Kontrol Sistemleri

Stability, Routh Criterion, Control Actions

Original notes: Control-Systems-lecture-8.pdf

UYARI Bu belge, el yazısı ders notlarından Claude (Anthropic) yardımıyla dijitale aktarılmıştır. Yardımcı kaynak olarak hazırlanmıştır; hata veya eksik içerebilir. Lütfen orijinal notları ve ders kaynaklarını esas alın.

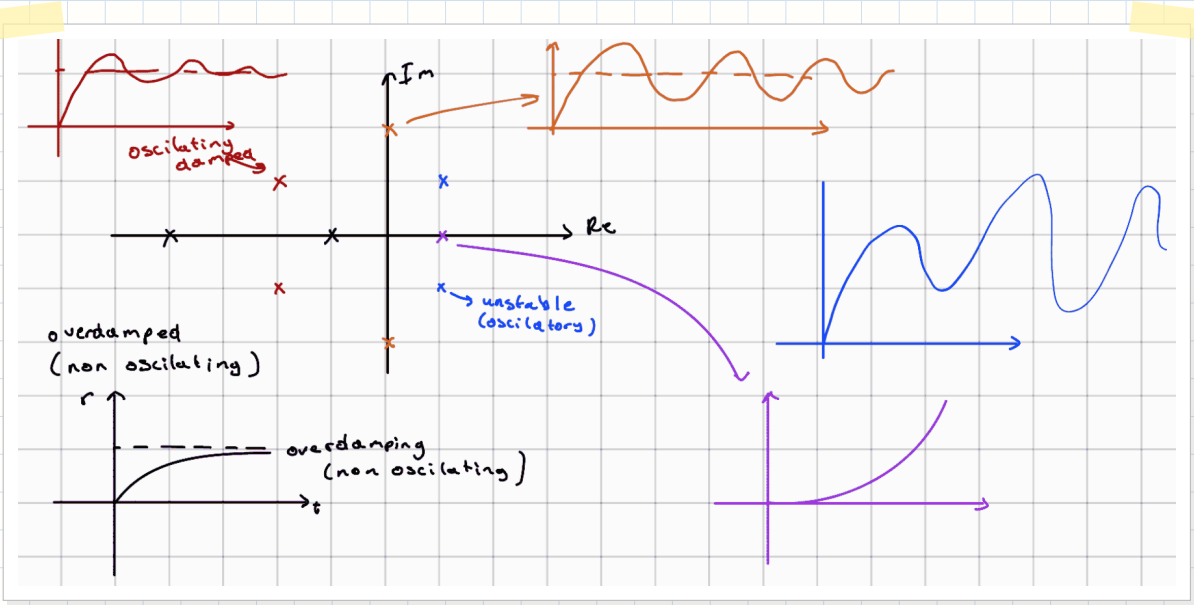
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orig. p. 1

1. Quality of response

$$\text{Closed Loop TF} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \Delta(s)$$

$\Delta(s) = 0 \rightarrow$ characteristic roots (poles)

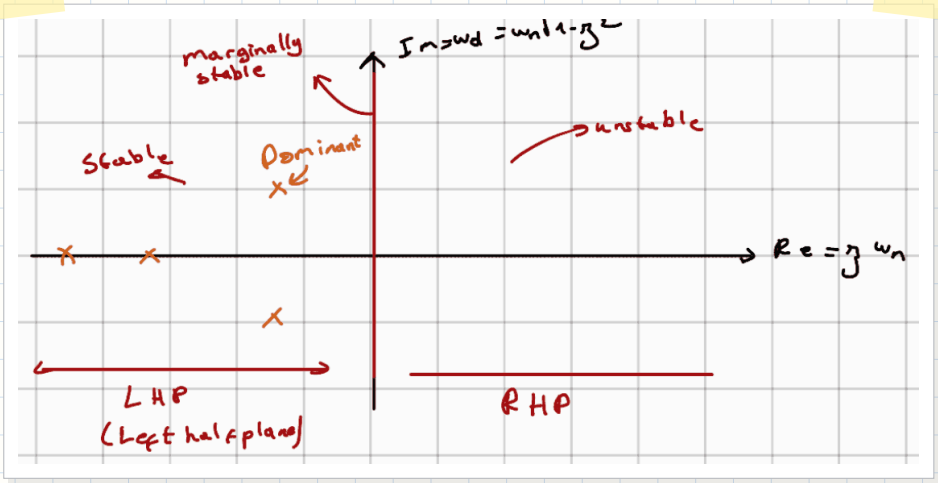


original drawing, p. 1

2. Stability

This is a characteristic of the system, and 'not' governed by the input signal.

⋆ States of stability



original drawing, p. 1

orig. p. 2

Routh methode

Gives a way of determining stability state without the requirement of finding the poles

– No specific pole is given but the numbers of poles on the RHP is found.

$$\frac{C(s)}{R(s)} = \frac{\overbrace{b_0 s^m + b_1 s^{m-1} + \cdots + b_m}^{\text{zeros}}}{\underbrace{a_0 s^n + a_1 s^{n-1} + \cdots + a_n}_{\text{poles}\quad \Delta(s)\text{ characteristic eq}}} \qquad m \le n$$

⋆ zero pole cancellation is not happening

$$\Delta(s) = a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n \quad : \quad \text{polynomial of degree 'n'}$$

if we can decompose it: $(a_1 s^2 + a_2 s + a_3)$ and $(\beta_1 s + \beta_2)$ by multiplying them

$s + 2 = 0 \rightarrow s = -2$

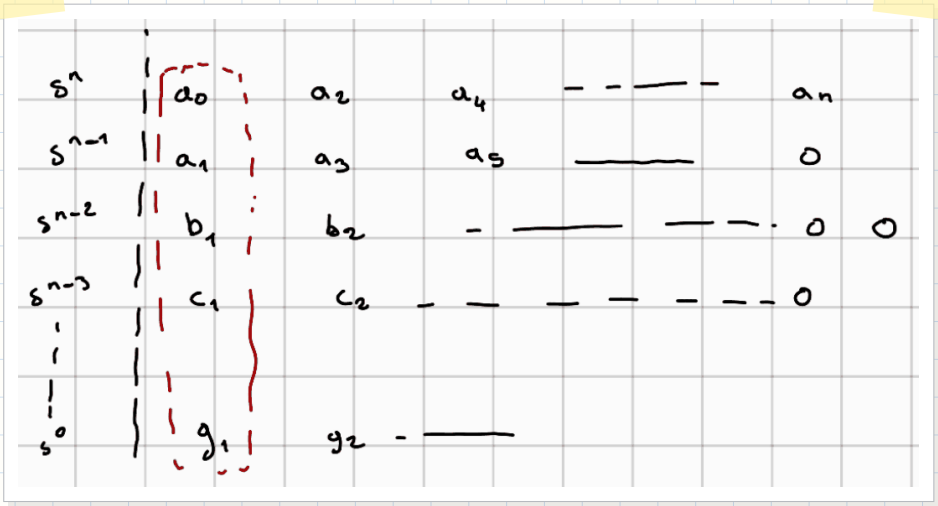
$s - 2 = 0 \rightarrow s = +2$

so β_1 and β_2 should be pozitive $\qquad \beta_1 = \frac{a_{n-1} a_n}{a_2}$

Requirement #1: all coefficients (a_n) must be positive and nonzero!!

Requirement #2: Construct the table of Routh

s^n	a_0	a_2	a_4	$\cdots$	a_n
s^{n-1}	a_1	a_3	a_5	$\cdots$	0
s^{n-2}	b_1	b_2	$\cdots$	0	0
s^{n-3}	c_1	c_2	$\cdots$	0	
$\vdots$					
s^0	g_1	g_2	$\cdots$		



original drawing, p. 2

orig. p. 3

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1}$$

$$b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1}$$

$$\vdots$$

$$c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1}$$

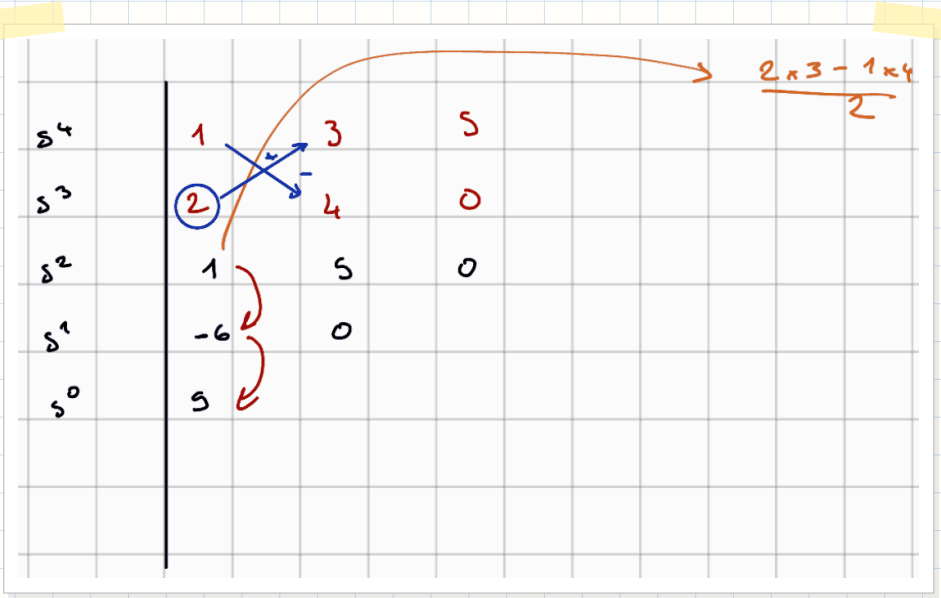
Conclusion: The number changes in signs = number of poles on the RHP (look to the first column)

Example

$s^4 + 2s^3 + 3s^2 + 4s + 5 = 0$

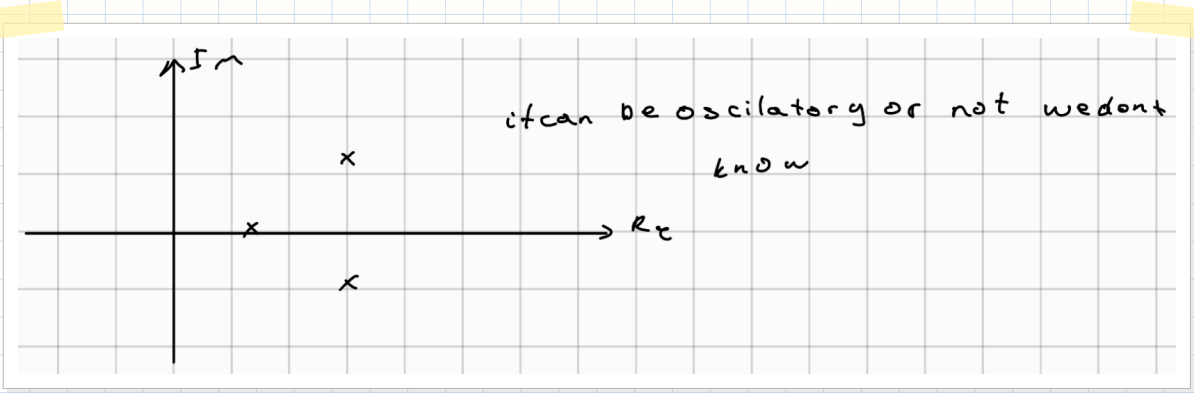
Pass the necessary condition

s^4	1	3	5
s^3	2	4	0
s^2	1	5	0
s^1	−6	0	
s^0	5		



original drawing, p. 3

2 changes of sign: two poles on the RHP.



original drawing, p. 3

it can be oscilatory or not, we don't know

orig. p. 4

Exception #1

The value of the cell in the first column at some specific row becomes zero

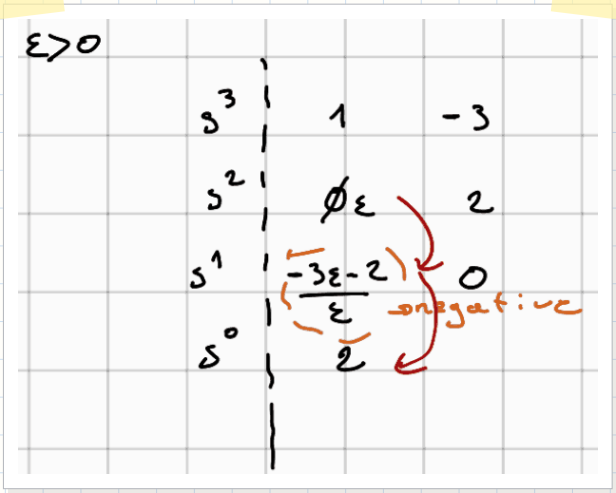
↪ just replace zero with $\varepsilon > 0$ and move on

$$\Delta(s) = s^3 - 3s + 2 = 0 \Rightarrow \boxed{s^3 + 0s^2 - 3s + 2} = 0$$

→ Definitely unstable because there is negative coefficient

$\varepsilon > 0$

s^3	1	-3
s^2	ε	2
s^1	$\frac{-3\varepsilon - 2}{\varepsilon}$	0
s^0	2	



original drawing, p. 4

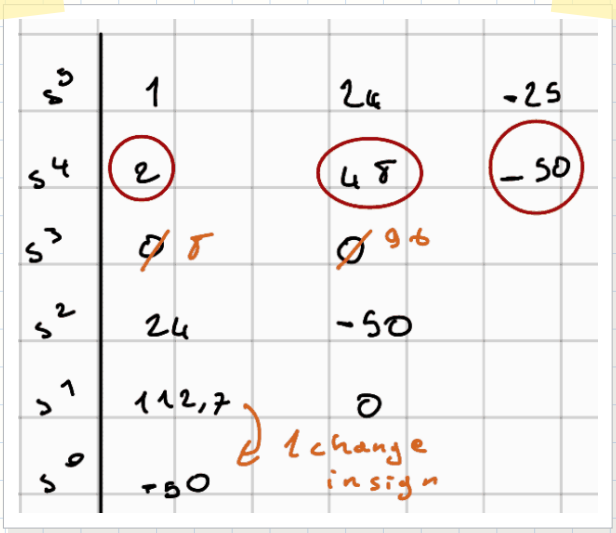
we have got two roots on the jω axis.[1]

Exception #2

getting a full zero row!

$$s^5 + 2s^4 + 24s^3 + 48s^2 - 25s - 50 = 0$$

s^5	1	24	−25
s^4	2	48	−50
s^3	8	96	
s^2	24	−50	
s^1	112,7	0	
s^0	−50		



original drawing, p. 4

[1] Claude notu: muhtemelen "RHP" olmalı (iki işaret değişimi var).

1 change in sign → 1 pole: it have to be real because complex pole have conjugate

construct an auxilary equation

$$P(s) = 2s^4 + 48s^2 - 50$$

$$\frac{dP}{ds} = 8s^3 + 96s \quad \rightarrow \quad \text{we have to use it}$$

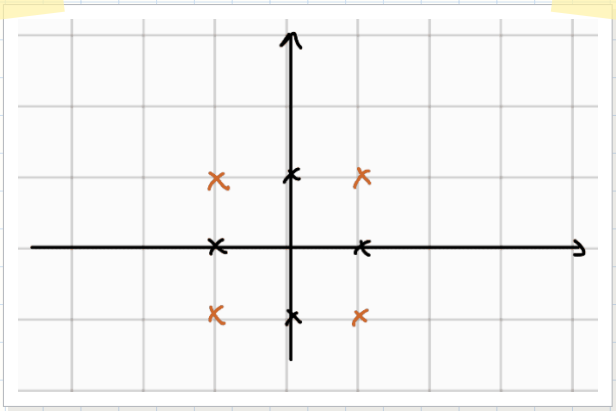
Solution of the auxilary eq. gives us some of the poles of the main system

$$2s^4 + 48s^2 - 50 = 0 \Rightarrow \begin{cases} s^2 = -25 & \rightarrow s_{1,2} = \pm 5j \\ s^2 = 1 & \rightarrow s_{3,4} = \pm 1 \end{cases}$$

orig. p. 5

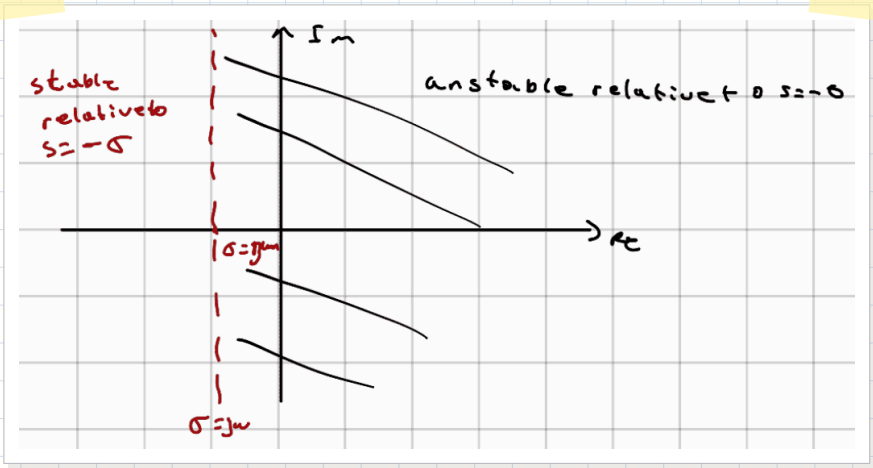
case 1 of exception → at least two poles on the imaginary axis

case 2 exception → some poles are symmetric with respect to the origin



original drawing, p. 5

Relative stability



original drawing, p. 5

$s + \sigma = \hat{s}$ at $s = -\sigma \rightarrow \hat{s} = 0$

$\hookrightarrow s = \hat{s} - \sigma$

Example

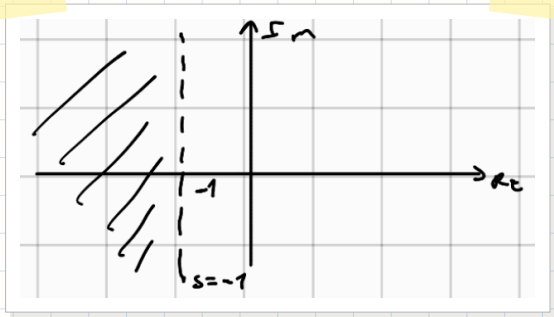
$\dddot{c} + 6\ddot{c} + 11\dot{c} + kc = \ddot{r} - r$

$$\frac{C(s)}{R(s)} = \frac{s^2 - 1}{s^3 + 6s^2 + 11s + k}$$

find possible k values so that all poles are in the shaded area?

$$\Delta(s) = s^3 + 6s^2 + 11s + k = 0$$

$s = \hat{s} - 1$



original drawing, p. 5

orig. p. 6

$$\Delta(\hat{s}) = (\hat{s} - 1)^3 + 6(\hat{s} - 1)^2 + 11(\hat{s} - 1) + k = 0$$
$$= \hat{s}^3 + 3\hat{s}^2 + 2\hat{s} + k - 6 = 0$$

necessary condition

$$\boxed{k > 6}$$

$\hat{s}^3$	1	2 0
$\hat{s}^2$	3	$k-6$ 0
$\hat{s}^1$	$\frac{6-(k-6)}{3}$	0
$\hat{s}^0$	$k-6$	

all elements have to be pozitive

$$\frac{6 - k + 6}{3} = \frac{12 - k}{3} > 0$$

$\Rightarrow k < 12$

$\hookrightarrow 6 < k < 12$

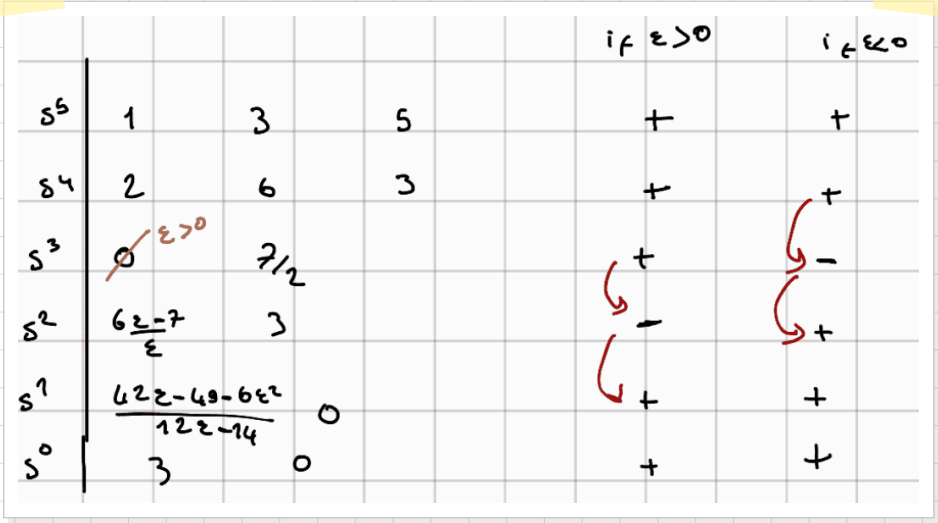
Method #2 for the first exception case

$$\begin{pmatrix} s_1 = +2 \\ \alpha = \frac{1}{2} \end{pmatrix} \Rightarrow s = \frac{1}{\alpha}$$

This change of variable gives us the reciprocal of the root

$$s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3 = 0$$

				if $\varepsilon > 0$	if $\varepsilon < 0$
s^5	1	3	5	+	+
s^4	2	6	3	+	+
s^3	ε	7/2		+	−
s^2	$\frac{6\varepsilon - 7}{\varepsilon}$	3		−	+
s^1	$\frac{42\varepsilon - 49 - 6\varepsilon^2}{12\varepsilon - 14}$	0		+	+
s^0	3	0		+	+



original drawing, p. 6

Two poles are on the RHP

orig. p. 7

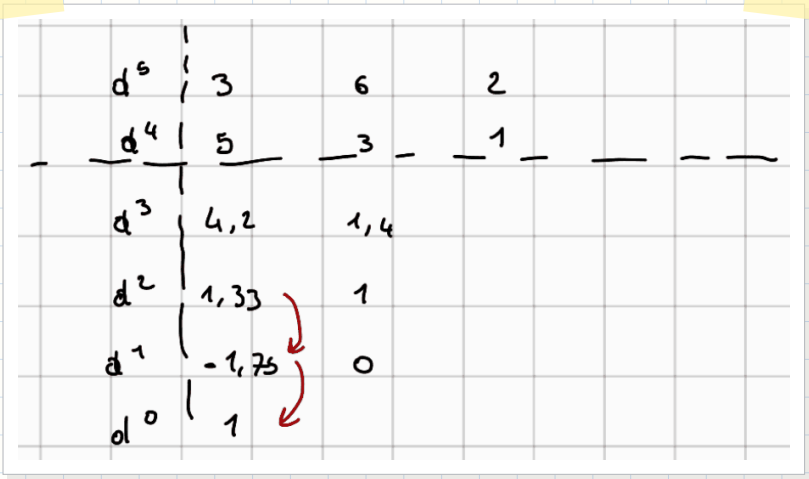
$$s = \frac{1}{\alpha}$$

$$\left(\frac{1}{\alpha}\right)^5 + 2\left(\frac{1}{\alpha}\right)^4 + 3\left(\frac{1}{\alpha}\right)^3 + 6\left(\frac{1}{\alpha}\right)^2 + 5\left(\frac{1}{\alpha}\right) + 3 = 0$$

$$\left(\frac{1}{\alpha}\right)^5 \left[1 + 2\left(\frac{1}{\alpha}\right)^{-1} + 3\left(\frac{1}{\alpha}\right)^{-2} + 6\left(\frac{1}{\alpha}\right)^{-3} + 5\left(\frac{1}{\alpha}\right)^{-4} + 3\left(\frac{1}{\alpha}\right)^{-5}\right] = 0$$

$$\Rightarrow 1 + 2\alpha + 3\alpha^2 + 6\alpha^3 + 5\alpha^4 + 3\alpha^5 = 0$$

α^5	3	6	2
α^4	5	3	1
α^3	4,2	1,4	
α^2	1,33	1	
α^1	−1,75	0	
α^0	1		



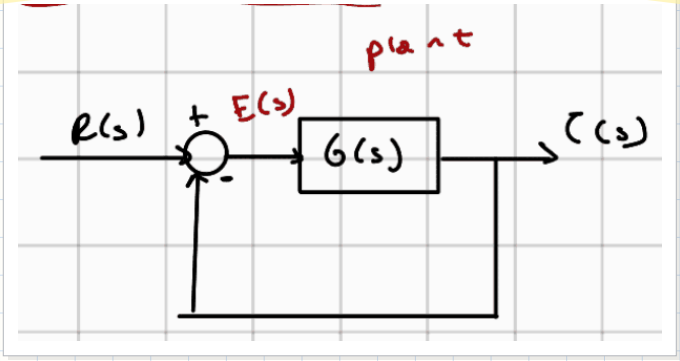
original drawing, p. 7

Solve this:[2]

$$s^7 + 3s^6 + 5s^5 + 9s^4 + 9s^3 + 7s^2 + 3s + 2 = 0$$

orig. p. 8

Control actions

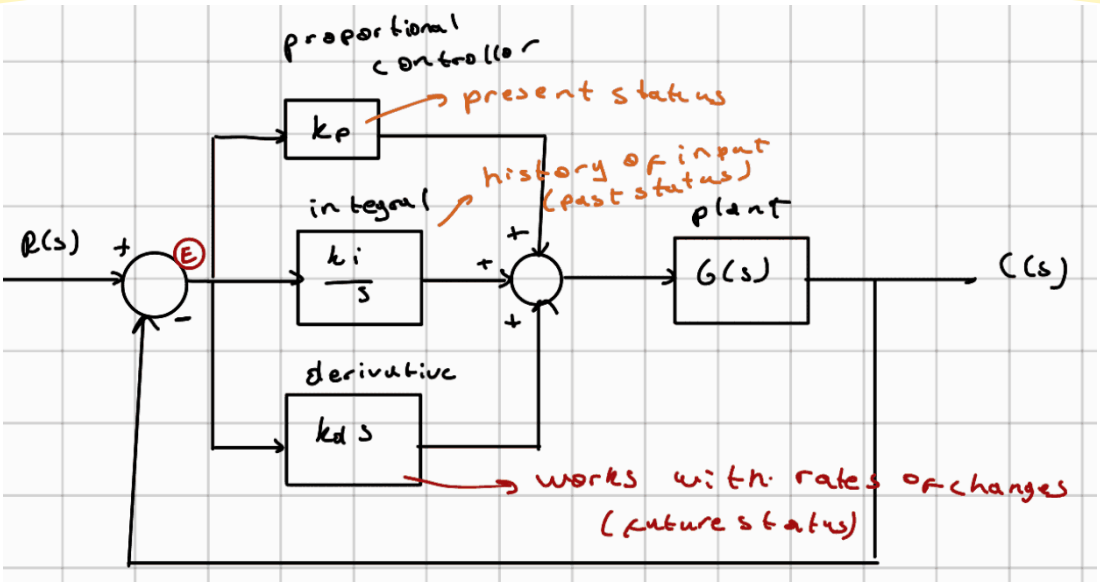


original drawing, p. 8

$$E(s) = R(s) - C(s)$$

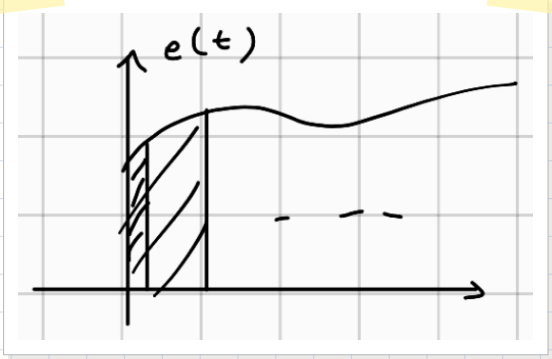
$R(s)$ is the input and "can not" affect stability and accuracy of the system

[2] Claude notu: üsler küçük yazıldığı için okunması zor; ardışık üsler (7'den 0'a) varsayıldı.

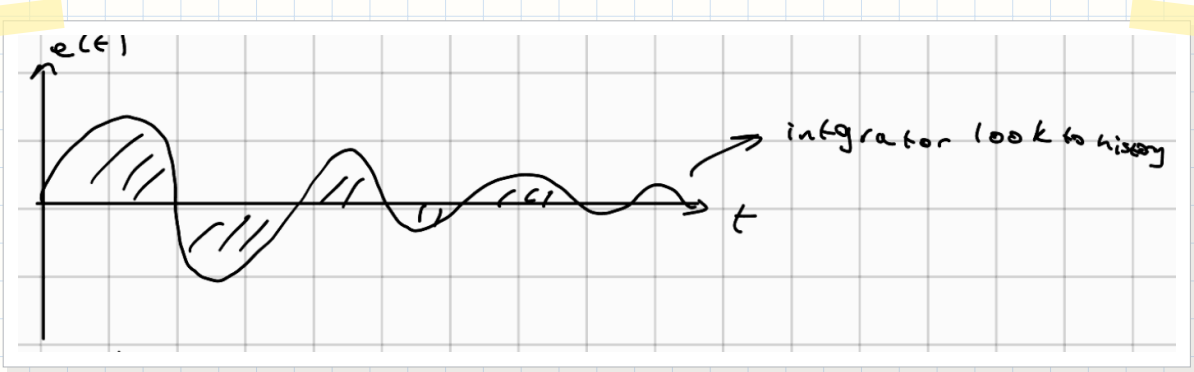


original drawing, p. 8

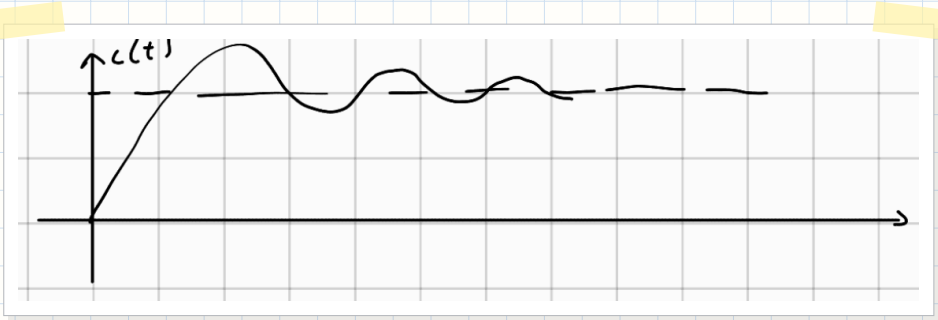
$$k_i \int e(t)\,dt$$



original drawing, p. 8

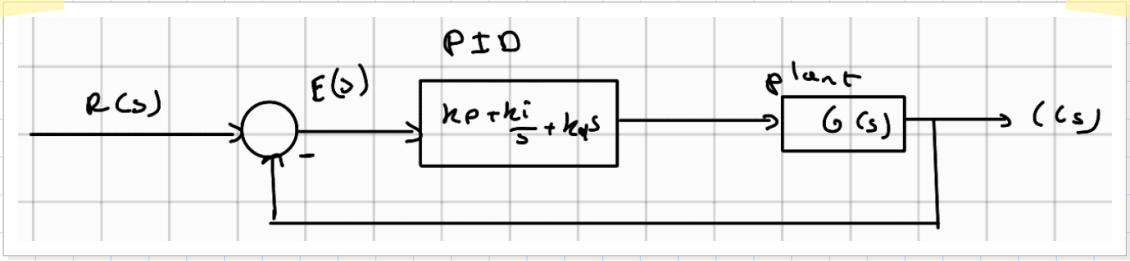


original drawing, p. 8



original drawing, p. 8

orig. p. 9 $k_d \dot{e}$ → rate of change of error



original drawing, p. 9

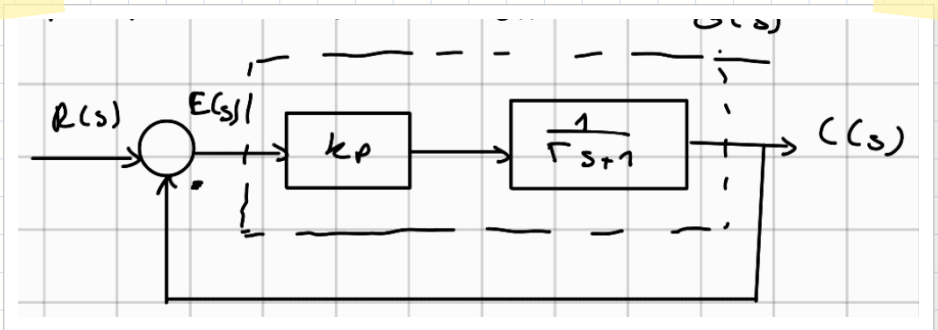
Derivative operator is always used with 'P' and 'I' operations

{ PD ✓ PI ✓ PID ✓
ID ✓

D ×

1) proportional action

→ responsible for stability



original drawing, p. 9

$$G(s) = \frac{k_p}{\tau s + 1}$$

$$\text{CLTF}: \quad \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)} = \frac{k_p}{\tau s + 1 + k_p}$$

k_p is also responsible for the location of the pole

$$\begin{aligned} E(s) &= R(s) - C(s) \\ &= R(s)\left(1 - \frac{C(s)}{R(s)}\right) \\ &= R(s)\left(1 - \frac{G(s)}{1+G(s)}\right) \\ &= R(s)\left(\frac{1}{1+G(s)}\right) \end{aligned}$$

Steady state error

Final value theorem

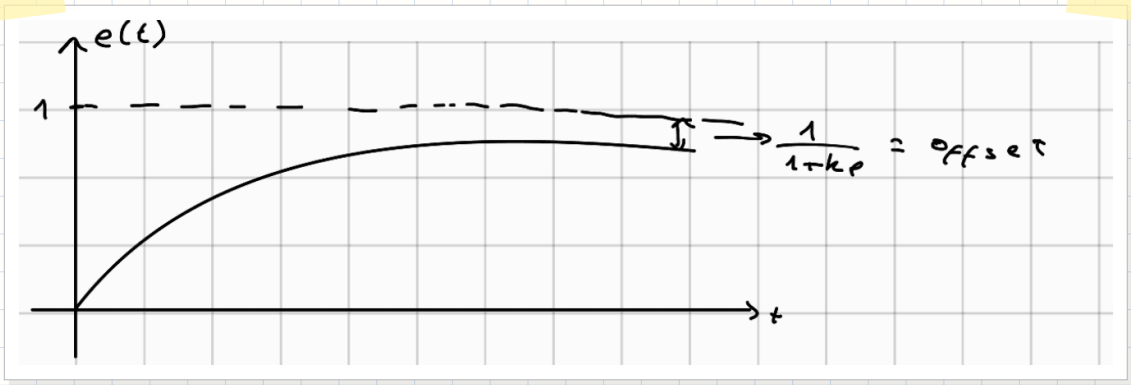
$$e_{ss} = ? = e(\infty) = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)$$

$$e_{ss} = \lim_{s\to 0} \frac{sR(s)}{1+G(s)}$$

if $R(s) = \frac{1}{s}$ step

$$\begin{aligned} e_{ss} &= \frac{1}{1+G(0)} \\ &= \frac{1}{1+k_p} \qquad k_p \uparrow^{\infty} \Rightarrow e_{ss} \downarrow 0 \end{aligned}$$

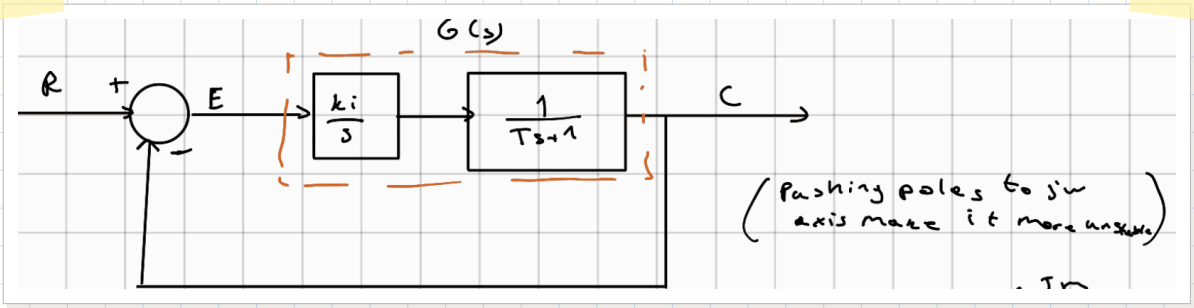
orig. p. 10



original drawing, p. 10

2) Integral action

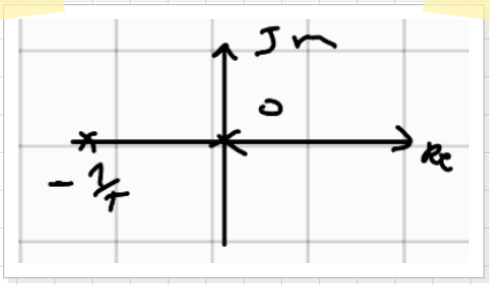
→ responsible for error



original drawing, p. 10

$$G(s) = \frac{k_i}{s(\tau s + 1)}$$

(A pole added to the open loop function)



original drawing, p. 10

$$\frac{C(s)}{R(s)} = \frac{k_i}{s(\tau s + 1) + k_i}$$

$$E(s) = R(s)\left(\frac{1}{1 + G(s)}\right)$$

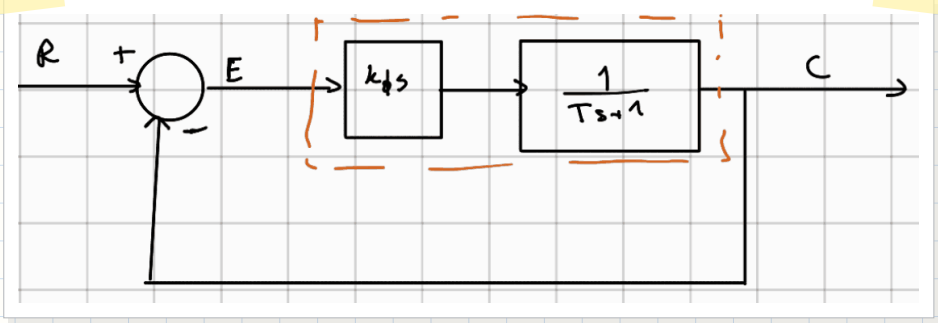
$$R(s) = \frac{1}{s}$$

$$e_{ss} = \lim_{s \to 0} s \frac{1}{s}\left(\frac{1}{1 + G(s)}\right) = \frac{1}{1 + G(0)} = 0$$

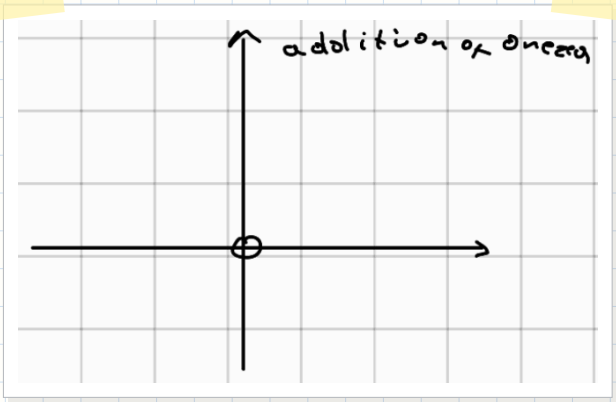
3) Derivative action

(responsible for sensitivity and increases stability in that sense)

$$G(s) = \frac{k_d s}{\tau s + 1} \qquad \text{OLTF}$$



original drawing, p. 10



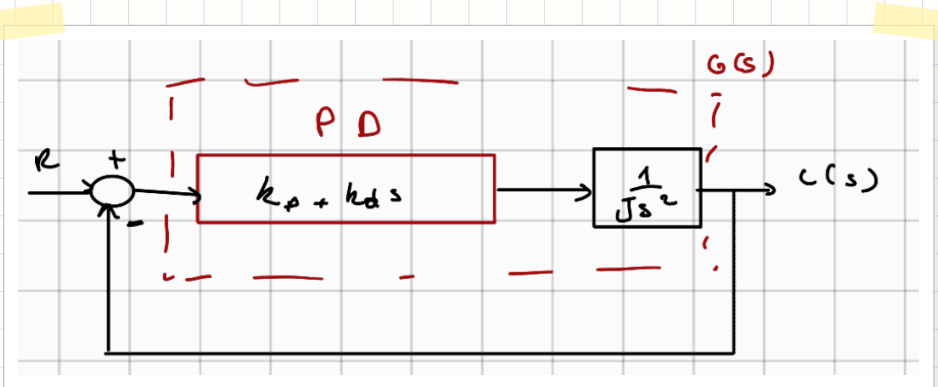
original drawing, p. 10

$$E(s) = R(s)\left(\frac{1}{1+G(s)}\right)$$

orig. p. 11

$$E(s) = R(s)\left(\frac{1}{1+G(s)}\right) = R(s)\left(\frac{\tau s+1}{k_d s+\tau s+1}\right)$$

$$R(s) = \frac{1}{s} \Rightarrow e_{ss} = \lim s\left(\frac{1}{s}\right)\left(\frac{\tau s+1}{k_d s+\tau s+1}\right) = 1$$



original drawing, p. 11

$$\frac{C(s)}{R(s)} = \frac{k_p + k_d s}{k_p + k_d s + Js^2} \qquad \Delta(s)$$

$$s^2 + \underbrace{\frac{k_d}{J}}_{2\zeta\omega_n} s + \underbrace{\frac{k_p}{J}}_{\omega_n^2} = 0$$

$$\omega_n = \sqrt{\frac{k_p}{J}}$$

$$\zeta = \frac{1}{2\omega_n}\frac{k_d}{J} = \frac{1}{2}\sqrt{\frac{J}{k_p}}\frac{k_d}{J}$$

$$e^{-\zeta\omega_n t}\left(- - \frac{}{}\right)$$

$$E(s) = R(s)\frac{1}{1+G(s)}$$

$$R(s) = \frac{1}{s}$$

$$e_{ss} = \lim_{s\to 0} s\left(\frac{1}{s}\right)\frac{1}{1+G(0)}$$

$$G(s) = \frac{k_p + k_d s}{Js^2} = \infty$$

$$e_{ss} = 0$$