Kontrol Sistemleri

Positive feedback root locus, lead/lag compensators

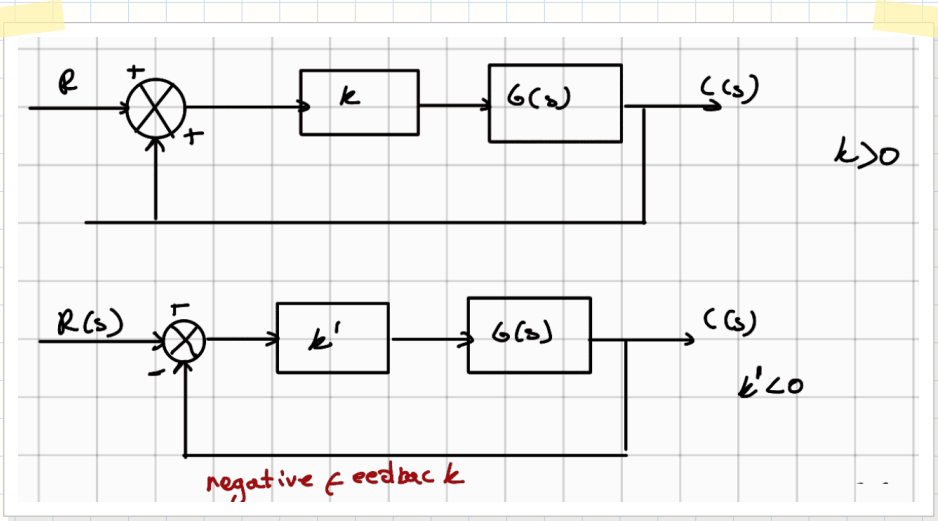
Original notes: Control-Systems-lecture-12.pdf

UYARI Bu belge, el yazısı ders notlarından Claude (Anthropic) yardımıyla dijitale aktarılmıştır. Yardımcı kaynak olarak hazırlanmıştır; hata veya eksik içerebilir. Lütfen orijinal notları ve ders kaynaklarını esas alın.

NOTICE *This document was transcribed from handwritten lecture notes with the help of Claude (Anthropic). It is meant as a supplementary resource and may contain errors or omissions. Always check against the original notes and official course material.*

orig. p. 1

Positive Feedback architecture



original drawing, p. 1

$$\frac{C(s)}{R(s)} = \frac{k\,G(s)}{1 - k\,G(s)H(s)}$$

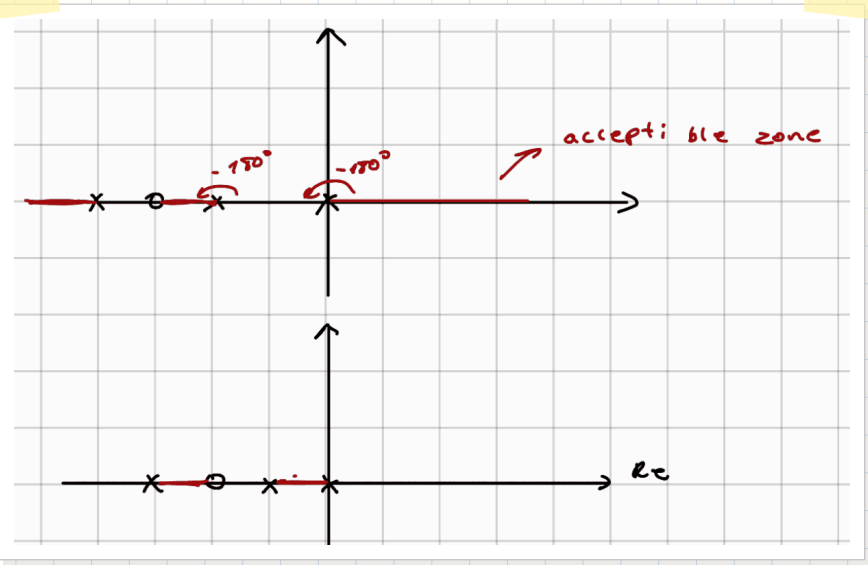
$$1 - k\,G(s)H(s) = 0$$

$$G(s)H(s) = \overbrace{\frac{1}{k}}^{\text{positive}} + 0j$$

$$\angle G(s)H(s) = \tan^{-1}\left(\frac{0}{\varepsilon^{+}}\right) = \pm 360\,i \quad \text{or} \quad \pm 180(2i)$$

$$\boxed{|G(s)H(s)| = \frac{1}{k}}$$

positive feedback



original drawing, p. 1

(2) Departure angles from complex poles
arrival " to complex zero

orig. p. 2

departure angle = 360 ?illegible + ∑ phase of c.c pole w.r.t. other poles
− ∑ phase of c.c pole w.r.t. other zeros

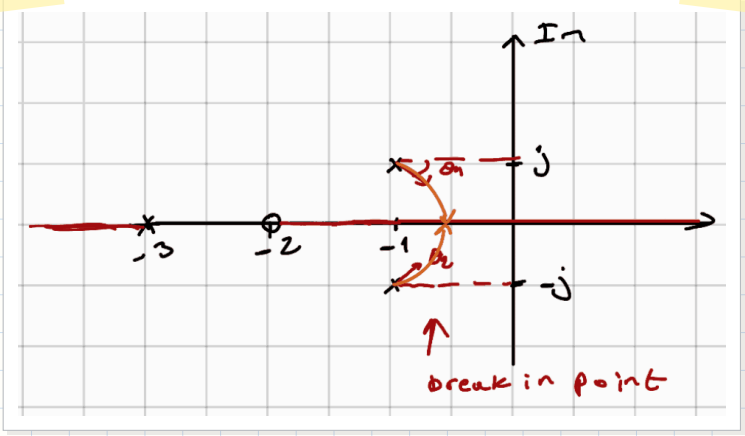
(3) Asymptote angles

$$\frac{\pm 360\, i}{n-m}$$

Example

$G(s) = \dfrac{k(s+2)}{(s+3)(s^2+2s+2)}$
Positive feedback system

$z_1 = -2$
$p_1 = -3$
$p_{2,3} = -1 \pm j$



original drawing, p. 2

branches: 3

$n - m = 3 - 1 = 2$

angle of asymptotes: $\dfrac{\pm 360\, i}{n-m} = \pm 180\, i$

for $s = -1 + j$

$$\theta_1 = 0^\circ - 27^\circ - 90^\circ + 45^\circ = -72^\circ$$

for $s = -1 - j$

$$\theta_2 = 0^\circ + 27^\circ + 90^\circ - 45^\circ = +72^\circ$$

To find characteristic equation

$$1 + \frac{k(s+2)}{(s+3)(s^2+2s+2)} = 0$$

$$(s+3)(s^2+2s+2) + k(s+2) = 0$$

$$k = -\frac{(s+3)(s^2+2s+2)}{(s+2)}$$

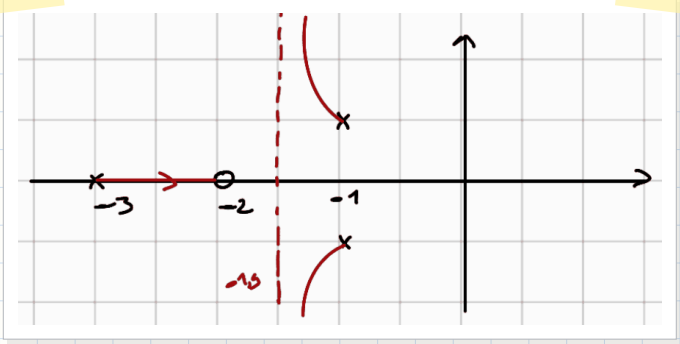
$$\frac{dk}{ds} = 0 \Rightarrow 2s^3 + 11s^2 + 20s + 10 = 0$$

$$\Rightarrow 2(s+0{,}8)(s+2{,}35+j0{,}77)(s+2{,}35-j0{,}77) = 0$$

orig. p. 3

$\Rightarrow s = -0{,}8$ break in

if its negative feedback



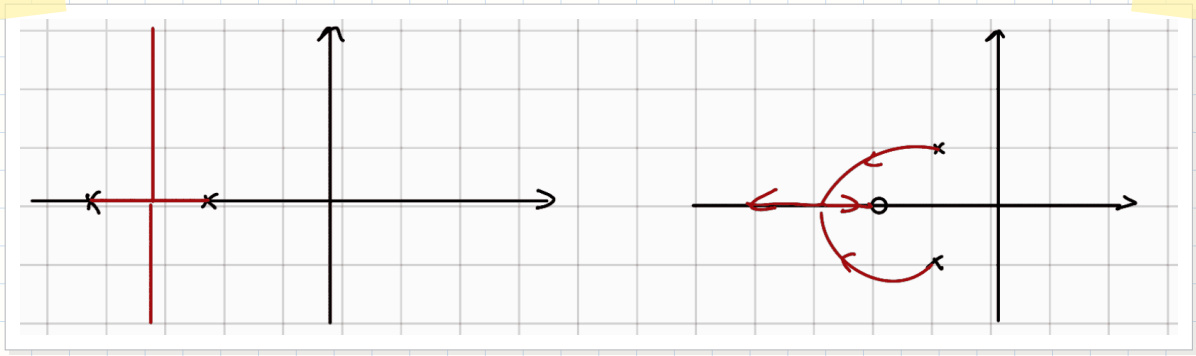
original drawing, p. 3

$$\text{centroid} = \frac{\sum \text{real part poles} - \sum \text{real part of zeroes}}{n-m} = -1{,}5$$

Example

Negative feedback

[0] Claude notu: pozitif geri besleme için karakteristik denklem muhtemelen $1 - kG(s) = 0$ olmalı; $dk/ds = 0$ kökleri bundan etkilenmez.



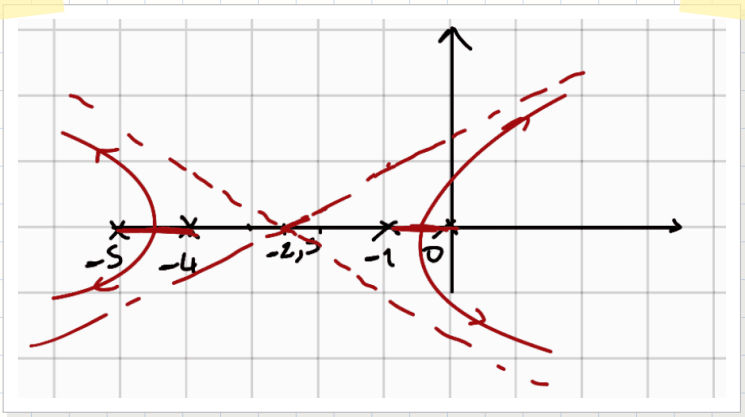
original drawing, p. 3

Pozitive feedback



original drawing, p. 3

Negative feedback



original drawing, p. 3

$$\frac{\pm 180(2i+1)}{n-m}$$

$$\pm 45(2i+1)$$

$$\pm 45 \qquad \pm 135$$

centroid = −2,5

orig. p. 4

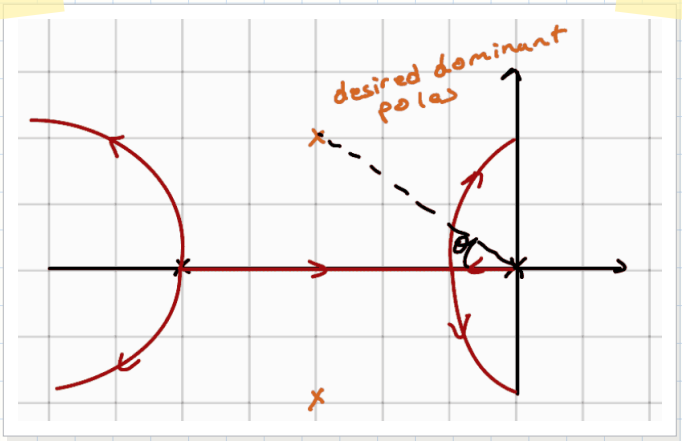
Pozitif f? *illegible*



original drawing, p. 4

$$\frac{\pm 360\,i}{4} = \pm 90\,i$$

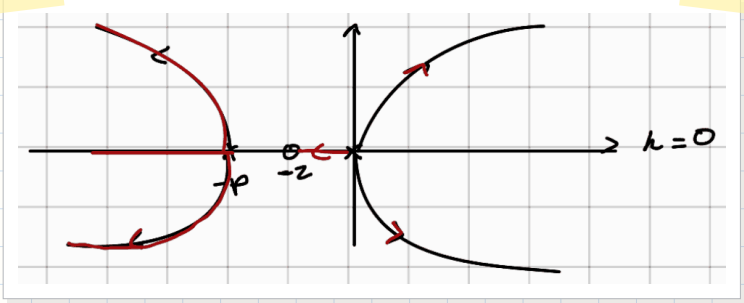
Compensator design



original drawing, p. 4

specific "k" which renders the system unstable

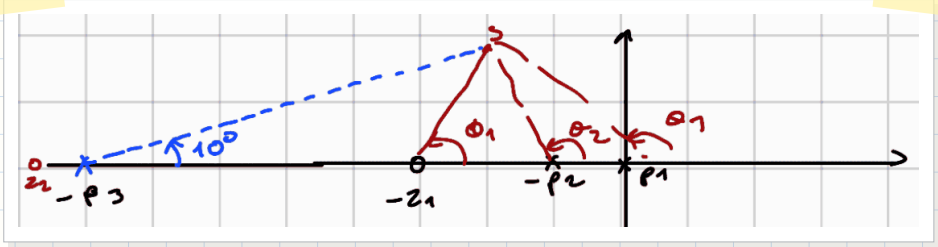
4 seperate roots of ch.e.q.



original drawing, p. 4

$$1 + \frac{k(s+2)}{s^3(s+p)^3}$$

$\theta = \cos^{-1}\zeta$ damping ratio



original drawing, p. 4

↘ $\frac{1}{s+p_3}$ → delayed integrator

$$F(s) = \frac{s+a}{s+b} = \frac{R e^{i\phi}}{M e^{i\theta}} = \left(\frac{R}{M}\right) e^{i(\phi-\theta)}$$

Phase of ?s points $\beta = \phi_1 - \theta_1 - \theta_2$

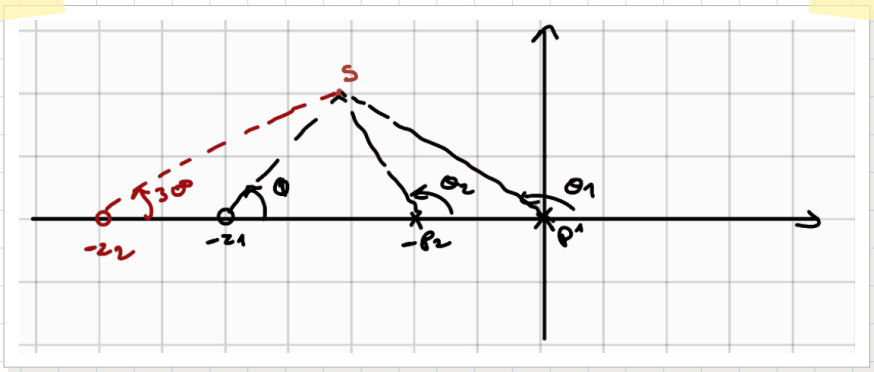
the original OLTF $\frac{s+z_1}{(s+p_1)(s+p_2)}$

if $\beta = 190°$ Negative $\frac{-10}{180}$ → to make it negative p_3 angle have to be bigger

?PI $\frac{s+z_2}{s+p_3} \simeq \frac{1+z_2}{\lim_{p_3 \to 0} s}$

orig. p. 5

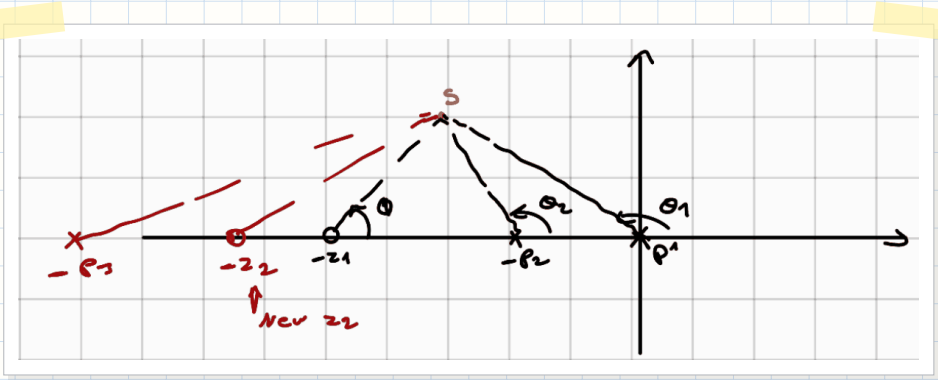
if $\beta = 150°$ $\frac{30°}{180°}$



original drawing, p. 5

$\underbrace{(s+z_2)}_{\text{derivative operator}}$ $\frac{(s+z_1)}{(s+p_1)(s+p_2)}$

Its unuseless itself so we have to add a pole

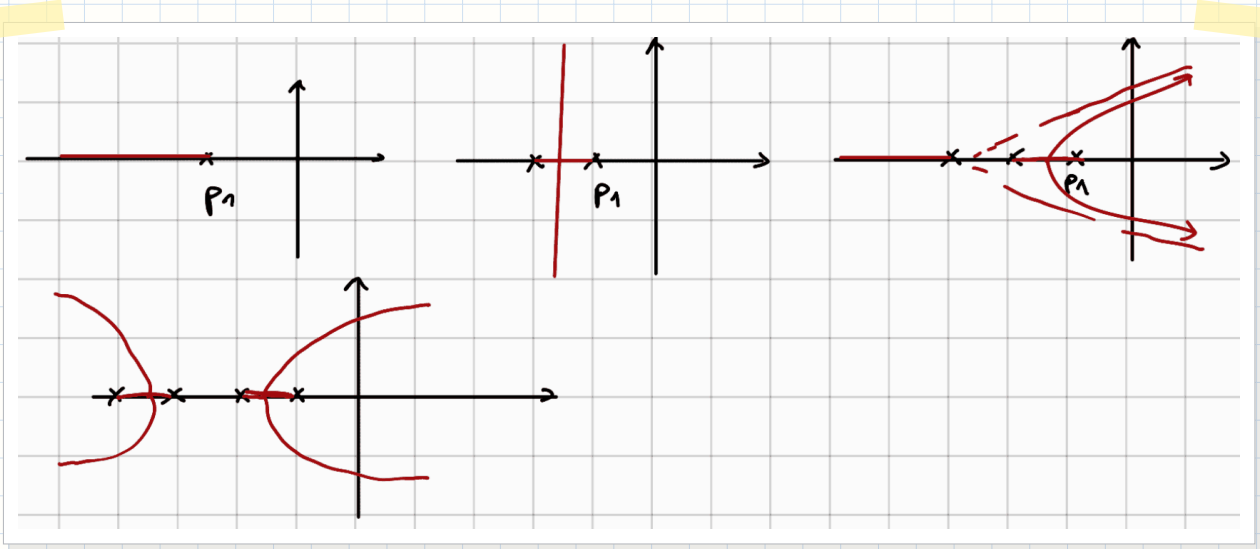


original drawing, p. 5

$$\frac{s+z_2}{s+p_3} \underset{\lim z_2 \to 0}{=} \frac{s}{s+p_3} = 1 - \frac{p_3}{s+p_3}$$

?? WTF

Adding poles



original drawing, p. 5

by adding poles the locus plot move to right, systems get more unstable. (lag compensator)

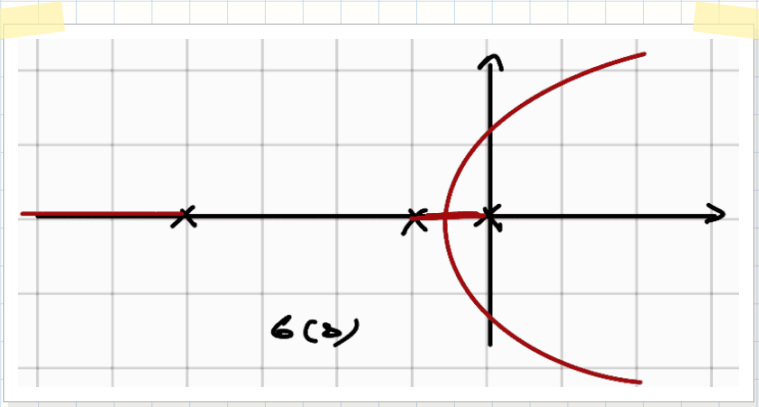
orig. p. 6

System get more unstable - slower! ($t_s \uparrow$)

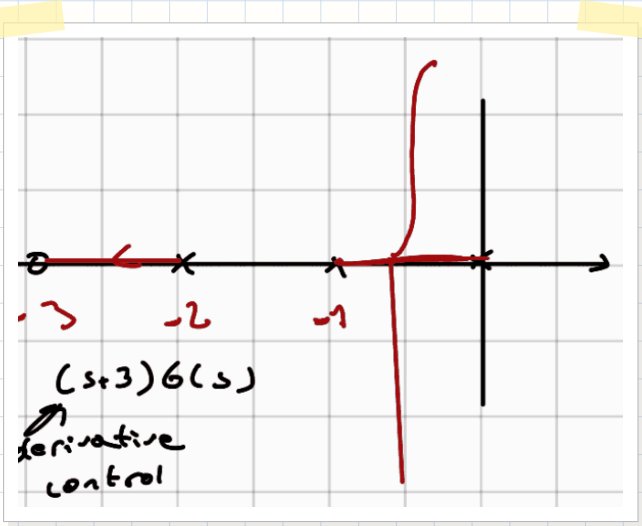
More accurate by adding integrator

lag compensation, The output angle is less than input

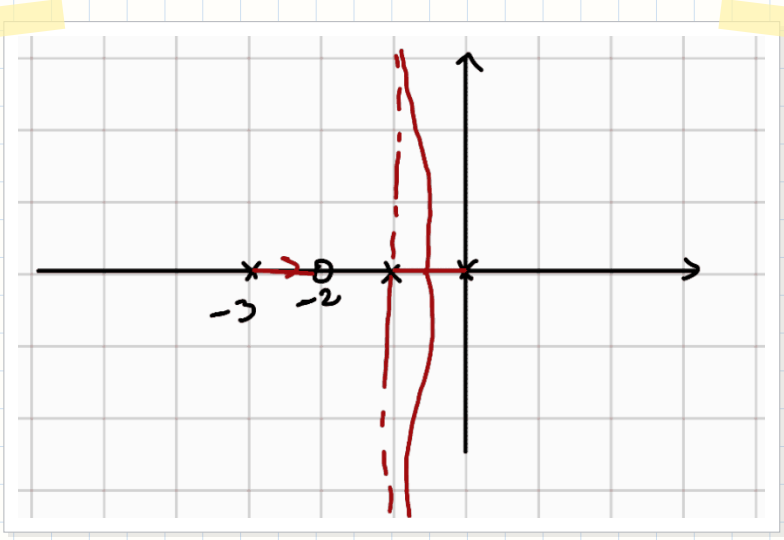
zero placement



original drawing, p. 6

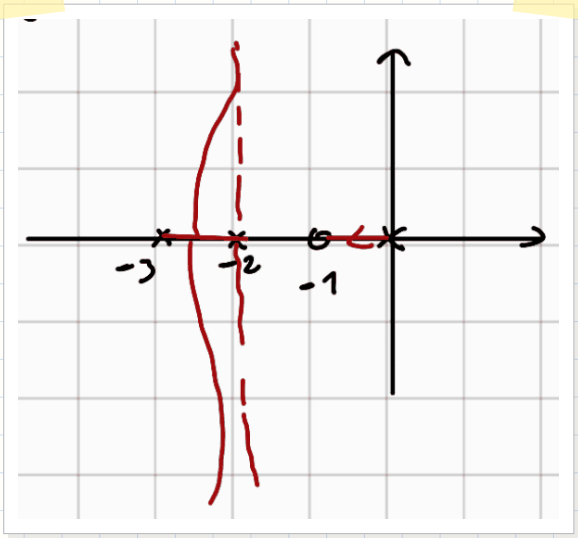


original drawing, p. 6



original drawing, p. 6

$$\frac{-1-3-(-2)}{2} = \frac{-4+2}{2} = -1$$



original drawing, p. 6

pulls the root locus plot to the left

→ more stable

→ faster response ($t_s \downarrow$)

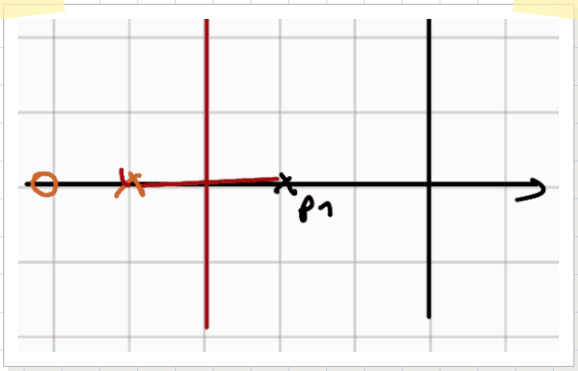
more sensitive to noise

The output angle is more than the input angle

its leading output

$$r(t) = \sin \omega t \quad \rightarrow \quad y(t) = \sin(\omega t + \phi)$$

orig. p. 7

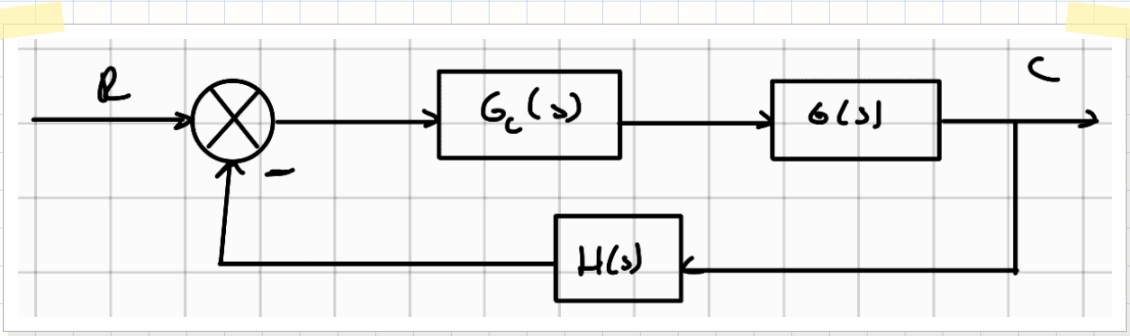


original drawing, p. 7

lag compensation ? $z > p > 0$

lead laging compensator

Series/parallel compensation



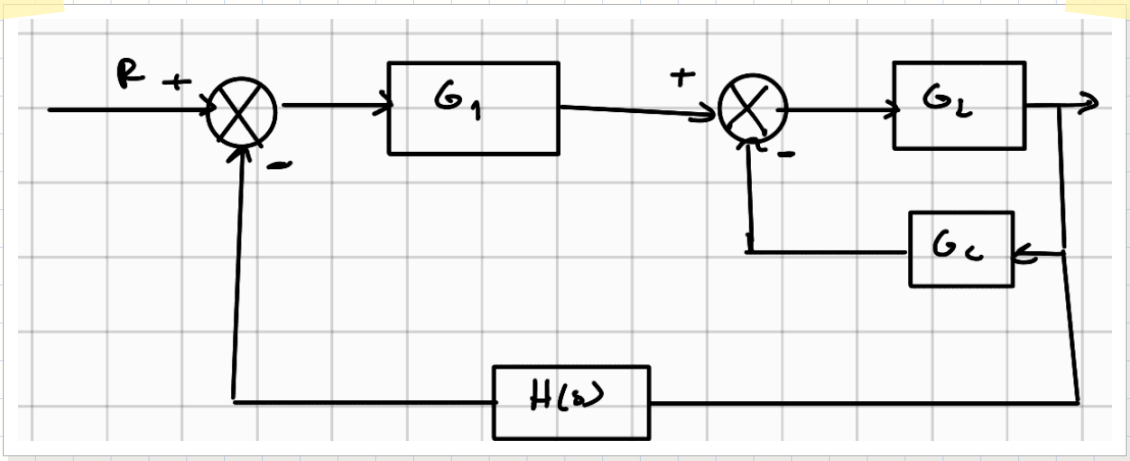
original drawing, p. 7

$$\frac{C(s)}{R(s)} = \frac{G_c G}{1 + G_c G H}$$

$$1 + G_c G H = 0 \Rightarrow G_c G H = -1$$

$\angle G_c G H$ = gives the root locus

parallel



original drawing, p. 7

$$\frac{C(s)}{R(s)} = \frac{G_1 G_2}{1 + G_2 G_c + G_1 G_2 H} \longrightarrow 1 + G_c \frac{G_2}{1 + G_1 G_2 H} = 0$$

$$1 + G_2 G_c + G_1 G_2 H = 0 \Rightarrow 1 + G_c G_p = 0$$

$$\times \frac{1}{1 + G_1 G_2 H}$$

$$= \frac{1 + G_1 G_2 H + G_2 G_c}{1 + G_1 G_2 H}$$

orig. p. 8

Lead compensator: zero is closer to jω axis

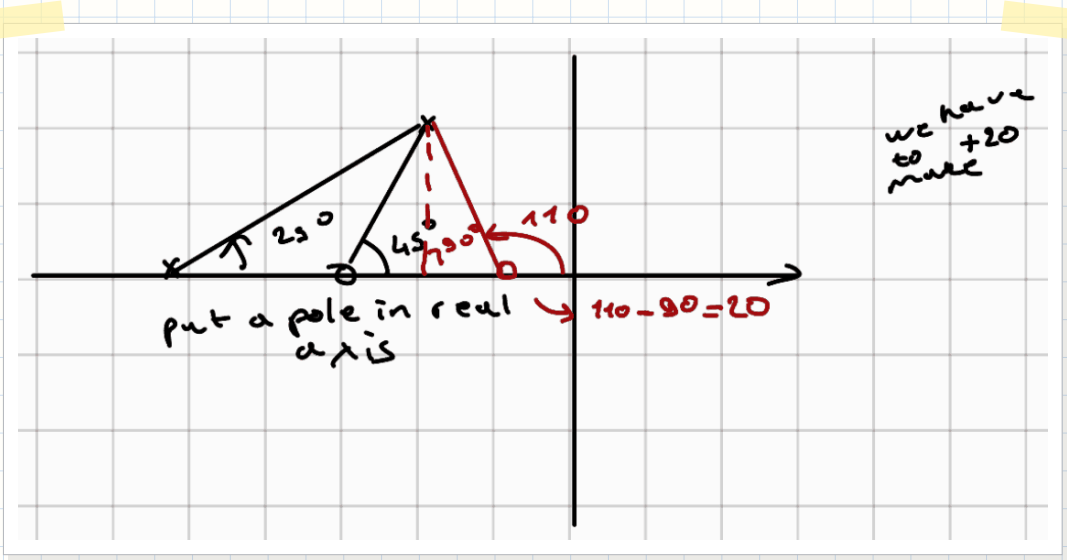
$$G_c(s) = k_c \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}} \rightarrow \text{specs} \qquad 0 < \alpha < 1$$

$$k_c \frac{s+z}{s+p}$$

if $\alpha > 1 \Rightarrow$ lag compensator

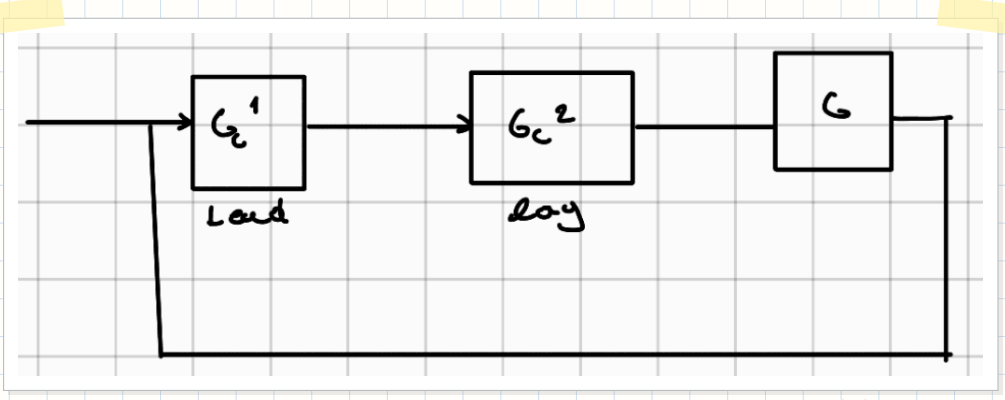
$$G_c(s) = k_c \frac{s + \frac{1}{T}}{s + \frac{1}{\beta T}} \qquad \beta > 1$$

$$\frac{1}{T} = 10 \qquad \begin{cases} z = -10 \\ p = -\frac{10}{\alpha} = -20 \end{cases} \qquad \alpha = \frac{1}{2}$$



original drawing, p. 8

Lead lag compensator

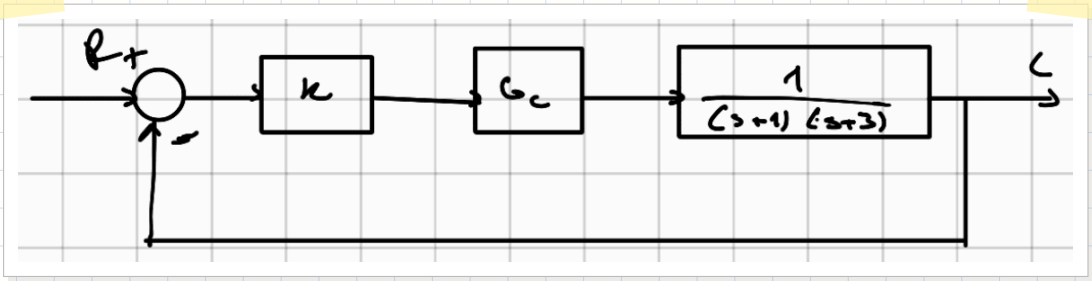


original drawing, p. 8

$$\left(k_c \frac{s + \frac{1}{T_1}}{s + \frac{1}{\alpha T_1}}\right)\left(\frac{s + \frac{1}{T_2}}{s + \frac{1}{\beta T_2}}\right)$$? 5 unknowns to be design

$$0 < \alpha < 1 \qquad \beta > 1$$

Ex:



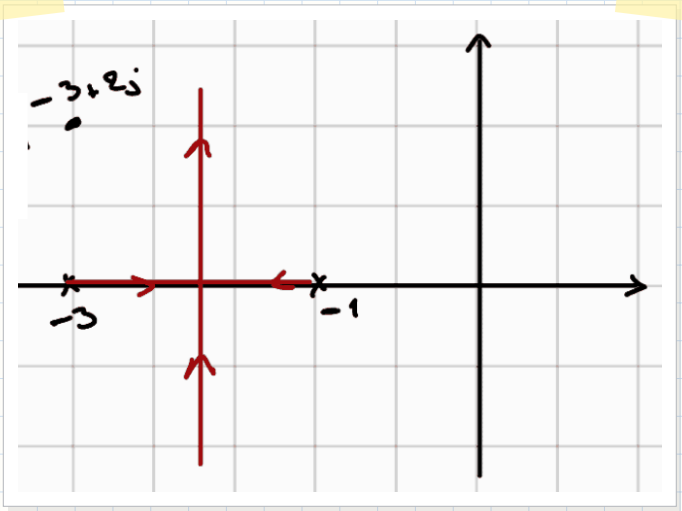
original drawing, p. 8

orig. p. 9

Dominant pole = $-3 + 2j$ → lead compensator

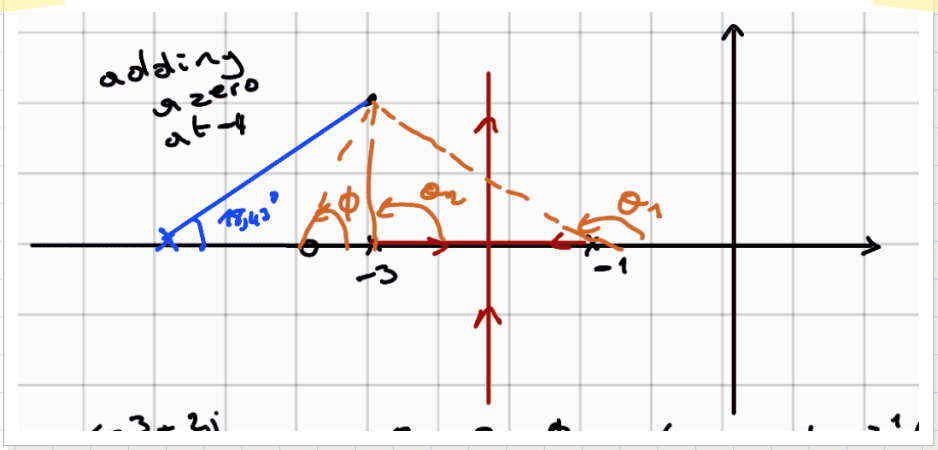
e_{ss} ← Steady state error = 10% (step input) → lag compensator

$G_c = 1$



original drawing, p. 9

$$G_c = \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}}$$



original drawing, p. 9

$$\angle(-3 + 2j) = -\theta_1 - \theta_2 + \phi = -\left(180 - \tan^{-1}\left(\frac{2}{2}\right)\right) - 90 + \tan^{-1}\left(\frac{2}{1}\right)$$

$$= -135 - 90 + 63{,}43^\circ = -161{,}56^\circ$$

phase deficiency $= -135 - 90 + 63{,}43^\circ = -161{,}56^\circ$

So compensator $G_c(s) = \dfrac{s+4}{s+9}$

$$E_{ss} = \frac{1}{1+k_p} \rightarrow \text{static position error constant}$$

$$k_p = \lim_{s\to 0} G(s)$$

$$E_{ss} = \lim_{s\to 0} \frac{sR(s)}{1+G(s)} = \lim_{s\to 0} \frac{1}{1+G(s)} = \frac{1}{1+\lim_{s\to 0} G(s)} = \frac{1}{1+k_p}$$

orig. p. 10

$$G(s) = k_c \left(\frac{s+4}{s+9}\right)\left(\frac{1}{(s+1)(s+3)}\right) \quad \text{compensator system}$$

$$E_{ss} = 0{,}1 = \frac{1}{1+k_p} \Rightarrow 1 + k_p = 10 \Rightarrow k_p = 9$$

$$9 = k_p = \lim_{s\to 0} G(s) = k_c \left(\frac{4}{9}\right)\left(\frac{1}{1\times 3}\right)$$

$$k_c \left(\frac{4}{27}\right) = 9$$

$$\Rightarrow k_c = \frac{9 \times 27}{4}$$

$$G(s) = \left| k \frac{s+4}{s+9} \frac{1}{(s+1)(s+3)} \right| = 1$$

k give dominant pole