ROC içinde kalır

→ $\mathrm{Re}\{s\} = \sigma < 0$ iken

$$\int_{T_1}^{T_2} |x(t)|\,e^{-\sigma t}\,dt < e^{-\sigma T_2} \int_{T_1}^{T_2} |x(t)|\,dt < \infty$$

dur ve $\mathrm{Re}\{s\} < 0$ s düzlemi ROC içinde kalır.

Örnek

$$x(t) = \begin{cases} e^{-at}, & 0 < t < T \\ 0, & \text{dışında} \end{cases}$$

$$X(s) = \int_{-\infty}^{\infty} x(t)\,e^{-st}\,dt = \int_{0}^{T} e^{-(a+s)t}\,dt = \frac{1}{s+a}\left(1 - e^{-(a+s)T}\right)$$

← 3)'e göre ROC bütün s düzlemi

← △ 2)'e göre ROC kutup içermez, $s = -a$ kutup var.

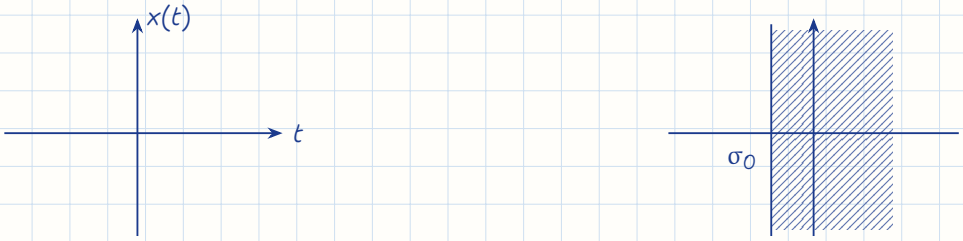
[6] Claude notu: sıfır payda yer aldığı için muhtemelen "pay'da" olmalı.

$s = -a$ iken $X(s) = \frac{0}{0}$

$$\lim_{s\to -a} X(s) = \lim_{s\to -a}\left(\frac{\frac{d}{ds}\left(1-e^{-(a+s)T}\right)}{\frac{d}{ds}(s+a)}\right)$$

$$= \lim_{s\to a} Te^{-aT}e^{-sT} \Rightarrow X(-a) = T$$ [7]

4) $x(t)$ sağ yanlı ve $\mathrm{Re}\{s\} = \sigma_0$ doğrusu ROC'da ise, $\mathrm{Re}\{s\} > \sigma_0$ için bütün s değerleri de ROC'da var?dır



orj. s. 11

5) *(orijinalde boş bırakılmış)*

6) $x(t)$ çift yanlı ve $\mathrm{Re}\{s\} = \sigma_0$ doğrusu ROC'da ise, ROC, s düzleminde $\mathrm{Re}\{s\} = \sigma_0$ doğrusunu içeren bir şeritten oluşur.



7) $X(s)$ rasyonel ise, ROC, ya kutuplarla sınırlıdır yada sonsuza genişler ve kutup içermez.

→ $x(t)$ sağ yanlı ise ROC, en sağdaki kutuların sağında kalan s düzlemidir

→ $x(t)$ sol yanlı ise, ROC en soldaki kutuların solunda kalan s düzlemidir.

orj. s. 12

Örnek

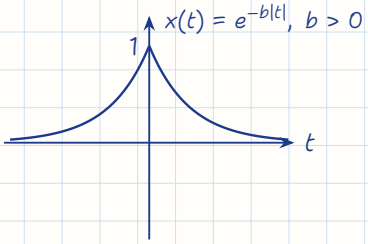
$x(t) = e^{-b|t|}$

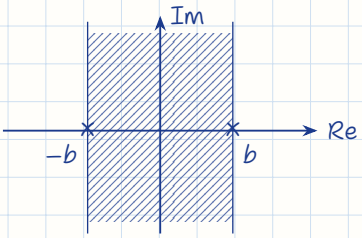
$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt = \int_{-\infty}^{0} e^{bt}e^{-st}dt + \int_{0}^{\infty} e^{-bt}e^{-st}dt$$

$$= \frac{-1}{s-b} + \frac{1}{s+b}, \quad \begin{matrix}\mathrm{Re}\{s-b\} < 0 \\ \mathrm{Re}\{s+b\} > 0\end{matrix}$$

$$X(s) = \frac{-2b}{s^2-b^2}, \quad -b < \mathrm{Re}\{s\} < b$$

[7] Claude notu: son satırdaki limitte muhtemelen $s \to -a$ olmalı.


x(t) = e^{−b|t|}, b > 0
1
t


Im
Re
−b
b