Kontrol Sistemleri

Block Diagram & Signal Flow Graph

Original notes: Control-Systems-lecture-3.pdf

UYARI Bu belge, el yazısı ders notlarından Claude (Anthropic) yardımıyla dijitale aktarılmıştır. Yardımcı kaynak olarak hazırlanmıştır; hata veya eksik içerebilir. Lütfen orijinal notları ve ders kaynaklarını esas alın.

NOTICE *This document was transcribed from handwritten lecture notes with the help of Claude (Anthropic). It is meant as a supplementary resource and may contain errors or omissions. Always check against the original notes and official course material.*

orig. p. 1

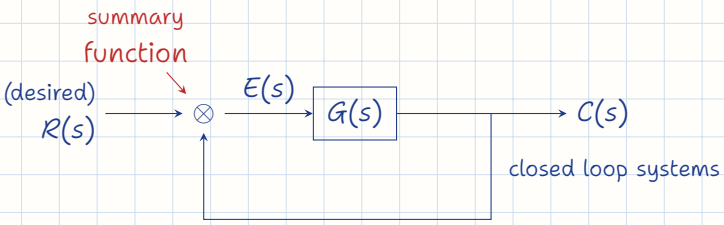
Representation methods in the Freq. domain

1- Block diagram ✓
2- Signal-flow plot

$$\text{Transfer function } (G(s)) = \frac{\text{Output}}{\text{Input}} = \frac{C(s)}{R(s)}$$

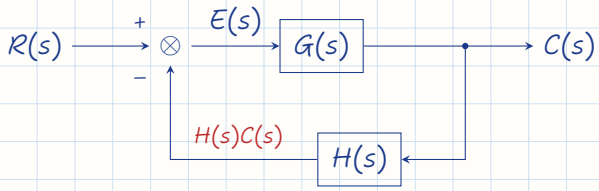


$$C(s) = R(s)\,G(s)$$



$$E(s) = R(s) - C(s)^{1}$$

Transducers are required



Feed forward T.F.

[1] Claude notu: Orijinalde eksi işareti görünmüyor.

F.F. = $G(s)$

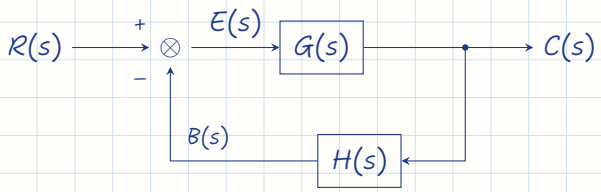
Closed loop T.F

$$\frac{C(s)}{R(s)}$$

$$E(s) = R(s) - H(s)\,C(s)$$
$$C(s) = E(s)\,G(s)$$
$$C(s) = [R(s) - H(s)\,C(s)]\,G(s)$$
$$C = RG - HCG$$
$$C[1 + HG] = RG$$
$$\frac{C}{R} = \frac{G}{1 + HG} = \frac{\text{feed forward T.F}}{1 \mp \text{open loop T.F}}$$ (for negative feedback: +)

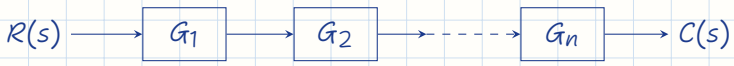
orig. p. 2

open Loop T.F.



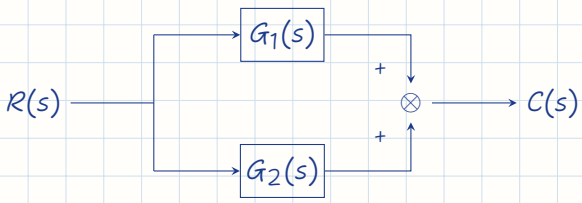
$$\frac{B(s)}{R(s)} = G(s)\,H(s)$$

Cascade configuration

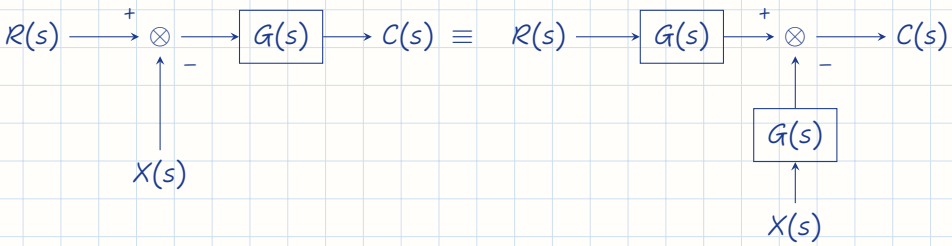


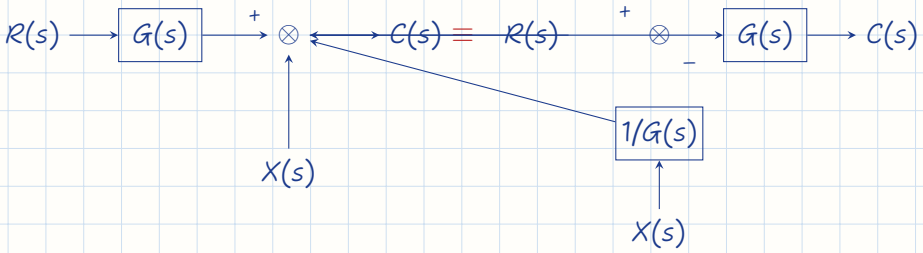
cascade config

$$\frac{C(s)}{R(s)} = G_1\,G_2 \cdots G_n$$



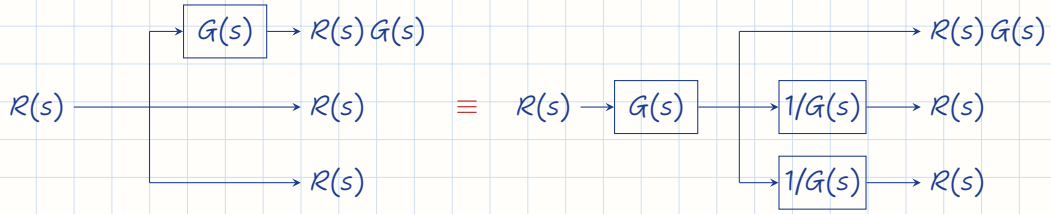
$$C(s) = R(s)\,G_1(s) + R(s)\,G_2(s)$$
$$\frac{C(s)}{R(s)} = G_1(s) + G_2(s)$$

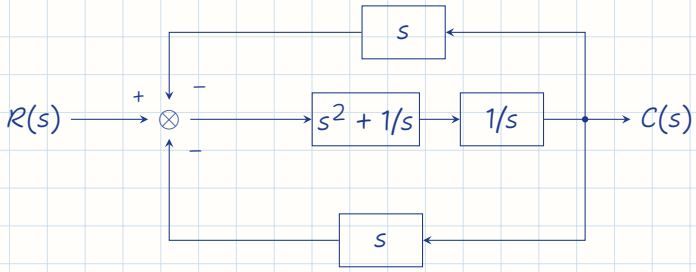




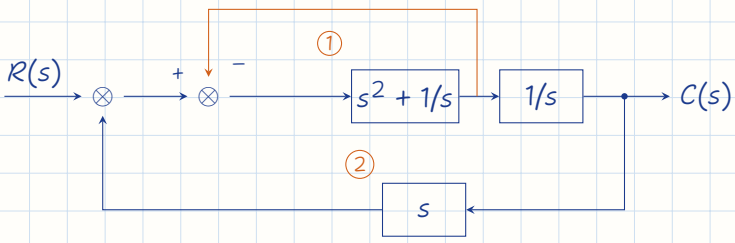
orig. p. 3

Pickoff points

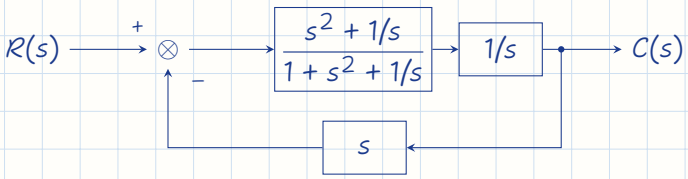




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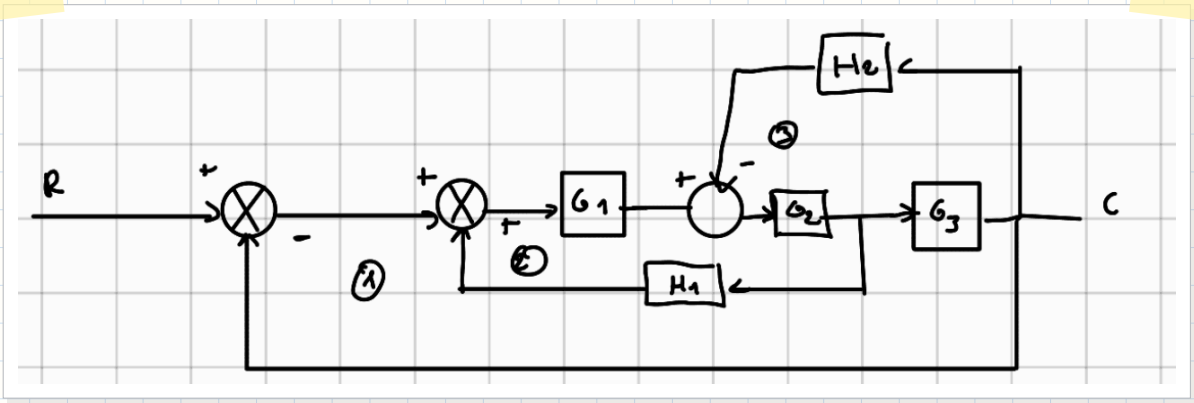
$$TF_1 = \frac{s^2 + 1/s}{1 + s^2 + 1/s}$$



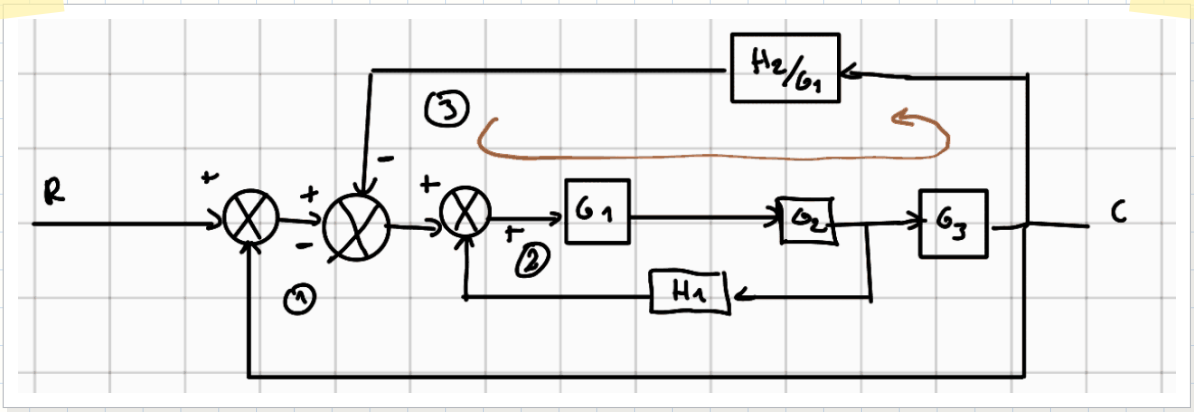
$$\frac{C(s)}{R(s)} = \frac{\frac{1}{s}\left(\frac{s^2+1/s}{1+s^2+1/s}\right)}{1 + \frac{1}{s}\left(\frac{s^2+1/s}{1+s^2+1/s}\right)s}$$ [2]

[2] Claude notu: Sayfanın alt kenarı kesik olduğundan son satır (payda) kısmen görünüyor; ifade tahmindir.

orig. p. 4



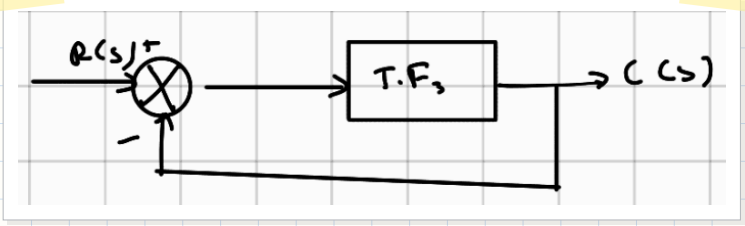
original drawing, p. 4



original drawing, p. 4

$$TF_2 = \frac{G_1 G_2}{1 - H_1 G_1 G_2}$$

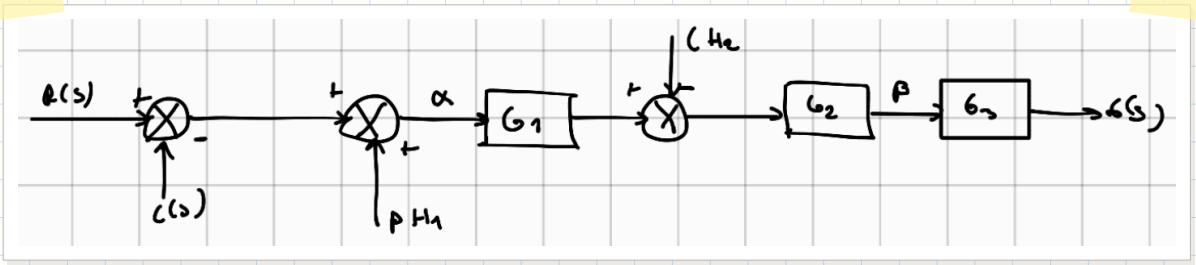
$$TF_3 = \frac{\left(\frac{G_1 G_2}{1 - H_1 G_1 G_2}\right) G_3}{1 + \frac{G_1 G_2}{1 - G_1 G_2 H_1} G_3 \frac{H_2}{G_1}}$$



original drawing, p. 4

$$\frac{C(s)}{R(s)} = \frac{TF_3}{1 + TF_3}$$

Transverse signal approach



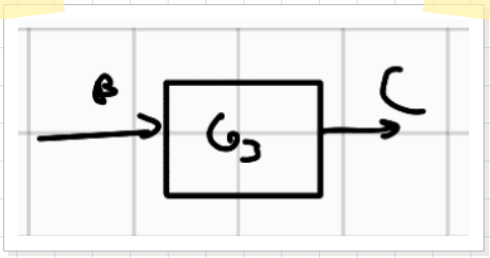
original drawing, p. 4

$$\Big[\underbrace{(R - C + \beta H_1)}_{\alpha} G_1 - CH_2\Big] G_2 = \beta$$

$$\big[(R - C) G_1 - CH_2\big] G_2 + \beta H_1 G_1 G_2 = \beta$$

orig. p. 5

$$\beta = \frac{\big[(R - C)G_1 - CH_2\big] G_2}{1 - H_1 G_1 G_2}$$



original drawing, p. 5

$$C = \beta G_3$$

$$\hookrightarrow \frac{\big[(R - C)G_1 - CH_2\big] G_2 G_3}{1 - H_1 G_1 G_2} = C$$

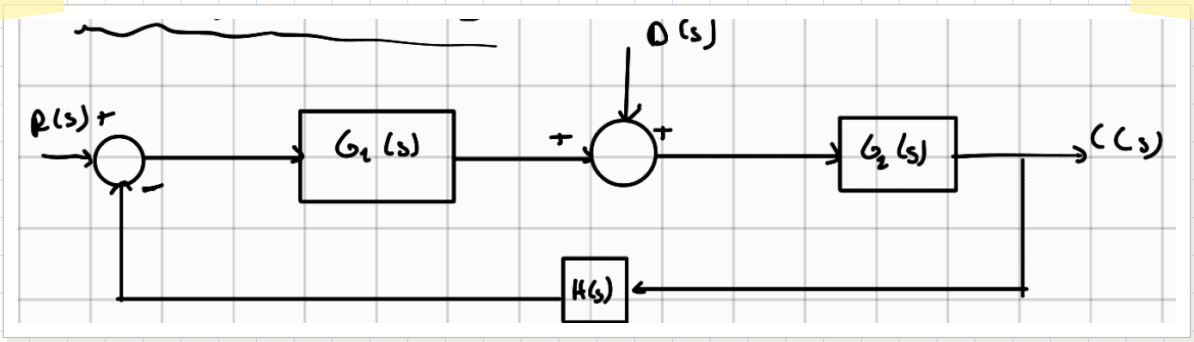
$$\Rightarrow \frac{R G_2 G_3}{1 - H_1 G_1 G_2}{}^{3} - \frac{C G_1 G_2 G_3 + CH_2 G_2 G_3}{1 - H_1 G_1 G_2} = C$$

$$\underbrace{C\left[1 + \frac{G_2 G_3}{1 - H_1 G_1 G_2}(G_1 + H_2)\right]}_{\Phi_1} = \left(\underbrace{\frac{G_2 G_3}{1 - H_1 G_1 G_2}}_{\Phi_2}\right) R^{4}$$

$$\rightarrow \frac{C}{R} = \frac{\Phi_2}{\Phi_1}$$

[3] Claude notu: muhtemelen pay $R G_1 G_2 G_3$ olmalı.
[4] Claude notu: muhtemelen pay $G_1 G_2 G_3$ olmalı.

Noises/distrubances



original drawing, p. 5

Superposition principle

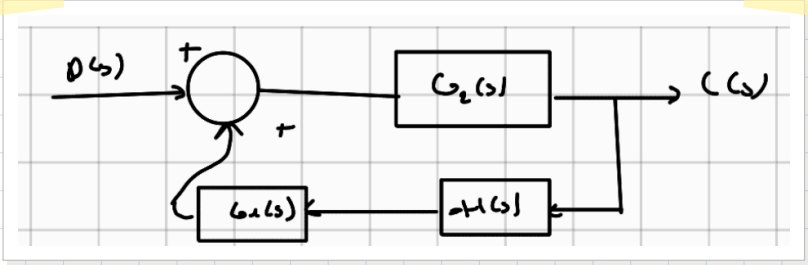
$$C(s) = \underbrace{C_R(s)}_{\text{Response due to } R(s)} + \overbrace{C_D(s)}^{\text{Response due to } D(s)}$$

case 1) $R(s) \neq 0$, $D(s) = 0$

$$\frac{C(s)}{R(s)} = \frac{G_1(s)G_2(s)}{1 + G_1G_2H} \rightarrow C_R(s) = \frac{G_1G_2}{1 + G_1G_2H} R$$

case 2) $R(s) = 0$, $D(s) \neq 0$

orig. p. 6



original drawing, p. 6

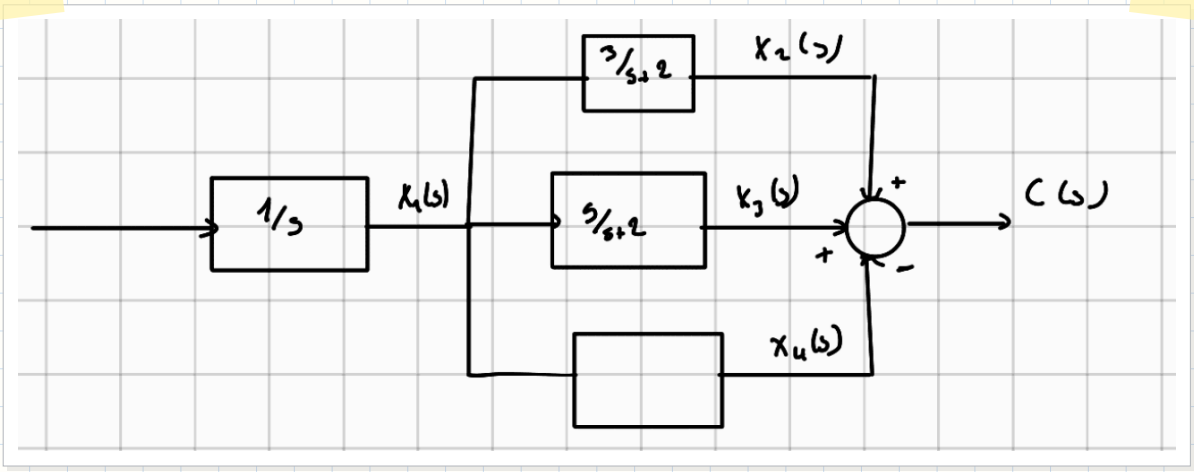
$$\frac{C(s)}{D(s)} = \frac{G_2}{1 + G_2G_1H}$$

$$\rightarrow C_D = \left(\frac{G_2}{1 + G_1G_2H}\right) D(s)$$

$$C(s) = \frac{G_1G_2}{1 + G_1G_2H} R + \frac{G_2}{1 + G_1G_2H} D$$

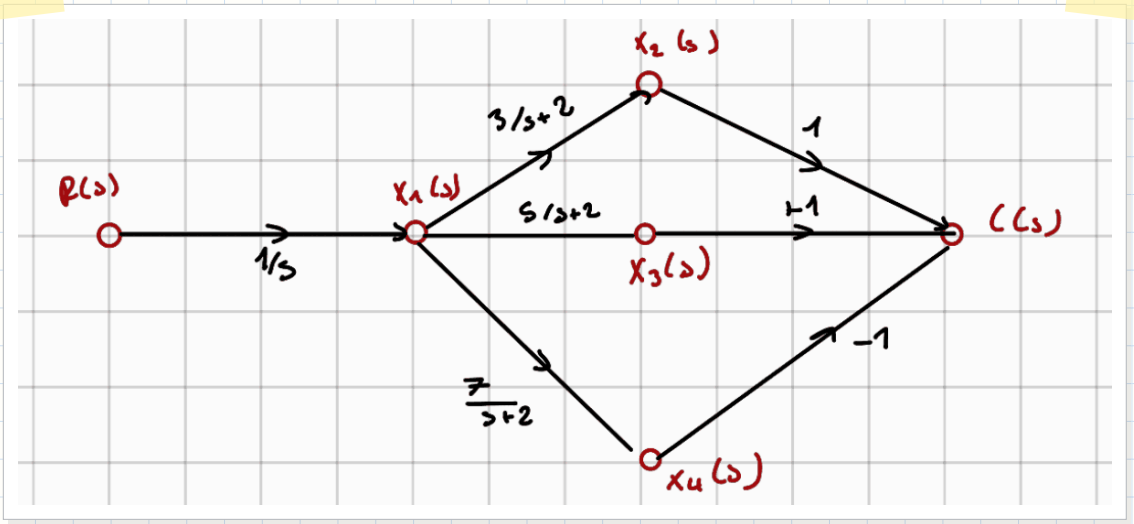
$$= \frac{G_2}{1 + G_1G_2H} (G_1R + D)$$

Signal Flow representation /graph



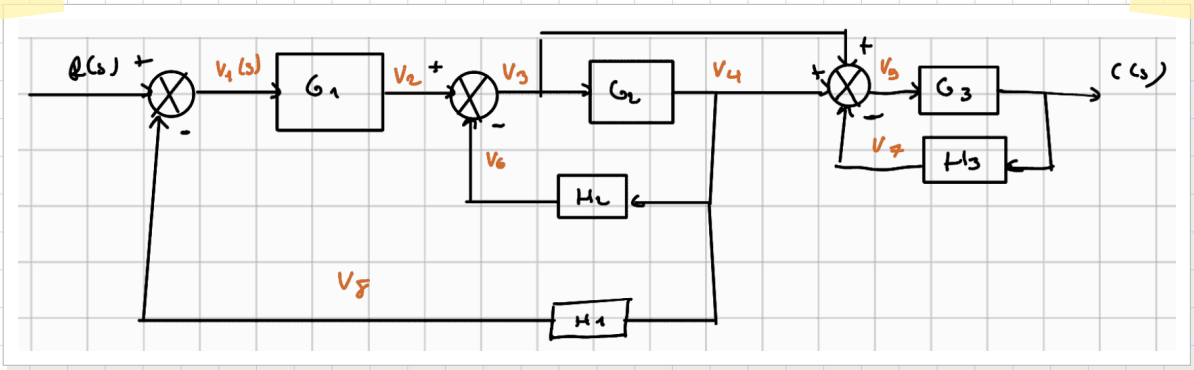
original drawing, p. 6

6 nodes

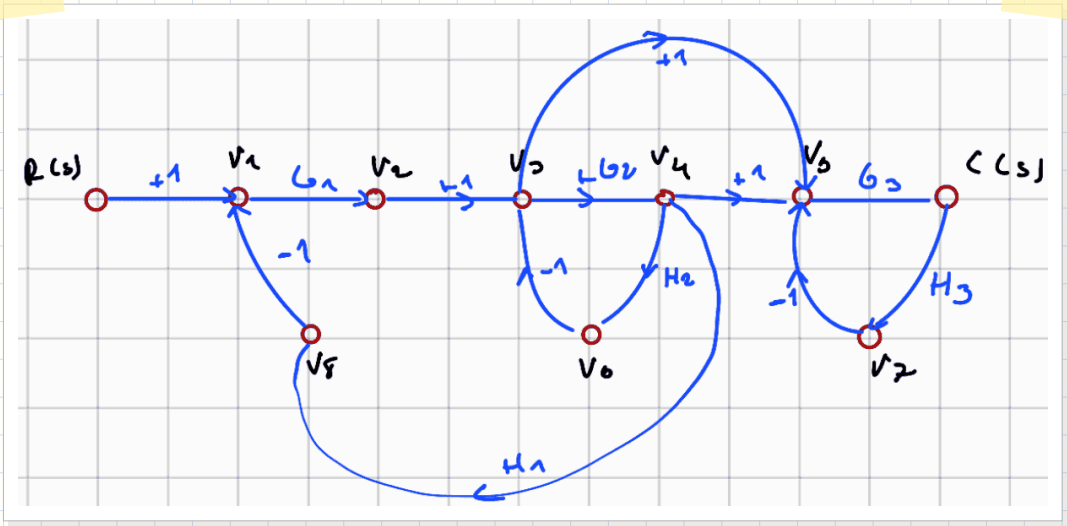


original drawing, p. 6

orig. p. 7



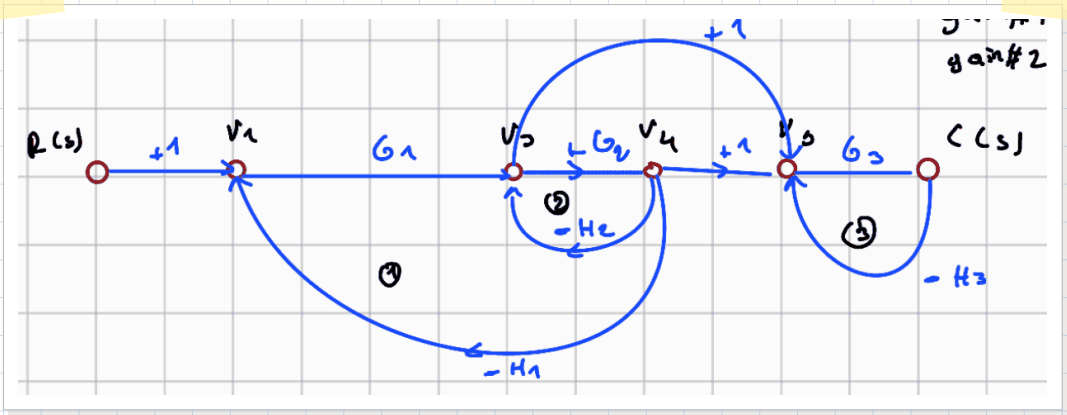
original drawing, p. 7



original drawing, p. 7

non touching loops (2 ?in katına)
⊗ Birbirine dokunmayan loopların gainleri (küçük loop gainleri çarpılır)

gain#1 : $G_2H_2G_3H_3$
gain#2 : $G_1G_2G_3H_1H_3$



original drawing, p. 7

Feed forward path
$R(s) \rightarrow C(s)$
T_1 = # 1 : $G_1G_2G_3$ first path
T_2 = # 2 : G_1G_3 second path

There is 3 loop

loop
1 : $-G_1G_2H_1$
2 : $-G_2H_2$
3 : $-G_3H_3$

Mason's rule

$$G(s) = \frac{C(s)}{R(s)} = \frac{\sum_{k=1} T_k \Delta_k}{\Delta}$$

orig. p. 8

k: number of forward paths

T_k: forward path gain for path # k

$$\Delta = 1 - \sum \text{loop gains} + \sum \begin{matrix}\text{non touching loop gains} \\ \text{taken 2 at a time}\end{matrix} - \sum \begin{matrix}\text{non touching loop gains} \\ \text{taken 3 at a time}\end{matrix}$$

$$+\sum_{\substack{\text{non touching loop gains}\\ \text{taken 4 at a time}}} \cdots$$

$$\Delta_k = \Delta - \sum_{\substack{\text{loop gains that touch}\\ \text{the } k^{th} \text{ forward path}}}$$

$$\Delta = 1 + G_1G_2H_1 + G_2H_2 + G_3H_3 + (G_2G_3H_2H_3 + G_1G_2G_3H_1H_3)$$

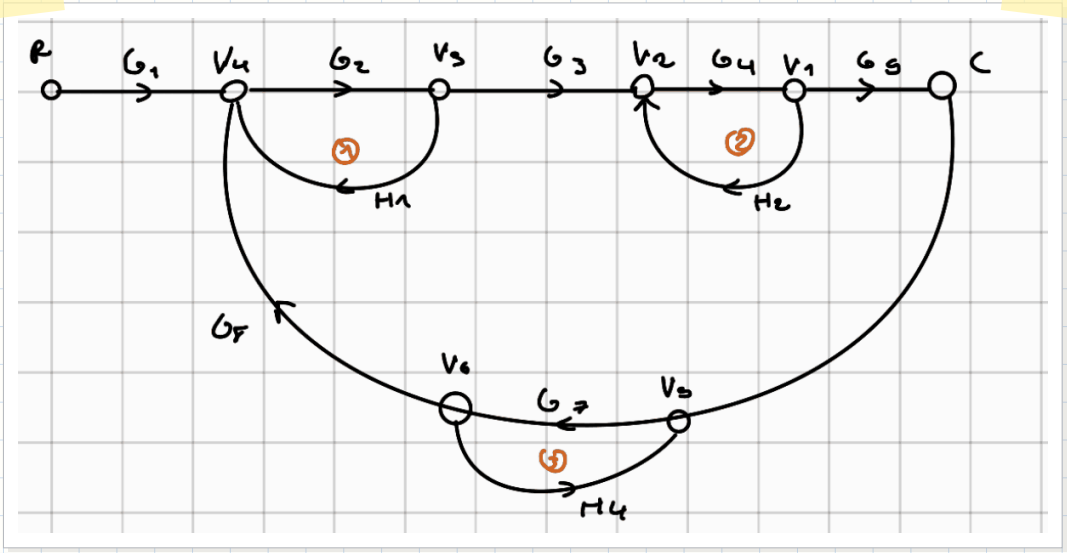
$$\Delta_k = \Delta - \sum_{\substack{\text{loop gains that touch}\\ \text{the } k^{th} \text{ forward path}}}$$

(make the touching gain 0 in Δ)

$$\Delta_1 = 1 \qquad (H_1, H_2, H_3 = 0)$$

$$\Delta_2 = 1$$

$$\frac{C(s)}{R(s)} = \frac{T_1\overset{1}{\Delta_1} + T_2\overset{1}{\Delta_2}}{\Delta} = \frac{G_1G_2G_3 + G_1G_3}{\Delta}$$



original drawing, p. 8

Feed Forward path : $G_1G_2G_3G_4G_5$

loops : # 1 : G_2H_1
2 : G_4H_2
3 : $G_2G_3G_4G_5G_6G_7G_8$
4 : G_7H_4

orig. p. 9

Non touching loop gains (2 taken at a time)

loops 1,2 : $G_2H_1\ G_4H_2$
loops 1,4 : $G_2H_1\ G_7H_4$
loops 2,4 : $G_4H_2\ G_7H_4$

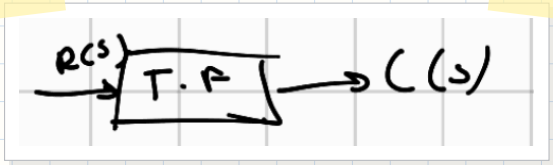
Non touching loop gains (3 taken at a time)

loops 1,2,4 : G_2H_1 G_4H_2 G_7H_4

$$\Delta = 1 - G_2H_1 - G_4H_2 - G_2G_3G_4G_5G_6G_7G_8 - G_7H_4 + G_2H_1G_4H_2 + G_2H_1G_7H_4 + G_4H_2G_7H_4 - G_2G_4G_7H_1H_2H_4$$

$$\frac{C(s)}{R(s)} = \frac{\sum T_k\Delta_k}{\Delta} = \frac{T_1\Delta_1}{\Delta}$$

$$\Delta_1 = 1 - G_7H_4$$



original drawing, p. 9