İşaretler ve Sistemler

Uygulama – Dönüşümler, sistem özellikleri, konvolüsyon, Fourier serisi (sayfa 1–21)

Orijinal not: isaretler-uygulama.pdf

UYARI Bu belge, el yazısı ders notlarından Claude (Anthropic) yardımıyla dijitale aktarılmıştır. Yardımcı kaynak olarak hazırlanmıştır; hata veya eksik içerebilir. Lütfen orijinal notları ve ders kaynaklarını esas alın.

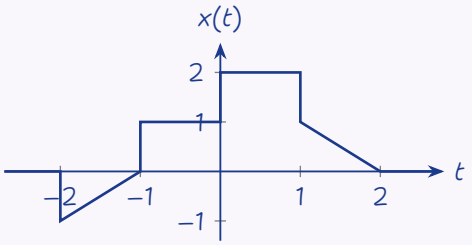
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Bağımsız değişken dönüşümleri

Örnek (Soru)

orj. s. 1

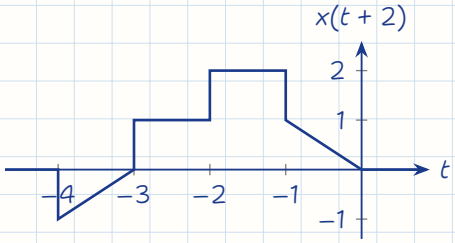
Soru: ? Verilen $x(t)$ işareti için aşağıdaki işaretleri çiziniz.



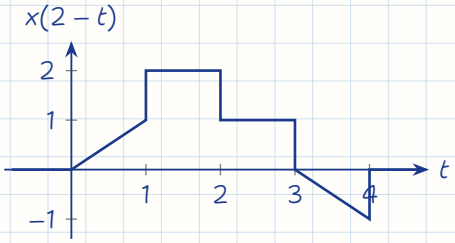
a) $x(2-t)$
b) $x(2t+1)$
c) $x\left(\frac{t-3}{2}\right)$

1) Öteleme
2) Ters Alma
3) Ölçekleme

a) 1) Öteleme



2) Ters Alma

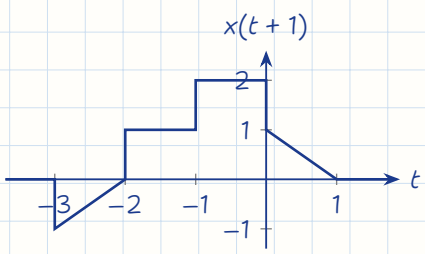


2. yol: $x(t)$, $t:[-2\ \ 2]$ $\quad 2-t:[-2\ \ 2]$

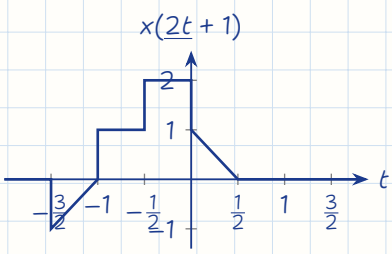
$x(2-t)$, $t:[0\ \ 4]$ ← $\quad t:[4\ \ 0]$

orj. s. 2

b) 1) Öteleme



2) Ölçekleme



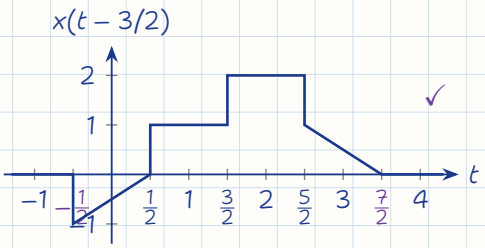
$\frac{t}{2}$ olsaydı, 2 katına ✓

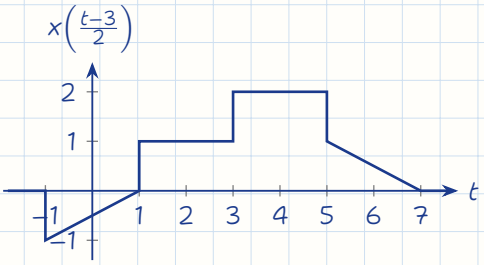
2. yol: $x(t)$, $t : [-2 \;\; 2]$ $\qquad 2t+1 : [-2 \;\; 2]$

$x(2t+1)$, $t : \left[-\frac{3}{2} \;\; \frac{1}{2}\right]$ $\qquad t : \left[-\frac{3}{2} \;\; \frac{1}{2}\right]$

orj. s. 3 $x\left(\frac{t-3}{2}\right)$

c) 1) Öteleme $x(t-3/2)$ ⟶ 2) Ölçekleme





2. yol: $x(t)$, $t : [-2 \;\; 2]$

$x\left(\frac{t-3}{2}\right)$, $t : [-1 \;\; 7]$

Sistem özellikleri

Örnek (Soru)

orj. s. 4

Soru: $y(t) = t \cdot x(t-2)$ sisteminin a) bellekli b) kararlı c) nedensel d) doğrusal e) zamanda değişmezlik özelliklerine sahip olup olmadığını inceleyiniz.

a) Bellekli: $t = 2 \rightarrow y(2) = 2 \cdot x(0)$ → $x(t)$'nin geçmiş değerine bağlı old. için → Bellekli ✓

b) Kararlılık: $|x(t)| \le B$ $\qquad y(t)$?

$|x(t-2)| \le B$ ✓

$|t \cdot x(t-2)| \le \infty \rightarrow -\infty < t < \infty$ old. için.

$|y(t)| < \infty \Rightarrow$ kararlı değildir. ✓[1]

c) Nedensellik: $t = -3 \rightarrow y(-3) = -3 \cdot x(-5) \rightarrow$ Nedensel ✓

d) Doğrusallık: $a \cdot x_1(t) + b \cdot x_2(t) \xrightarrow{?} a \cdot y_1(t) + b \cdot y_2(t)$

$x_1(t) \longrightarrow y_1(t)$ $\qquad y_1(t) = t \cdot x_1(t-2)$

$x_2(t) \longrightarrow y_2(t)$ $\qquad y_2(t) = t \cdot x_2(t-2)$

$x_3(t) \longrightarrow y_3(t)$

- $y_3(t) = t \cdot x_3(t-2) = a \cdot t \cdot x_1(t-2) + b \cdot t \cdot x_2(t-2)$

orj. s. 5

(Bu sayfa 4. sayfanın biraz ilerletilmiş ekran görüntüsü: b)–d) satırları tekrar ediyor, üzerlerine pembe altı çizili vurgular eklenmiş – ör. "kararlı değildir", "Nedensel". Yeni kısımlar aşağıda.)

$$y(t) = t \cdot x(t-2)$$

[1] Claude notu: muhtemelen $|y(t)|$'nin sınırlı olmadığı (ör. $|y(t)| \rightarrow \infty$) kastedilmiş olmalı.

d) Doğrusallık (devam):

$$x_3(t) = a \cdot x_1(t) + b \cdot x_2(t)$$

- $y_3(t) = t \cdot x_3(t-2) = a \cdot t \cdot x_1(t-2) + b \cdot t \cdot x_2(t-2)$
- $y_3(t) = a \cdot y_1(t) + b \cdot y_2(t)$
  $= a \cdot t \cdot x_1(t-2) + b \cdot t \cdot x_2(t-2)$

} ↕ = ✓ Doğrusal. ✓

orj. s. 6

e) Zamanda değişmezlik: $x(t) \longrightarrow y(t) = t \cdot x(t-2)$

$x(t-t_0) \longrightarrow y(t-t_0)$

$x_1(t) \longrightarrow y_1(t)$
$y_1(t) = t \cdot x_1(t-2)$
$x_1(t) = x(t-t_0)$ ↑
$y_1(t) = t\,x(t-t_0-2)$

→ $\boxed{y(t) = t \cdot x(t-2)}$ ✓

$x_1(t) = x(t-t_0)$

① $y_1(t) = t \cdot x_1(t-2) = t\,x(t-t_0-2)$
② $y(t-t_0) = (t-t_0)\,x(t-t_0-2)$

} ↕ ≠ Zamanda değişir. ✓

Örnek (Soru)

orj. s. 7

Soru: Aşağıda giriş-çıkış ilişkisi verilen sistemleri lineerlik bakımından sınıflandırın.

a) $y(t) = 2x(t) - 3$

b) $y(t) = e^{-t} \cdot x(t)$

Tanım (Lineerlik Şartı) ★

$x_1(t) \xrightarrow{T[\cdot]} y_1(t)$
$x_2(t) \xrightarrow{T[\cdot]} y_2(t)$

$$T[a \cdot x_1(t) + b \cdot x_2(t)] = a \cdot T[x_1(t)] + b \cdot T[x_2(t)]$$

a) $x(t) \xrightarrow{T[\cdot]} y(t) = 2 \cdot x(t) - 3$

$x_1(t) = a \cdot x(t) \longrightarrow y_1(t) \stackrel{?}{=} T[x_1(t)]$ $\quad T[x(t)] = y(t)$

① $y_1(t) = 2 \cdot a \cdot x(t) - 3$
② $T[x_1(t)] = T[a \cdot x(t)] = a \cdot T[x(t)] = a \cdot y(t) = 2a\,x(t) - 3a$

} $\boxed{y_1(t) \neq T[x_1(t)]}$ ↓ Lineer değil.

$x_1(t) \longrightarrow y_1(t) = 2\,x_1(t) - 3$
$x_1(t) = a \cdot x(t) \rightarrow y_1(t) = 2 \cdot a \cdot x(t) - 3$ ✓

$T[x_1(t)] = T[a \cdot x(t)] = a \cdot \underbrace{T[x(t)]}_{y(t)}$

$= a \cdot y(t)$
$= a \cdot 2x(t) - 3a$ ✓

b) $x(t) \longrightarrow y(t) = e^{-t} \cdot x(t)$

$x_2(t) = a \cdot x(t)$ $\quad y_2(t) \stackrel{?}{=} T[x_2(t)]$

orj. s. 8 (Sayfanın üst kısmı 7. sayfadaki a) çözümünün tekrarı; yeni kısım b) çözümü.)

b) $x(t) \longrightarrow y(t) = e^{-t} \cdot x(t)$

$x_2(t) \longrightarrow y_2(t) = e^{-t} \cdot x_2(t)$

$x_2(t) = a \cdot x(t) \longrightarrow y_2(t) = e^{-t} \cdot a \cdot x(t)$ ①

$x_2(t) = a \cdot x(t)$ $\quad y_2(t) \stackrel{?}{=} T[x_2(t)]$

$T[x_2(t)] = T[a\,x(t)] = a \cdot T[x(t)] = a \cdot y(t) = a \cdot e^{-t}x(t)$ ②

① $y_2(t) = e^{-t} \cdot a \cdot x(t)$

② $T[x_2(t)] = T[a \cdot x(t)] = a \cdot T[x(t)] = a \cdot e^{-t} \cdot x(t)$

$y_2(t) = T[x_2(t)]$ ↓ Lineer. ✓

Örnek (Soru)

orj. s. 9

Soru: Aşağıda giriş-çıkış ilişkisi verilen sistemleri zamanda değişmezlik bakımından sınıflandırın.

a) $y(t) = 2x(t) + 3$

b) $y(t) = (t + 2) \cdot x(t)$

Tanım (Zamanda Değişmezlik Şartı)

$$x(t) \longrightarrow y(t)$$
$$x(t - t_0) \longrightarrow y(t - t_0)$$

a) $x(t) \longrightarrow y(t) = 2x(t) + 3$

$x(t) \longrightarrow y(t) = 2x(t) + 3$

$x_1(t) \longrightarrow y_1(t) = 2x_1(t) + 3$

$= 2x(t - t_0) + 3$ ①

$x_1(t) = x(t - t_0)$ $\quad y_1(t) \stackrel{?}{=} y(t - t_0)$?

① $y_1(t) = 2x_1(t) + 3 = 2x(t - t_0) + 3$

② $y(t - t_0) = 2x(t - t_0) + 3$

$y_1(t) = y(t - t_0)$ ↓ Zamanda değişmez ✓ (time invariant)

b) $x(t) \longrightarrow y(t) = (t + 2) \cdot x(t) \longrightarrow$ $\quad x_2(t) \longrightarrow y_2(t) =$

$x_2(t) = x(t - t_0)$ $\quad y_2(t) \stackrel{?}{=} y(t - t_0)$

$y_2(t) = (t + 2)x_2(t) = (t + 2) \cdot x(t - t_0)$

$y(t - t_0) = (t - t_0 + 2)x(t - t_0)$

$y_2(t) \neq y(t - t_0)$ ↓ zamanda değişir. (time variant)

orj. s. 10 (9. sayfadaki b) çözümünün ilerletilmiş hâli: satırlar ① ve ② diye numaralanmış, $y(t - t_0) = (t - t_0 + 2)x(t - t_0)$ ve "zamanda değişir" vurgulanmış. Pembe ek:)

$x_2(t) \longrightarrow y_2(t) = (t + 2) \cdot x_2(t)$

$y_2(t) = (t + 2)x(t - t_0)$ ①

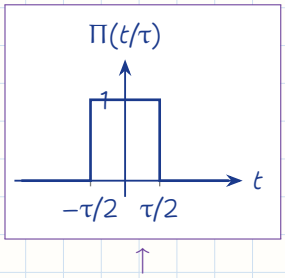
LTI sistemlerde nedensellik

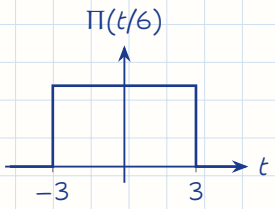
Örnek (Soru)

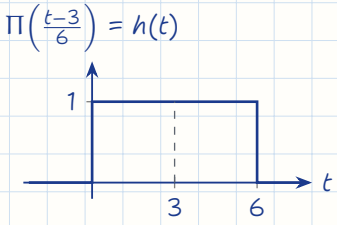
orj. s. 11

Soru: $h(t) = \Pi\left(\frac{t-3}{6}\right)$ Birim dürtü cevabı verilen LZD LTI sistem nedensel midir?

Nedensellik Şartı: $t < 0$, $h(t) = 0$ ise sistem nedenseldir!







$t < 0$ iken $h(t) = 0$ olduğu için sistem Nedenseldir!

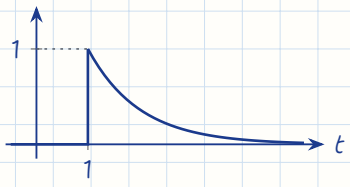
Konvolüsyon

Örnek (Soru)

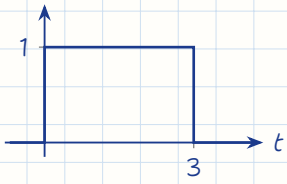
orj. s. 12

?okunamadı ...rından LTI sistemin çıkışını konvolüsyon kullanarak hesaplayın. *(sorunun ilk satırı ekran görüntüsünde kesik)*

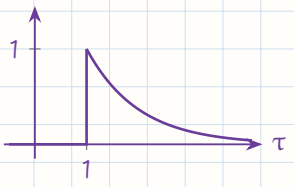
$h(t) = e^{1-t} \cdot u(t-1)$



$x(t) = u(t) - u(t-3)$

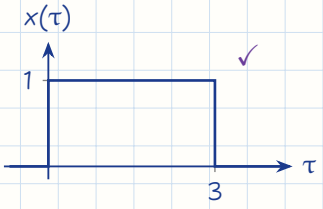


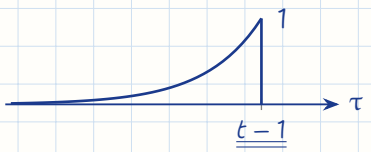
$h(\tau) = e^{1-\tau}u(\tau-1)$



$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$h(t-\tau) = e^{-(t-\tau-1)} \cdot u(t-\tau-1)$$

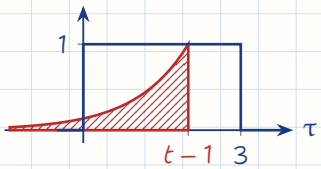




$t - 1 < 0$ ise $y(t) = 0$

$0 \le t - 1 < 3 \rightarrow 1 \le t < 4$

$x(\tau), h(t-\tau)$



$$y(t) = \int_0^{t-1} e^{-(t-\tau-1)}\, d\tau = e^{-(t-1)} \cdot \int_0^{t-1} e^{\tau}\, d\tau$$
$$= e^{-(t-1)}\left(e^{t-1} - 1\right) = 1 - e^{-(t-1)}$$

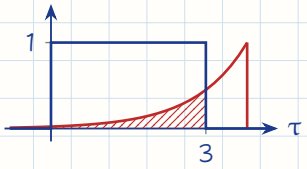
orj. s. 13

(Üst kısım 12. sayfanın devamı; pembe eklerle: $t-1<0$ ise $y(t)=0$ ✓; $0 \le t-1 < 3 \rightarrow 1 \le t < 4$; şekilde örtüşmeyen kısımlar (soldaki $\tau < 0$ bölgesi ve $t-1$ ile 3 arası) pembe X ile işaretlenmiş.)

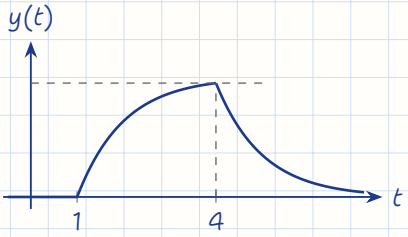
$$y(t) = \int_0^{t-1} 1 \cdot e^{-(t-\tau-1)}\, d\tau = e^{-(t-1)} \cdot \int_0^{t-1} e^{\tau}\, d\tau \qquad e^{\tau}\Big|_0^{t-1}$$
$$= e^{-(t-1)}\left(e^{t-1} - 1\right) = 1 - e^{-(t-1)}, \quad 1 \le t < 4$$

$t - 1 > 3 \;\rightarrow\; t > 4$

$x(\tau), h(t-\tau)$



$$y(t) = \int_0^3 e^{-(t-\tau-1)}\, d\tau = e^{-(t-1)} \int_0^3 e^{\tau}\, d\tau = e^{-(t-1)} \cdot \left(e^3 - 1\right)$$
$$= e^{-t}\left(e^4 - e^1\right)$$



$$y(t) = \begin{cases} 0, & t < 1 \\ 1 - e^{-(t-1)}, & 1 \le t < 4 \\ e^{-t}\left(e^4 - e^1\right), & t \ge 4 \end{cases}$$

Ayrık zamanda konvolüsyon

Örnek (Soru)

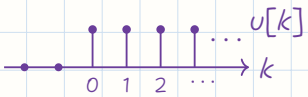
orj. s. 14

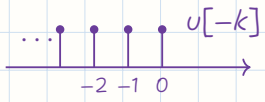
Soru: Ayrık zamanda aşağıdaki giriş ve dürtü cevapları için konvolüsyon hesaplayın ve grafiğini çizin.

a) $x[n] = (0.8)^n \cdot \upsilon[n]$ ve $h[n] = (0.9)^n \cdot \upsilon[n]$

b) $x[n] = \delta[n] - \delta[n-2]$ ve $h[n] = \upsilon[n]$

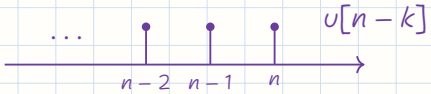






Hatırlatma:

$$\sum_{k=0}^{n} \alpha^k = \begin{cases} \dfrac{1-\alpha^{n+1}}{1-\alpha}, & \alpha \ne 1 \\ n+1, & \alpha = 1 \end{cases}$$



a)

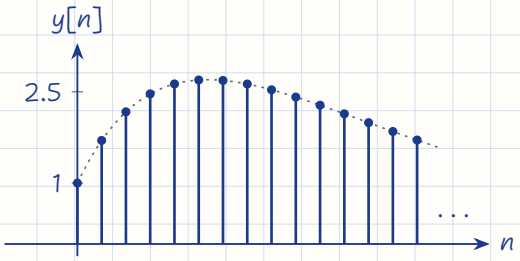
$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k] = \sum_{k=-\infty}^{\infty} \underbrace{(0.8)^k \cdot u[k]}_{x[k]} \cdot \underbrace{(0.9)^{n-k} \cdot u[n-k]}_{h[n-k]}$$

$$= (0.9)^n \sum_{k=0}^{n} \left(\frac{0.8}{0.9}\right)^k, \qquad n \geq 0$$

$$= (0.9)^n \cdot \frac{1 - \left(\frac{0.8}{0.9}\right)^{n+1}}{1 - \frac{8}{9}}, \qquad n \geq 0$$

$$= \frac{\cancel{(0.9)^n}\left[(0.9)^{n+1} - (0.8)^{n+1}\right]}{\frac{1}{9} \cdot \cancel{(0.9)^n} \cdot (0.9)}, \qquad n \geq 0$$

$$= \frac{\left((0.9)^{n+1} - (0.8)^{n+1}\right)}{0.1} \cdot u[n]$$



orj. s. 15

(Üst kısım 14. sayfadaki a) çözümünün ve y[n] grafiğinin tekrarı; sadeleştirilen kesir pembe ile çerçevelenmiş, son satırın başına pembe ile "y[n] =" yazılmış ve u[n] vurgulanmış. Pembe ekler:)

$$y[n] = \frac{\left((0.9)^{n+1} - (0.8)^{n+1}\right)}{0.1} \cdot u[n] \qquad n \geq 0 \;\rightarrow\; u[n]$$

$$x[n] * h[n] = \sum_k x[k]\,h[n-k]$$

$$\delta[n] * u[n] = \sum_k \delta[k]\,u[n-k]$$

$$x[n] = \delta[n] - \delta[n-2] \qquad h[n] = u[n]$$

b)[2]

$$y[n] = \sum_{k=\langle\infty\rangle} x[k]\,h[n-k] = \sum_k \overbrace{(\delta[k] - \delta[k-2])}^{x[k]} \cdot \overbrace{u[n-k]}^{h[n-k]} \longrightarrow \delta[k] * u[k]$$

$$= \boxed{\sum_k \delta[k]\,u[n-k]} - \sum_k \delta[k-2]\,u[n]$$

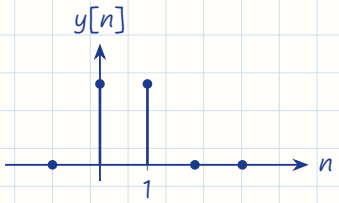
$$= u[n] - u[n-2] = \delta[n] + \delta[n-1]$$

[2] Claude notu: ikinci satırdaki ikinci toplamda muhtemelen $u[n-k]$ olmalı (sonuçta $u[n-2]$ elde ediliyor); 16. sayfada pembe ile düzeltilmiş.

$$\sum_{k} = \sum_{k=\langle\infty\rangle}$$

$$x[n]\cdot\delta[n-n_0] = x[n_0]\,\delta[n-n_0]$$

$$x[n] \circledast \delta[n-n_0] = x[n-n_0]$$



orj. s. 16

(b) çözümünün düzeltilmiş/ilerletilmiş hâli; sağ üstte a) şıkkının y[n] grafiğinin ucu görünüyor.)

$$\begin{cases} x[n] = \delta[n] - \delta[n-2] \\ h[n] = u[n] \end{cases}$$

b)

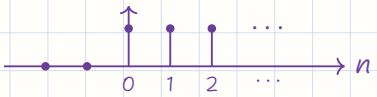
$$y[n] = x[n]*u[n] = \sum_{k=\langle\infty\rangle} x[k]\,h[n-k] = \sum_{k} \overbrace{(\delta[k]-\delta[k-2])}^{x[k]} \cdot \overbrace{u[n-k]}^{h[n-k]}$$

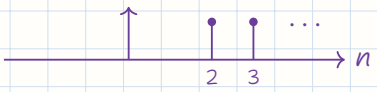
$$= \underbrace{\sum_{k} \delta[k]\,u[n-k]}_{\swarrow} - \underbrace{\sum_{k} \delta[k-2]\,u[n-k]}_{\longrightarrow\, u[n-2]} \quad \checkmark$$

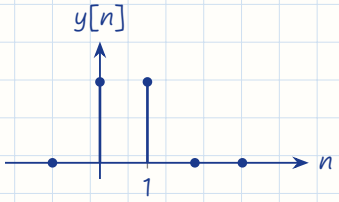
$$= u[n] - u[n-2] = \delta[n] + \delta[n-1]$$

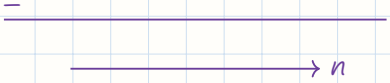
$$\sum_{k} \delta[k]\,u[n-k] = \delta[n]*u[n] = u[n]$$

$$x[n]*\delta[n-n_0] = x[n-n_0] \quad \leftarrow \bigstar$$









Örnek (Soru)

orj. s. 17

Soru:

→ $x[n] = u[n-3]$

→ $h[n] = -\delta[n] + \delta[n+2]$ ise $y[n]$ =?

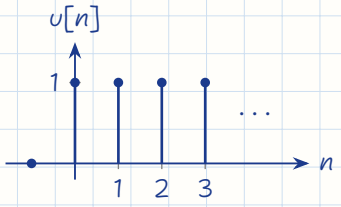
$x[n] \rightarrow \boxed{h[n]} \rightarrow y[n]$

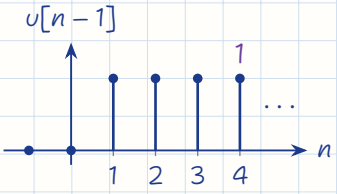
$x(t)*\delta(t-t_0) = x(t-t_0)$

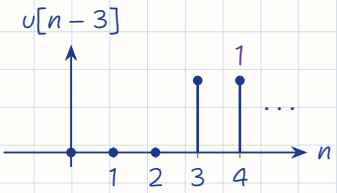
$x[n]*\delta[n-n_0] = x[n-n_0]$

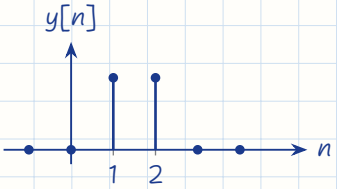
$$y[n] = x[n]*h[n] = \overbrace{u[n-3]}^{x[n]} * \overbrace{(-\delta[n]+\delta[n+2])}^{h[n]} = -u[n-3] + u[n-1]$$

$$u[n-3]*\delta[n+2] = u[n+2-3]$$
$$= u[n-1]$$









$$\Rightarrow\ y[n] = \delta[n-1] + \delta[n-2]$$

Seri bağlı LTI sistemler

Örnek (Soru)

orj. s. 18

Soru: Seri bağlanmış iki LTI sistemi düşünün. Bunlardan ilkinin dürtü cevabı $h_1[n] = \sin(\underline{8n})$ ve ikincisinin dürtü cevabı $h_2[n] = \underline{a^n}\,\upsilon[n]$, $|a| < 1$ ve giriş işareti $x[n] = \underline{\delta[n]} - a \cdot \underline{\delta[n-1]}$'dir. Çıkış işareti $y[n]$'yi bulunuz.



$$\underline{y[n]} = (x[n] * h_1[n]) * h_2[n]$$
$$= \underbrace{(x[n] * h_2[n])}_{\hookrightarrow\ \delta[n]} * h_1[n]$$

$$x[n]\,\delta[n-n_0] = x[n_0]\cdot\delta[n-n_0]$$

$$\underline{x[n]} * \underline{h_2[n]} = (\underline{\delta[n]} - a\,\delta[n-1]) * \boxed{a^n \cdot \upsilon[n]}$$
$$= a^n\cdot\upsilon[n] - \underbrace{a\cdot a^{n-1}}_{a^n}\cdot\upsilon[n-1] = a^n\underbrace{(\upsilon[n]-\upsilon[n-1])}_{\delta[n]} = \boxed{a^n}\,\delta[n] \underset{n=0}{=} \underline{\delta[n]}$$

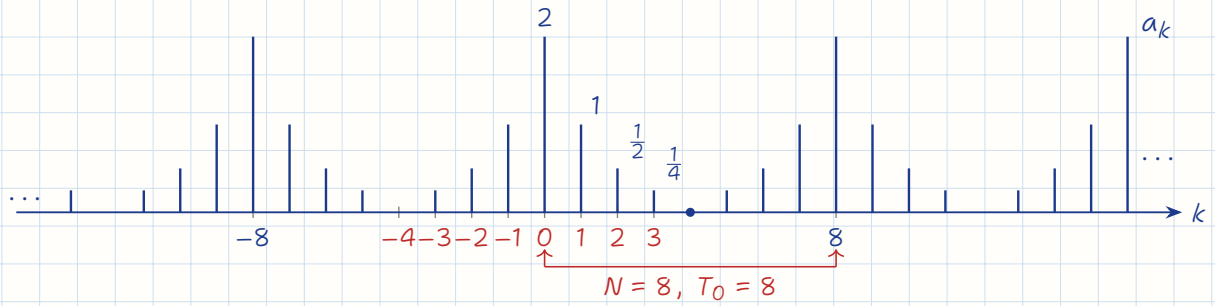
$$\underline{y[n]} = \underline{\delta[n] * h_1[n]} = \underline{h_1[n]} = \underline{\sin 8n}\ \checkmark$$

Ayrık zamanlı Fourier serisi

Örnek (Soru)

orj. s. 19

Soru: Aşağıda Fourier serisi katsayıları verilen $x[n]$ işaretini bulunuz.



$$x[n] = \sum_{k=\langle N \rangle} a_k e^{-jk\omega_0 n}$$ [3]

$$\omega_0 = 2\pi f_0 = \frac{2\pi}{N} \qquad \frac{2\pi}{T_0}$$

$$\omega_0 = \frac{2\pi}{8} = \frac{\pi}{4} \checkmark$$

$$x[n] = \sum_{k=-4}^{4} a_k \cdot e^{jk\omega_0 n} = \underset{\uparrow\, k=0}{\overset{a_0}{2}} + \overset{a_1=a_{-1}}{1} \cdot \underline{\left[e^{-j\omega_0 n} + e^{j\omega_0 n}\right]} + \overset{a_2=a_{-2}}{\frac{1}{2}} \left[e^{-j2\omega_0 n} + e^{j2\omega_0 n}\right]$$

$$+ \underset{a_3=a_{-3}}{\frac{1}{4}} \left[e^{-j3\omega_0 n} + e^{j3\omega_0 n}\right]$$

$$= 2 + 2\cos(\omega_0 n) + \cos(2\omega_0 n) + \tfrac{1}{2}\cos(3\omega_0 n)$$

$$= 2 + 2 \cdot \cos\left(\frac{\pi n}{4}\right) + \cos\left(\frac{\pi n}{2}\right) + \frac{1}{2}\cos\left(\frac{3\pi n}{2}\right)$$

Claude notu: son terimde $3\omega_0 = 3\pi/4$ olduğundan muhtemelen $\cos\left(\frac{3\pi n}{4}\right)$ olmalı.

orj. s. 20

(19. sayfanın pembe eklerle tekrarı: grafikte k = 4'teki sıfır değerine "4" yazılmış; kutudaki ve açılımdaki üsler pembe + ile düzeltilmiş; ilk köşeli parantezin altına $2\cos(\omega_0 n)$ yazılmış; $\frac{1}{2}\cos(3\omega_0 n)$ terimi okla gösterilmiş. Pembe ekler:)

$$x[n] = \sum_{k=\langle N \rangle} a_k e^{+jk\omega_0 n} \qquad \underbrace{\left[e^{-j\omega_0 n} + e^{j\omega_0 n}\right]}_{2\cos(\omega_0 n)}$$

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2} \qquad \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

✓ $$x[n] = 2 + 2 \cdot \cos\left(\frac{\pi n}{4}\right) + \cos\left(\frac{\pi n}{2}\right) + \frac{1}{2}\cos\left(\frac{3\pi n}{2}\right)$$

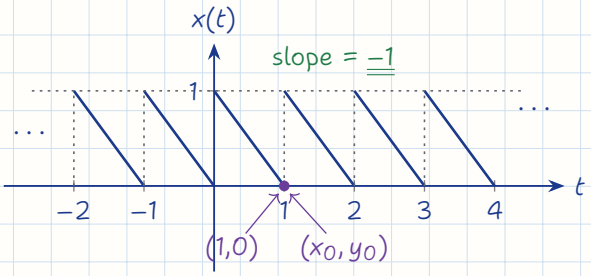
Sürekli zamanlı Fourier serisi

Örnek (Soru)

orj. s. 21

Soru: Aşağıda ?verilen periyodik işaretin Fourier serisi açılımını bulunuz. *(satır ekran görüntüsünde kesik)*

[3] Claude notu: sentez denkleminde üs $+jk\omega_0 n$ olmalı; 20. sayfada pembe ile + olarak düzeltilmiş.



m (slope) $(x - x_0) = y - y_0$

Doğru denk.:
$(-1) \cdot (x - 1) = y \rightarrow \boxed{y = 1 - x}$

periyot: $T_0 = 1 \rightarrow \omega_0 = \frac{2\pi}{T_0} = 2\pi$ ✓

Üstel seri Açılımı: $x(t) = \sum_{k=-\infty}^{\infty} x_k \cdot e^{jk\omega_0 t}$, $\quad \boxed{x_k = \frac{1}{T_0} \int_{T_0} x(t) \cdot e^{-jk\omega_0 t}\, dt}$

$$x_k = \int_0^1 (1-t) \cdot e^{-jk2\pi t}\, dt = \underbrace{\int_0^1 e^{-jk2\pi t}\, dt}_{I_1} - \underbrace{\int_0^1 t \cdot e^{-jk2\pi t}\, dt}_{I_2} = I_1 - I_2$$

$$I_1 = \int_0^1 e^{-jk2\pi t}\, dt = \frac{-1}{jk2\pi} e^{-jk2\pi t}\Big|_0^1 = \frac{-1}{jk2\pi}\left[e^{-j2\pi k} - 1\right] = \frac{1}{j2\pi k}\left(1 - e^{-j2\pi k}\right)$$

$I_2 = \int_0^1 t \cdot e^{-jk2\pi t}\, dt$

Kısmi integrasyon: $\int u\, dv = u \cdot v - \int v\, du$