Kontrol Sistemleri

Modeling: electrical systems, transfer function, state space

Original notes: Control-Systems-lecture-4.pdf

UYARI Bu belge, el yazısı ders notlarından Claude (Anthropic) yardımıyla dijitale aktarılmıştır. Yardımcı kaynak olarak hazırlanmıştır; hata veya eksik içerebilir. Lütfen orijinal notları ve ders kaynaklarını esas alın.

NOTICE *This document was transcribed from handwritten lecture notes with the help of Claude (Anthropic). It is meant as a supplementary resource and may contain errors or omissions. Always check against the original notes and official course material.*

orig. p. 1

Modeling

Differential eq for any dynamical system describes the exact response of the system for any desired input

DE → Represented (analized) in Time domain → State Space representation

→ Freq domain representation (Transfer function)

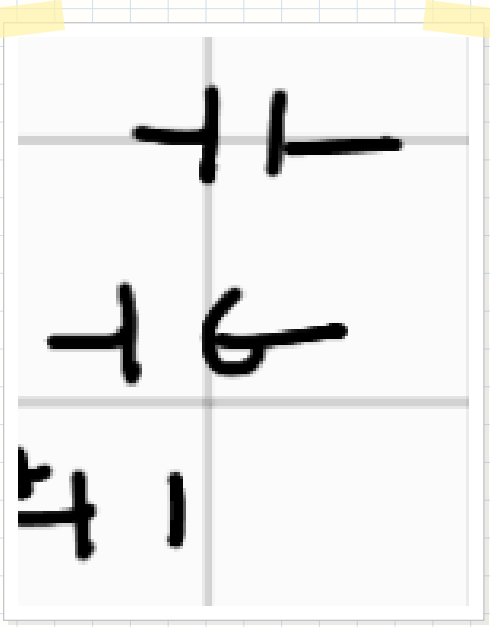
↪ graphically respresented by block-diagram or signal flow graphs

General forms appearing in engineering (DE)

Dynamical Systems

- Electrical Systems (circuits, opamp)
- Mechanical Systems (Vibration) (oscillating, gears)
- Thermal and fluid system (hydrolic, pneumatic, fluid surface control, thermal)
- Electromechanical Systems

Passive elements



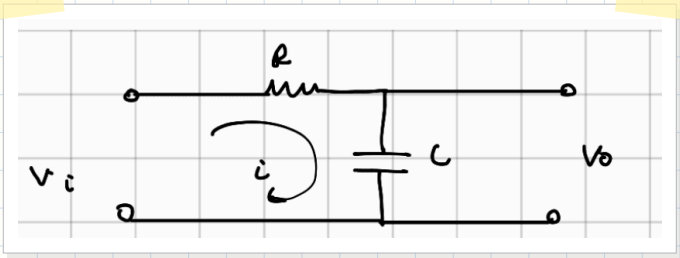
original drawing, p. 1



original drawing, p. 1

	Resistors	Capacitors	Inductors
Voltage-current	$V(t) = R\,i(t)$	$V(t) = \frac{1}{C}\int_0^t i(\tau)\,d\tau$	$V(t) = L\frac{di}{dt}$
Current-Voltage	$i(t) = \frac{V(t)}{R}$	$i(t) = C\frac{dv(t)}{dt}$	$i(t) = \frac{1}{L}\int_0^t v(\tau)\,d\tau$
Voltage-charge	$V(t) = R\frac{dq(t)}{dt}$	$V(t) = \frac{q}{C}$	$V(t) = L\frac{d^2q(t)}{dt^2}$
Impedance: $\frac{\text{Output}}{\text{input}}$	$Z = R$	$\frac{1}{Cs}$	Ls
Admittance $= \frac{1}{Z}$	$Y = \frac{1}{R}$	Cs	$\frac{1}{Ls}$

orig. p. 2



original drawing, p. 2

Voltage Law

$$\begin{cases} V_i = Ri + \frac{1}{C}\int i\,dt \\ V_o = \frac{1}{C}\int i\,dt \end{cases} \xrightarrow{\text{Laplace}} \begin{cases} V_i = RI(s) + \frac{1}{Cs}I(s) \\ V_o = \frac{1}{Cs}I(s) \end{cases}$$

$$\frac{V_o}{V_i} = \frac{\frac{1}{Cs}\cancel{I(s)}}{\left(R + \frac{1}{Cs}\right)\cancel{I(s)}} = \frac{1}{RCs+1} = \frac{1/RC}{s+1/RC} = \frac{T}{s+T}$$

$$\frac{1}{RC} = T \text{ constant}$$

$$\xrightarrow{\mathcal{L}^{-1}}$$

$$\frac{dV_o(t)}{dt} + T\,V_o(t) = T\,V_i(t)$$

First order O.E

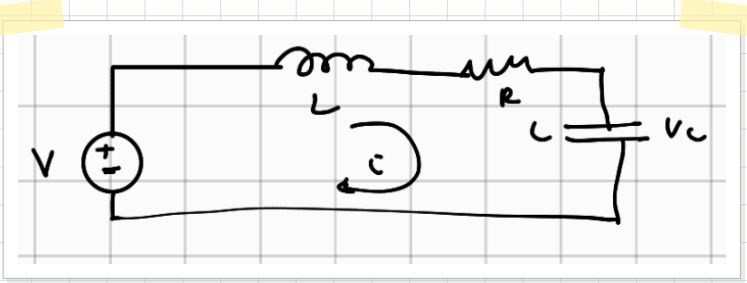
The order of input is less than the output

V_o: output according to the order of derivative

→ integration in time gives the response
@ $t = 0$, one IC
$V_o(0) = X$

You have to know initial conditions

RLC circuit in series



original drawing, p. 2

$$V(t) - Ri(t) - L\frac{di(t)}{dt} - \frac{1}{C}\int_0^t i(\tau)\,d\tau = 0$$

$$V(t) = R\,i(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int_0^t i(\tau)\,d\tau \qquad \text{integro differential eq } (*)$$

orig. p. 3

$$\begin{cases} v(t) : \text{input} \\ i(t) : \text{output} \end{cases}$$

$$\xrightarrow{\mathcal{L}} V(s) = R\,I(s) + Ls\,I(s) + \frac{1}{Cs}I(s)$$

$$\longrightarrow \frac{I(s)}{V(s)} = \frac{1}{R + Ls + \frac{1}{Cs}}$$

$$= \frac{Cs}{RCs + LCs^2 + 1} = \frac{s/L}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$\frac{I}{V} = \frac{s/L}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$\Rightarrow I\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right) = \frac{s}{L}(V)$$

$$\xRightarrow{\mathcal{L}^{-1}} \frac{d^2i(t)}{dt^2} + \frac{R}{L}\frac{di(t)}{dt} + \frac{i(t)}{LC} = \underbrace{\frac{1}{L}\frac{dV(t)}{dt}}_{\text{Derivative of the input}}$$

Second order O.E

$$\text{I.C are 2:} \quad \begin{cases} i(0) = \checkmark \\ \dfrac{di(0)}{dt} = \checkmark \end{cases}$$

$\boxed{i(t),\ \frac{di(t)}{dt}}$ → State of the System

Eq (?*)

$$\begin{cases} i(t) = \dfrac{dq(t)}{dt} \\ q(t) = C\,V_c(t) \end{cases} \Rightarrow i(t) = C\frac{dV_c(t)}{dt}$$

$$V(t) = RC\frac{dV_c(t)}{dt} + LC\frac{d^2V_c(t)}{dt^2} + \frac{1}{C}(C\,V_c(t))$$

$$\Rightarrow V = RC\,\dot{V}_c + LC\,\ddot{V}_c + V_c$$

2nd order O.E

I.C. $\begin{cases} V_c(0) \\ \dot{V}_c(0) \end{cases}$

Input : V
output : V_c

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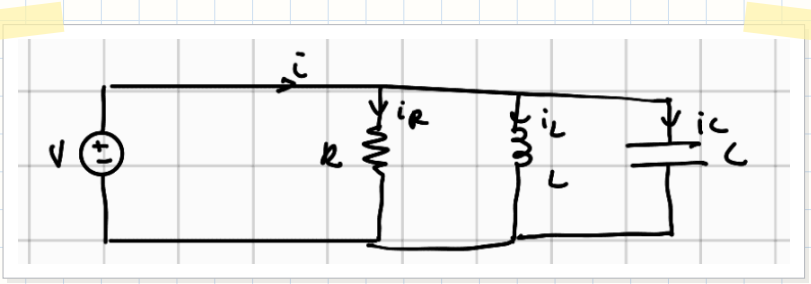
$$\xrightarrow{\mathcal{L}} V(s) = (RCs + LCs^2 + 1)\,V_c(s)$$

$$\longrightarrow \frac{V_c}{V} = \frac{1}{LCs^2 + RCs + 1}$$

$$= \frac{1/LC}{s^2 + \frac{R}{L}s + 1^1}$$

$$\frac{V_c}{V} = \frac{\alpha_1}{s^2 + \alpha_2 s + \alpha_3} \qquad \begin{cases} \alpha_1 = 1/LC \\ \alpha_2 = R/L \\ \alpha_3 = 1/LC \end{cases}$$

RLC circuit in parallel



original drawing, p. 4

$$i = i_R + i_L + i_C$$
$$= \frac{V}{R} + \frac{1}{L}\int_0^t V(\tau)\,d\tau + C\frac{dV(t)}{dt} \qquad **$$

input : $i(t)$
output : $V(t)$

$$I(s) = \frac{V(s)}{R} + \frac{V(s)}{Ls} + Cs\,V(s)$$
$$\longrightarrow \frac{V(s)}{I(s)} = \frac{1}{\frac{1}{R} + \frac{1}{Ls} + Cs}$$

Here we have the derivative of input

Eq (?**)

$$V(t) = \frac{d\Phi(t)}{dt} \quad \rightarrow \text{ magnetic flux (Faraday law)}$$

$$i(t) = \frac{d\Phi(t)}{R\,dt} + \frac{\Phi(t)}{L} + C\frac{d^2\Phi(t)}{dt^2}$$

2nd order
input: i
output: $\Phi(t)$

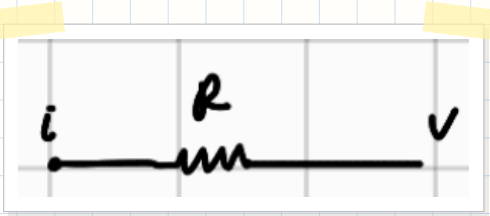
$$\Rightarrow I(s) = \left(\frac{s}{R} + \frac{1}{L} + Cs^2\right)\Phi(s)$$

$$\rightarrow \frac{\Phi(s)}{I(s)} = \frac{1}{Cs^2 + \frac{s}{R} + \frac{1}{L}} = \frac{1/C}{s^2 + \frac{s}{RC} + \frac{1}{LC}} = \frac{\beta_1}{s^2 + \beta_2 s + \beta_3} \qquad \begin{cases} \beta_1 = 1/C \\ \beta_2 = 1/RC \\ \beta_3 = 1/LC \end{cases}$$

orig. p. 5

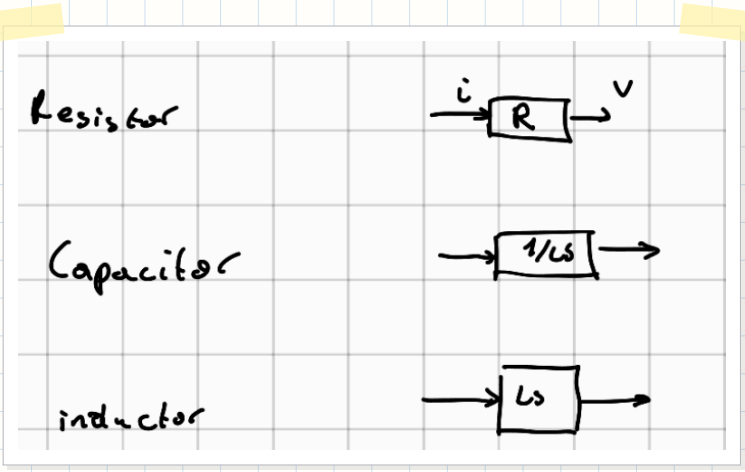
[1] Claude notu: muhtemelen 1/LC olmalı.

Methods of Complex impedences

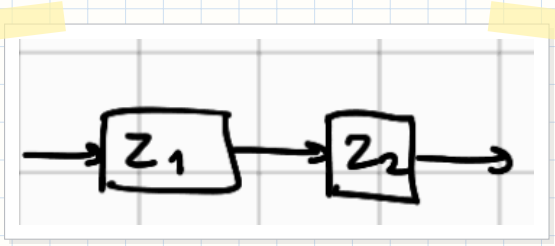


original drawing, p. 5

$$V = iR$$



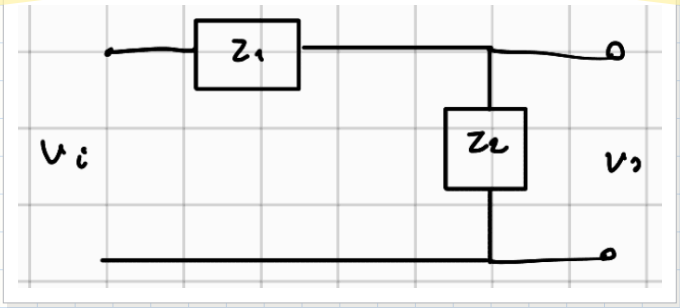
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original drawing, p. 5

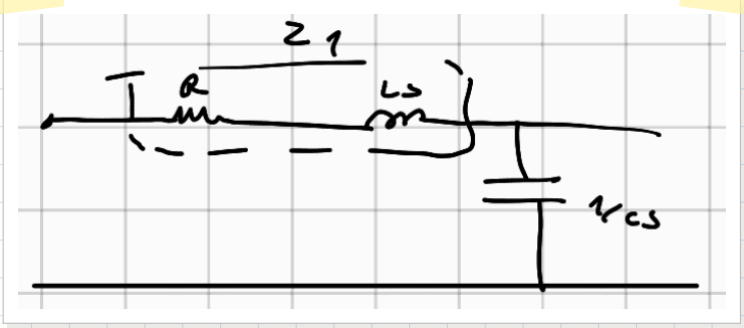
Impedance in Series $Z_{eq} = \sum Z_i$

" " parallel $Z_{eq} = \dfrac{1}{\sum \frac{1}{Z_i}}$



original drawing, p. 5

$$\frac{V_o(s)}{V_i(s)} = \frac{Z_2 I(s)}{Z_1 I(s) + Z_2 I(s)} = \frac{Z_2}{Z_1 + Z_2}$$

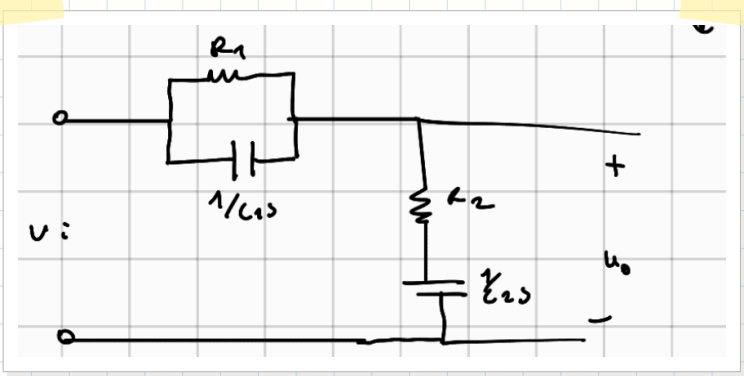


original drawing, p. 5

$$Z_2 = \frac{1}{Cs}$$
$$Z_1 = R + Ls$$

$$\frac{V_o}{V_i} = \frac{1/Cs}{R + Ls + 1/Cs} = \frac{1}{RCs + LCs^2 + 1}$$

$$\xrightarrow{\mathcal{L}^{-1}} V_i = RC\frac{dV_o}{dt} + LC\frac{d^2V_o}{dt^2} + V_o$$



original drawing, p. 5

$$Z_1 = \frac{1}{\frac{1}{R_1} + C_1 s} \qquad Z_2 = R_2 + \frac{1}{C_2 s}$$

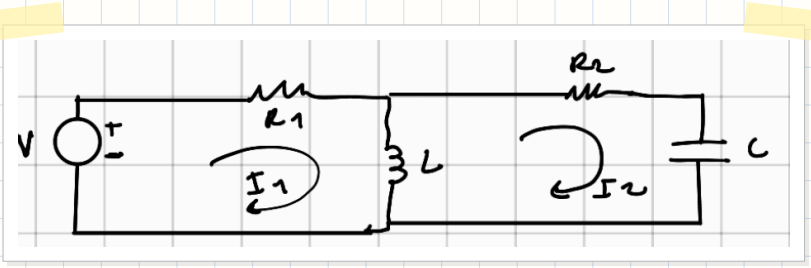
orig. p. 6

$$\frac{V_o}{V_i} = \frac{R_1 C_1 R_2 C_2 s^2 + (R_1 C_1 + R_2 C_2) s + 1}{R_1 C_1 R_2 C_2 s^2 + (R_1 C_1 + R_2 C_2 + R_1 C_2) s + 1}$$

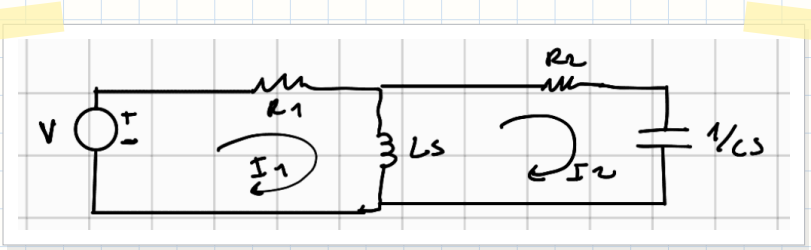
$$\frac{V_o}{V_i} = \frac{\alpha_1 s^2 + \alpha_2 s + 1}{\alpha_3 s^2 + \alpha_4 s + 1}$$

$$\xRightarrow{\mathcal{L}^{-1}} \alpha_1 \frac{d^2 v_i}{dt^2} + \alpha_2 \frac{dv_i}{dt} + v_i = \alpha_3 \frac{d^2 v_o}{dt^2} + \alpha_4 \frac{dv_o}{dt} + v_o$$

There is no dynamic behavior there is ? statics



original drawing, p. 6



original drawing, p. 6

Mesh #1 $\quad R_1 I_1 + Ls\, I_1 - I_2 Ls = V \Rightarrow (R_1 + Ls) I_1 - Ls\, I_2 = V$

summation of impedances in mesh: $(R_1 + Ls)$
impedance of the common element: Ls
Voltage source in mesh: V

Mesh #2 $\quad Ls\, I_2 + R_2 I_2 + \frac{1}{Cs} I_2 - Ls\, I_1 = 0$

$$\underbrace{\left(Ls + R_2 + \frac{1}{Cs}\right)}_{\text{summation of all impedances in mesh 2}} \qquad I_2 - Ls\, I_1 = 0$$

impedance on the common element $(Ls\, I_1)$

$$(R + Ls) I_1 - Ls\, I_2 = V$$

$$\left(R_2 + \frac{1}{Cs} + Ls\right) I_2 - Ls\, I_1 = 0 \quad \rightarrow \quad I_1 = \frac{R_2 + Ls + 1/Cs_2}{Ls}$$

$$\begin{cases} I_1 = ? \\ I_2 = ? \end{cases}$$

substitution gives

$$(R + Ls)\frac{R_2 + Ls + 1/Cs}{Ls} I_2 - Ls I_2 = V$$

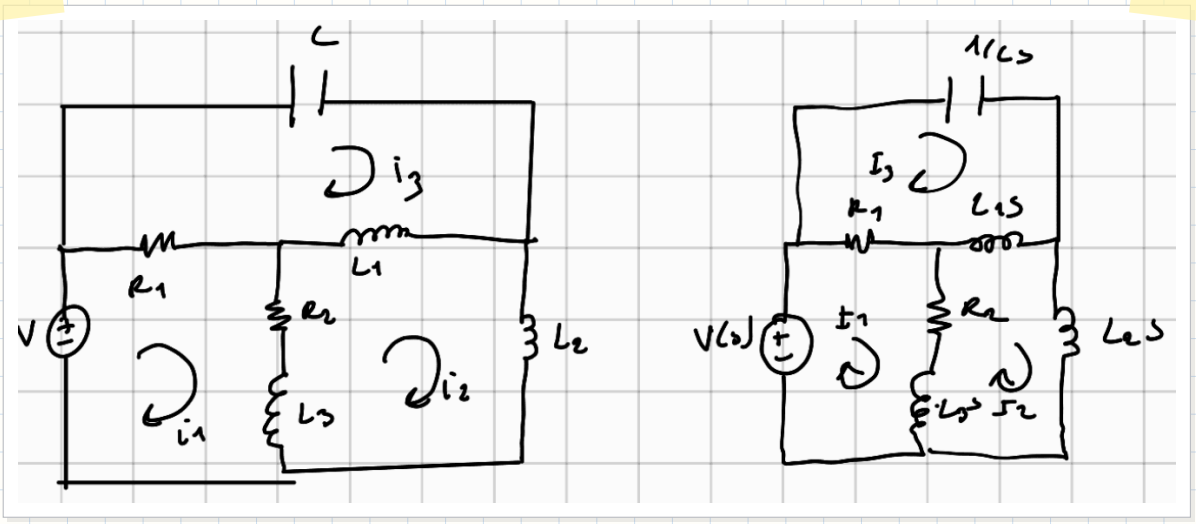
orig. p. 7

$$\longrightarrow \frac{I_2}{V} = \frac{1}{\left(\frac{R}{Ls} + 1\right)\left(R_2 + \left(s + \frac{1}{Cs}\right)\right) - Ls}$$ [3]

$$\frac{I_2}{V} = \frac{LCs^2}{(R_1 + R_2)CLs^2 + (R_1R_2C + L)s + R_1}$$

$$\Rightarrow LC\frac{d^2v}{dt^2} = (R_1 + R_2)CL\frac{d^2i_2}{dt^2} + (R_1R_2C + L)\frac{di_2}{dt} + R_1 i_2$$

$$\frac{I_1}{V} = \frac{Cs\left(Ls + R_2 + \frac{1}{Cs}\right)}{(R_1 + R_2)CLs^2 + (R_1R_2C + L)s + R_2}$$ [4]



original drawing, p. 7

mesh #1

$$\left(\begin{array}{c}\text{sum of impedances}\\ \text{in the loop}\end{array}\right) I_1 - \left(\begin{array}{c}\text{sum of common}\\ \text{impedances}\\ \text{between loops}\\ \text{1 and 2}\end{array}\right) I_2$$

$$- \left(\begin{array}{c}\text{sum of impedances}\\ \text{common between}\\ \text{loops 1,3}\end{array}\right) I_3 = \begin{array}{c}\text{sum of voltage}\\ \text{sources in the}\\ \text{loop}\end{array}$$

[2] Claude notu: muhtemelen I_2 ile çarpılmalı.
[3] Claude notu: muhtemelen $Ls + 1/Cs$ olmalı.
[4] Claude notu: muhtemelen R_1 olmalı.

$$\#1: \quad (R_1 + R_2 + L_1 s) I_1 - (R_2 + L_3 s) I_2 - R_1 I_3 = V^{5}$$
$$\#2: \quad (L_1 s + L_2 s + L_3 s + R_2) I_2 - (R_2 + L_3 s) I_1 - L_1 s I_3 = 0$$
$$\#3: \quad \left(\frac{1}{Cs} + L_1 s + R_1\right) I_3 - L_1 s I_2 - R_1 I_1 = 0$$

Solve it for $I_1, I_2, I_3 = f(U)$

$$\ddot{x} + \alpha_1 \dot{x} + \alpha_2 x = \underbrace{\beta_0 \ddot{u}}_{\text{very rare}} + \beta_1 \dot{u} + \beta_2 u$$

orig. p. 8

State space equation

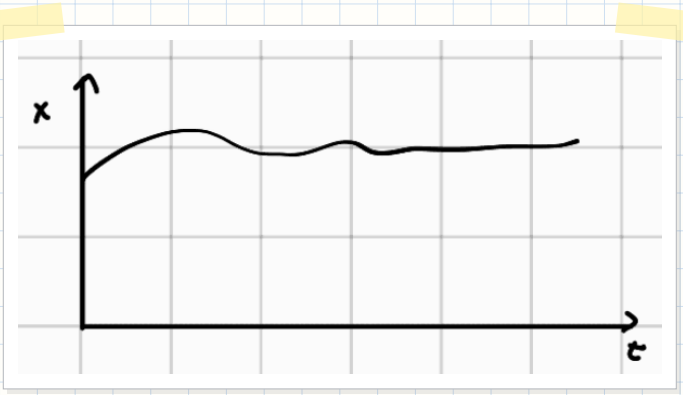
2^{nd} order O.E $\xrightarrow{\text{s.s.}}$ 2 first order O.E

sys of { 2^{nd} order equation (n) number } $\xrightarrow{\text{s.s}}$ 2n " " "

Example

$$\ddot{x} + \alpha_1 \dot{x} + \alpha_2 x = u$$

x: output 2^{nd} order O.E
u: input



original drawing, p. 8

Initial conditions
$x,\ \dot{x}$

State variables

$$y_1 = x$$
$$y_2 = \dot{x}$$

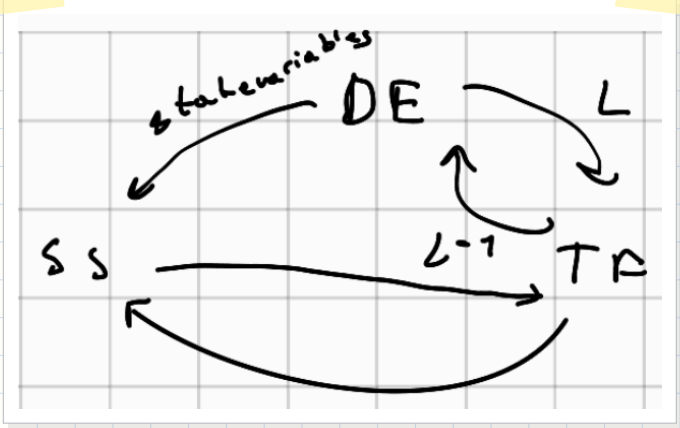
$$\dot{y}_1 = \dot{x} = y_2$$
$$\dot{y}_2 = \ddot{x} = u - \alpha_1 \dot{x} - \alpha_2 x = u - \alpha_1 y_2 - \alpha_2 y_1$$

[5] Claude notu: muhtemelen $L_3 s$ olmalı.

$$\underbrace{\frac{d}{dt}\begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix}}_{\substack{\text{state variable} \\ \text{vector} \\ \dot{\vec{X}}}} = \underbrace{\begin{bmatrix} 0 & 1 \\ -\alpha_2 & -\alpha_1 \end{bmatrix}}_{A} \underbrace{\begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix}}_{\vec{X}} + \underbrace{\begin{Bmatrix} 0 \\ 1 \end{Bmatrix}}_{\substack{\text{input} \\ \text{vector} \\ \vec{B}}} \underbrace{u}_{\text{input}}$$

$$\dot{\vec{X}} = A\vec{X} + \vec{B}u$$

orig. p. 9



original drawing, p. 9