Kontrol Sistemleri

Kök yer eğrileri (final çalışması)

Orijinal not: Notes.pdf

UYARI Bu belge, el yazısı ders notlarından Claude (Anthropic) yardımıyla dijitale aktarılmıştır. Yardımcı kaynak olarak hazırlanmıştır; hata veya eksik içerebilir. Lütfen orijinal notları ve ders kaynaklarını esas alın.

NOTICE *This document was transcribed from handwritten lecture notes with the help of Claude (Anthropic). It is meant as a supplementary resource and may contain errors or omissions. Always check against the original notes and official course material.*

orj. s. 1

Kök yer eğrileri

1) poles-zeros
2) $\alpha = n - m$
3) centroid : $\sum$ kutup $-$ $\sum$ sıfır

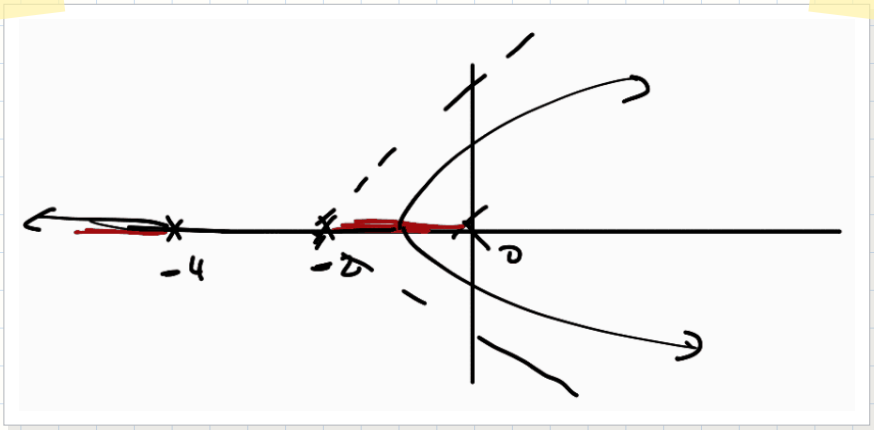
$$G(s)H(s) = \frac{k}{s(s+2)(s+4)}$$

zeros yok $\quad m = 0$

poles $\rightarrow 0, -2, -4 \quad n = 3$

$$\alpha = n - m \checkmark \qquad \frac{180(2i+1)}{3} = 60, -60, 180$$

$$\sigma_c = \frac{\sum \text{kutup} - \sum \text{sıfır}}{\alpha} = \frac{-2-4}{3} = -2$$



orijinal çizim, s. 1

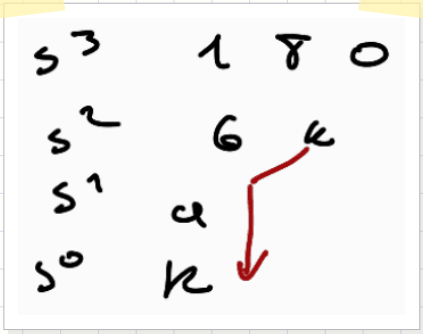
$$\Delta = 1 + G(s)H(s)$$

$$\Delta = 1 + \frac{k}{s(s+2)(s+4)}$$

$$\Delta = s^3 + 6s^2 + 8s + k = 0$$

Routh table

s^3	1	8	0
s^2	6	k	
s^1	a		
s^0	k		



orijinal çizim, s. 1

Çaprazın bir altına iniyor

$$\left.\begin{aligned} a &= -\frac{\begin{vmatrix} 1 & 8 \\ 6 & k \end{vmatrix}}{6} \\ k &= -\begin{vmatrix} 6 & k \\ a & 0 \end{vmatrix} \end{aligned}\right\} \quad 0 < k < 48 \downarrow \text{kritik kazanç}$$

$$k_c = 48$$

$$6s^2 + \overset{\checkmark 48}{k_c} = 0$$

$$s^2 = \sqrt{-8}^{1}$$

$s = \pm 2\sqrt{2}j$ eksenleri kestiği nokta

orj. s. 2

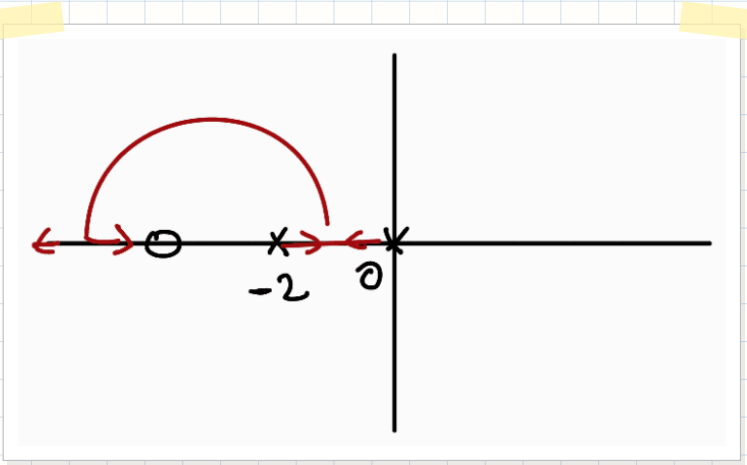
$$G(s) = \frac{s+4}{s(s+2)}k$$

zeros $= -4 \quad (m)$

poles $= 0, -2 \quad (n)$

$\alpha = n - m = 1 \qquad \sigma_c$

[1] Claude notu: muhtemelen $s^2 = -8$ (yani $s = \sqrt{-8}$) olmalı.



orijinal çizim, s. 2

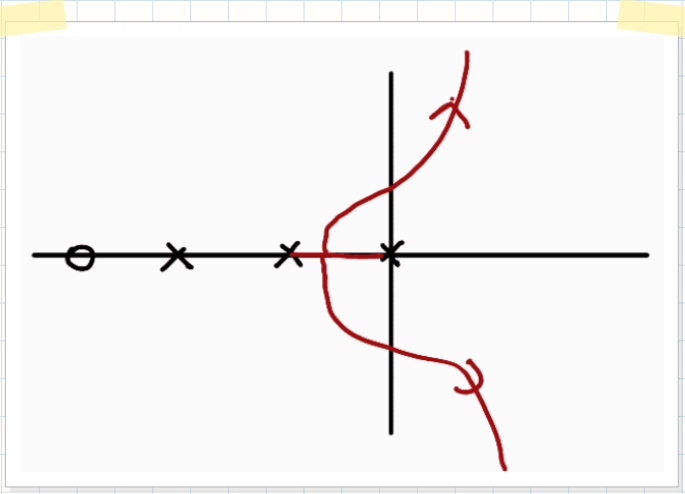
$$G(s) = \frac{(s+4)k}{s(s+1)(s+2)}$$

zeros $= -4 \quad m = 1$

poles $= 0, -1, -2 \quad n = 3$

$\alpha = n - m = 2 \rightarrow 90°$

$$\sigma_c = \frac{\sum \text{kutup} - \sum \text{sıfır}}{\alpha} = \frac{-3+4}{\alpha} = \frac{-1-2+4}{2} = \frac{1}{2}$$



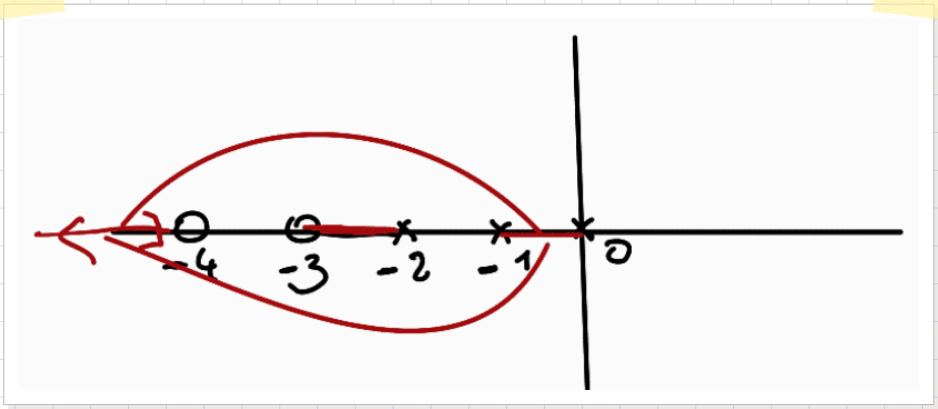
orijinal çizim, s. 2

$$G(s) = \frac{(s+4)(s+3)}{s(s+1)(s+2)}k$$

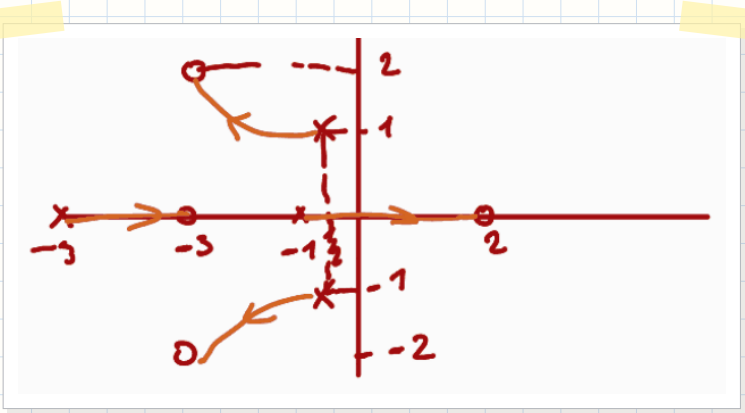
$z = -4, -3 \rightarrow m = 2 \quad n - m = 1$

$p = 0, -1, -2 \rightarrow n = 3$

$$-1 - 2 + \underbrace{4+3}_{7} = 4$$



orijinal çizim, s. 2



orijinal çizim, s. 2

$$\frac{\text{zeroes}}{\text{poles}} = \frac{(s-2)(s+3)(s+3+2j)(s+3-2j)}{(s+5)(s+1)(s+\frac{1}{2}-j)(s+\frac{1}{2}+j)}$$

$n - m = 0$

	step	ramp	
0	$\frac{1}{1+k_p}$		
1		$\frac{1}{k_v}$	
2			$\frac{1}{k_a}$

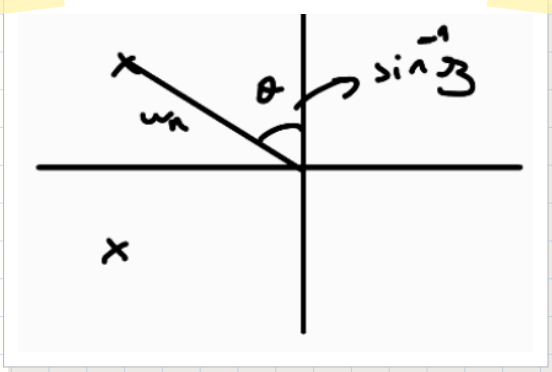
$$k_p = \lim_{s\to 0} G(s)$$
$$= \frac{-2 \cdot 3\,(3+2j)(3-2j)}{5 \cdot 1\,(\frac{1}{2}-j)(\frac{1}{2}+j)}$$
$$= \underline{-4{,}16}$$

$$\frac{1}{1+k_p} \to -0{,}316$$

orj. s. 3

$6 - 3 = 3$ ✓

$-60 \quad +60 \quad +180$

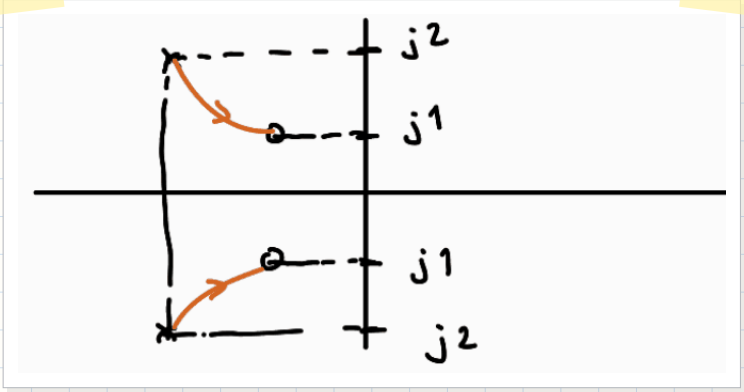


orijinal çizim, s. 3

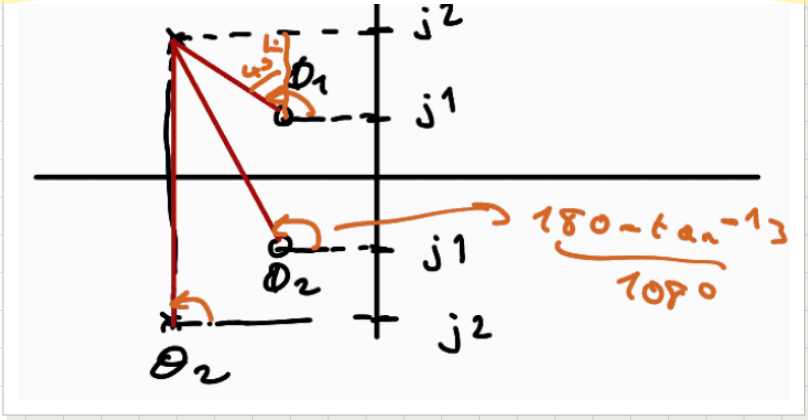
$$G(s)H(s) = \frac{k\,(s+1-j)(s+1+j)}{(s+2+2j)(s+2-2j)}$$

zeros: $-1+j$, $-1-j$

poles = $-2-2j$, $-2+2j$



orijinal çizim, s. 3



orijinal çizim, s. 3

$$\underbrace{(\Phi_1+\Phi_2)}_{\text{sıfırlar}} - \underbrace{(\Theta_1+\Theta_2)}_{\text{poles}} = \mp 180$$

$$\Theta_1 = \mp 180 + \underbrace{\Theta_2}_{90} - \underbrace{\Phi_1}_{135^\circ} - \Phi_2$$

Tekrar

$$M_p = \%25 \qquad t_p = 2 \text{ sn}$$

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

$$M_p = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} = 0{,}25$$

$$\frac{-\pi\zeta}{\sqrt{1-\zeta^2}} = \ln 0{,}25$$

$$\frac{\zeta}{\sqrt{1-\zeta^2}} = 0{,}441$$

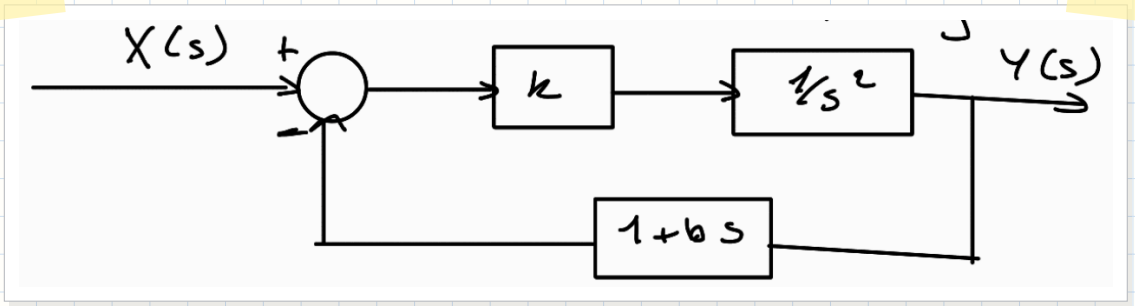
$$\frac{\zeta^2}{1-\zeta^2} = 0{,}194 \qquad \zeta^2(1+0{,}194) = 1^2$$

$$\zeta = 0{,}40$$

orj. s. 4

$$\overset{2s}{\nearrow} t_p = \frac{\pi}{\omega_d} \qquad \omega_d = \frac{\pi}{2} \qquad \frac{\pi}{2} = \omega_n\sqrt{1-\zeta^2}$$

$$\omega_n = \frac{\frac{\pi}{2}}{\sqrt{1-\zeta^2}} = 1{,}71$$



orijinal çizim, s. 4

$$G(s) = \frac{k\frac{1}{s^2}}{1+\frac{k(1+bs)}{s^2}}$$

$$= \frac{k}{s^2+k+kbs}$$

$$= \frac{k}{s^2+kbs+k} \qquad \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$k = \omega_n^2 \qquad k = \omega_n^2 = 2{,}92$$

$$kb = 2\zeta\omega_n$$

$$b = \frac{2\zeta\omega_n}{\omega_n^2} = 0{,}46$$

[2] Claude notu: muhtemelen sağ taraf 0,194 olmalı.

2)

$$G(s)H(s) = \frac{k}{s(s+2)(s+4)}$$

zeroes ∅

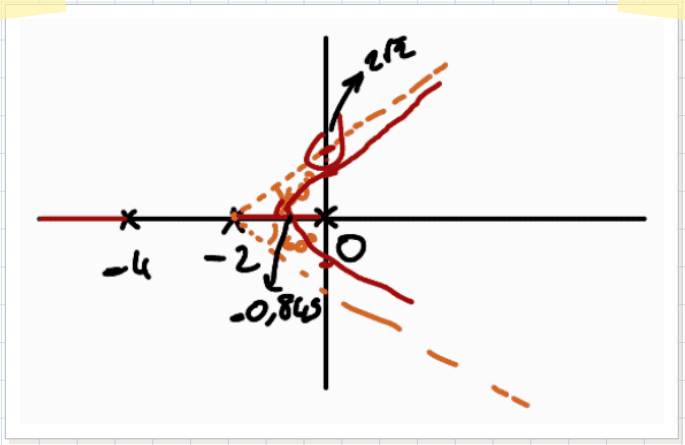
poles $p_1 = 0$ $p_2 = -2$ $p_3 = -4$

$$\alpha = n - m = 3 - 0 = 3$$

$$\text{centroid} = \frac{\sum \text{poles} - \sum \text{zeros}}{\alpha} = \frac{-2-4}{\alpha} = \frac{-6}{\alpha} = -2$$

$$\text{Assymptotes angles} = \pm\frac{180(2i+1)}{3} \qquad i = 0, \ldots$$

$$= 60,\ 180,\ \underbrace{-60}_{300}$$



orijinal çizim, s. 4

orj. s. 5

$$\Delta = \frac{s(s+2)(s+4)+k}{s(s+2)(s+4)} = 0$$

$$s(s+2)(s+4) + k = 0$$

$$k = -\underbrace{s(s+2)}_{s^2+2s}(s+4)$$

$$\underbrace{s^3 + 2s^2 + 4s^2 + 8s}_{s^3+6s^2+8s}$$

$$\frac{dk}{ds} = -(3s^2 + 12s + 8)$$

kökleri $(3s^2 + 12s + 8)$

$$\underbrace{s_1 = -0{,}845}_{\text{departure point}} \quad , \quad \underbrace{s_2 = -3{,}154}_{\times \text{ non acceptable}}$$

$$\downarrow \quad k = 3{,}07$$

? k range

$$s^3 + 6s^2 + 8s + k = 0$$

s^3	1	8	0
s^2	6	k	$48 - k > 0$
s^1	$\frac{48-k}{6}$	0	$0 < k < 48$
s^0	k		

$$k = 48$$

$$s^3 + 6s^2 + 8s + 48 = 0$$

$$s_1 = -6 \qquad \underline{s_2 = 2{,}82j} \qquad \underline{s_3 = -2{,}82j}$$

orj. s. 6

$$G(s) = \frac{s+4}{s(s+2)}k$$

zeroes

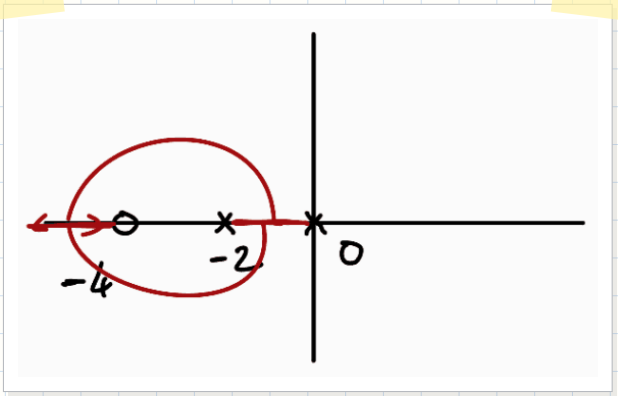
$s = -4$

$p_1 = 0 \qquad p_2 = -2$

$$\alpha = n - m = 1$$

$$\downarrow$$

$$\pm 180 \qquad \sigma_c \times$$



orijinal çizim, s. 6

-1'e kutup ekleyelim

$$G(s) = \frac{(s+4)k}{s(s+1)(s+2)}$$

zeroes

$z_1 = -4$

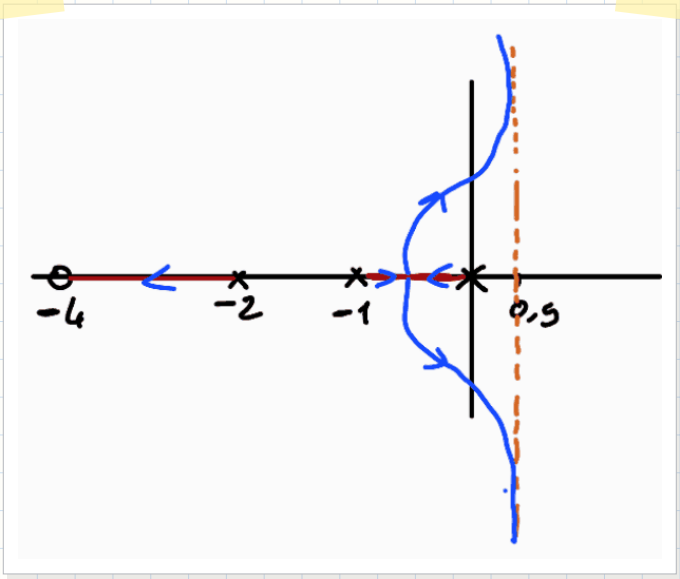
$p_1 = 0 \qquad p_2 = -1 \qquad p_3 = -2$

$$n - m = 3 - 1 = 2$$

$$\text{centroid} = \frac{-1 - 2 + 4}{2}$$

$$= \frac{1}{2} = 0{,}5$$

$$\text{angles} \quad \frac{180(2i+1)}{2} \quad \begin{matrix} +90 \\ -90 \end{matrix}$$



orijinal çizim, s. 6

$$s(s+1)(s+2)+(s+4)k=0$$

$$(s+4)k=-((s^2+s)(s+2))$$

$$k=\frac{-(s^3+3s^2+2s)}{(s+4)}$$

$$\frac{dk}{ds}=\frac{-(3s^2+6s+2)(s+4)+(s^3+3s^2+2s)(1)}{(s+4)^2}=0$$

$$=-3s^3-12s^2-6s^2-24s-2s-8+s^3+3s^2+2s=0$$

$$-2s^3-21s^2-24s-8=0$$[3]

$$2s^3+21s^2+24s+8=0$$

$s_1=-5{,}42$ × $\boxed{s_2=-0{,}45}$ $s_3=-1{,}62$ ×

orj. s. 7

$$s(s+1)(s+2)+(s+4)k=0$$

$$(s+4)k=-((s^2+s)(s+2))$$

$$k=\frac{-(s^3+3s^2+2s)}{(s+4)}$$

$$s^3+3s^2+2s+ks+4k=0$$

$$s^3+3s^2+(2+k)s+4k=0$$

s^3	1	$2+k$
s^2	3	$4k$
s^1	$\frac{6+3k-4k}{3}$	
s^0	0	

$$6-k>0$$

[3] Claude notu: muhtemelen $-15s^2$ olmalı ($-12-6+3=-15$).

$k < 6$ critical gain

$0 < k < 6$ // ↗

$s^3 + 3s^2 + 8s + 24 = 0$

$s_1 = -3$ $s_2 = 2{,}82j$

$s_3 = -2{,}82j$

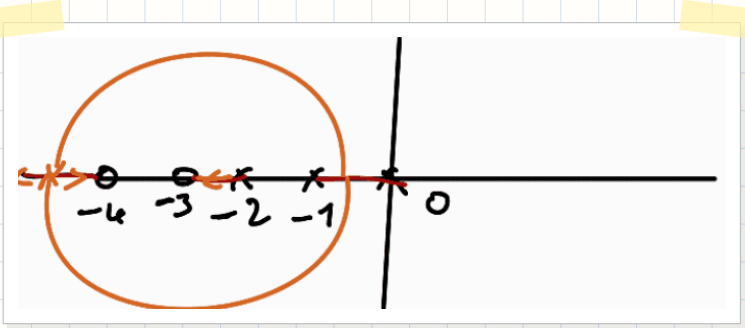
$\Rightarrow -3$'e zero ekle

$$G(s) = \frac{(s+4)(s+3)k}{s(s+1)(s+2)}$$

$z = -4, -3, \; m = 2$
$p = 0, -1, -2, \; n = 3$

$\alpha = n - m = 1$

angle $= \pm 180$



orijinal çizim, s. 7

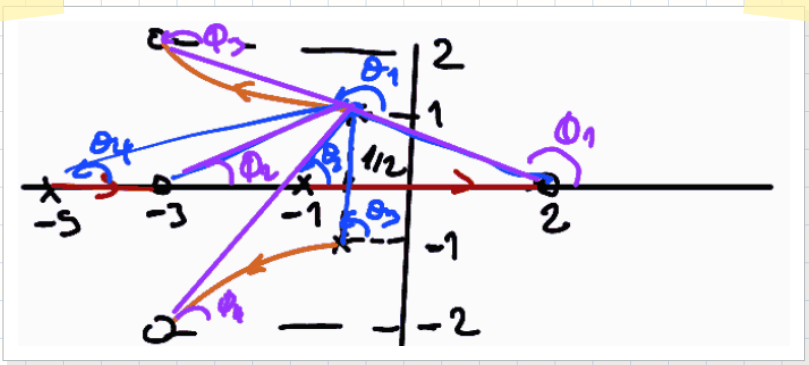
$$s(s+1)(s+2) + \underbrace{(s+4)(s+3)}_{\substack{s^2+3s+4s+12 \\ s^2+7s+12}} k = 0$$

$s^3 + 3s^2 + 2s + ks^2 + 7ks + 12k = 0$

$k(s^2 + 7s + 12) = -$

$s^3 + (3+k)s^2 + (2+7k)s + 12k = 0$

orj. s. 8



orijinal çizim, s. 8

$n - m = 0$ 0 sonsuz

$$G(s) = \frac{(s-2)(s+3)(s+3+2j)(s+3-2j)}{(s+5)(s+1)(s+\frac{1}{2}+j)(s+\frac{1}{2}-j)}$$

$$E(s) = \frac{R(s)}{1+G(s)} \rightarrow \frac{1}{s}$$

$$\lim_{s\to 0} sE(s) = \frac{1}{1+\underbrace{\lim_{s\to 0} G(s)}_{-12,48}} = -0,0871$$

$$E_{ss} = -0,0871$$

Skill Assessment Ex 8,4

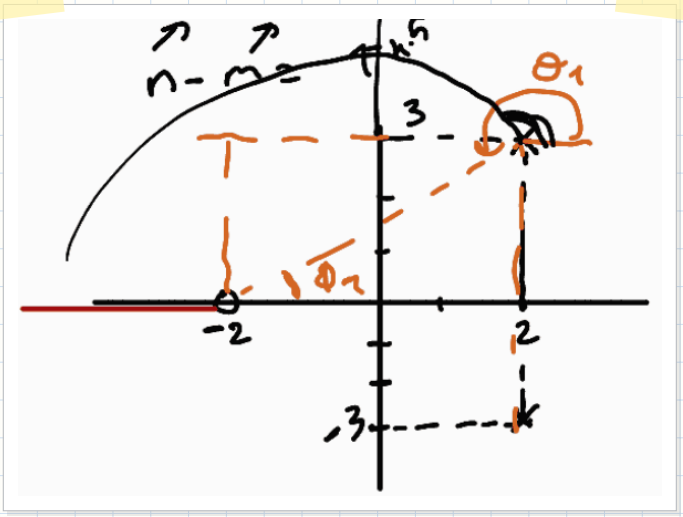
Unity feedback system

$$G(s) = \frac{k(s+2)}{s^2-4s+13}$$

$$z_1 = -2$$

a) $p_1 = 2+3j$

$p_2 = 2-3j$



orijinal çizim, s. 8

$$\phi_1 - \theta_1 - \theta_2 = 180$$

$$\underbrace{36,83}_{\arctan(3/4)} - \theta_1 - 90^\circ = 180^\circ$$

$$\theta_1 = 36,83 - 90 - 180$$

$$= -233$$

$$\Delta = 1 + G(s) = 0$$

$$s^2 - 4s + 13 + k(s+2) = 0$$

$$k = -\frac{s^2-4s+13}{(s+2)}$$

$$\frac{dk}{ds} = \frac{-(2s-4)(s+2)+(s^2-4s+13)}{(s+2)^2} = 0$$

orj. s. 9

$$-(2s^2 \cancel{+4s-4s} - 8) + (s^2 - 4s + 13) = 0$$

$$-2s^2 + 8 + s^2 - 4s + 13 = 0$$

$$-s^2 - 4s + 5 = 0$$[4]

$$\Rightarrow s^2 + 4s - 21 = 0$$

$$\boxed{\begin{aligned} s_1 &= 3 \\ s_2 &= -7 \end{aligned}}$$ break in point

$$s^2 - 4s + 13 + k(s+2) = 0$$

$$s^2 + (-4+k)s + 13 + 2k = 0$$

s^2	1	$13+2k$	
s^1	$k-4$	0	0
s^0	$13+2k$	0	

$$k - 4 > 0$$

$$\boxed{k > 4}$$

$$k - 4 = 0$$

$$k = 4 \text{ olsun}$$

$$s^2 + 13 + \underbrace{2k}_{8}^{4} = 0$$

$$s^2 = -21$$

$$s = \pm j\sqrt{21}$$

orj. s. 10

Şekil: Basılı soru 2: Verilen sistemde K_1 ve K_2 kazançları, birim basamak bozucu girişte kalıcı durum hatası $-0{,}00001$ ve birim basamak girişte kalıcı durum hatası $0{,}002$ olacak şekilde belirlenecek. Blok diyagram ($\frac{K_1(s+2)}{s+3}$ ve $\frac{K_2}{s(s+4)}$ blokları, $D(s)$ bozucu girişi) basılıdır; el yazısıyla $E(s)$, A_1, A_2 etiketleri eklenmiş. Dijitalleştirilmedi.
(orijinal notta, s. 10)

$$E(s) = R(s) - C(s) \Rightarrow C = R - E$$

$$C = (EA_1 + D)A_2$$

$$R - E = EA_1A_2 + DA_2$$

$$R - DA_2 = E(1 + A_1A_2)$$

[4] Claude notu: muhtemelen +21 olmalı.

$$E = \frac{R}{1 + A_1A_2} - \frac{DA_2}{1 + A_1A_2}$$

$$\text{if } R = 0 \qquad D \neq 0 \qquad D = \frac{1}{s}$$

$$E = -\frac{DA_2}{1 + A_1A_2}$$

$$E_{ss} = \lim_{s\to 0} sE(s) = \frac{-A_2}{1 + A_1A_2}$$

$$= \lim_{s\to 0} -\frac{\dfrac{k_2}{s(s+4)}}{1 + \dfrac{k_2}{s(s+4)}\dfrac{k_1(s+2)}{(s+3)}}$$

$$\Rightarrow 0{,}00001 = \lim_{s\to 0} \frac{k_2(s+3)}{s(s+3)(s+4) + k_1k_2(s+2)} = \frac{3k_2}{2k_1k_2} = \frac{3}{2k_1}$$

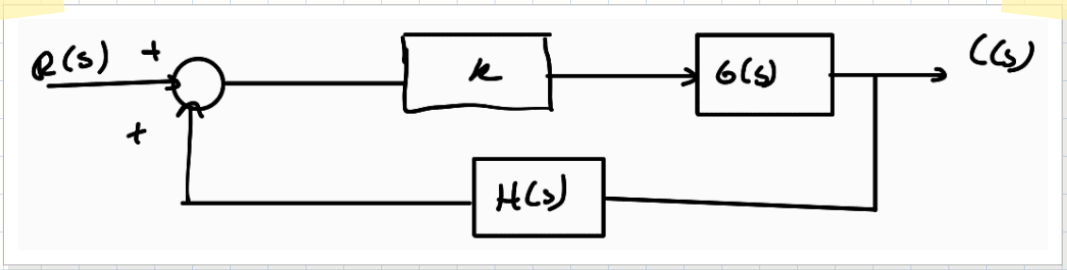
$$k_1 = 150.000$$

b) $R(s) = \frac{1}{s}$ $\quad D(s) = 0$ is 0,002

$$0{,}002 = \lim_{s\to 0} \frac{1}{1 + G_1G_2} = \lim_{s\to 0} \frac{s(s+3)(s+4)}{s(s+3)(s+4) + k_1k_2(s+2)} \to 0$$

orj. s. 11

Positive Feedback Architecture



orijinal çizim, s. 11

$$\frac{C(s)}{R(s)} = \frac{kG(s)}{1 - kG(s)H(s)}$$

$$1 - kG(s)H(s) = 0$$

$$|G(s)H(s)| = \frac{1}{k}$$

Arrival angle

$$360 = \text{zeros angle} - \text{poles angle}$$

③ Asymptotic angle

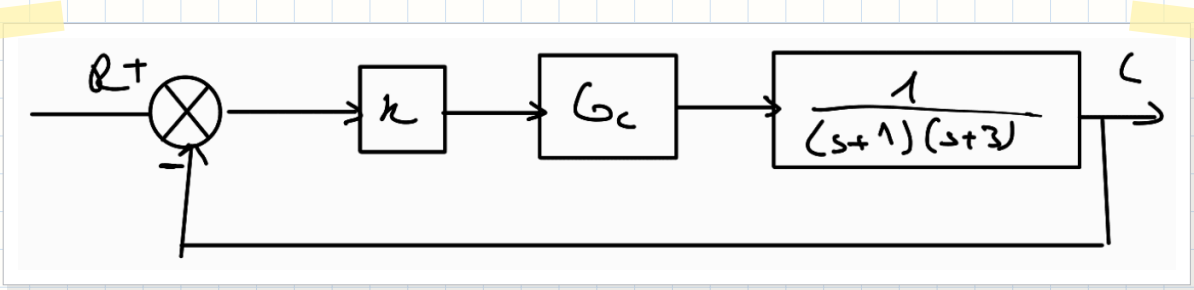
$$\pm\frac{360\,i}{n - m}$$

zero : lead ⟶ sola kayar root ? eğrisi
daha stabil, hızlı cevap
accuracy kötü

poles : lag

$$G_c = k_c \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}} \qquad 0 < \alpha < 1 : \text{lead} \qquad \alpha > 1 : \text{lag}$$

Ex



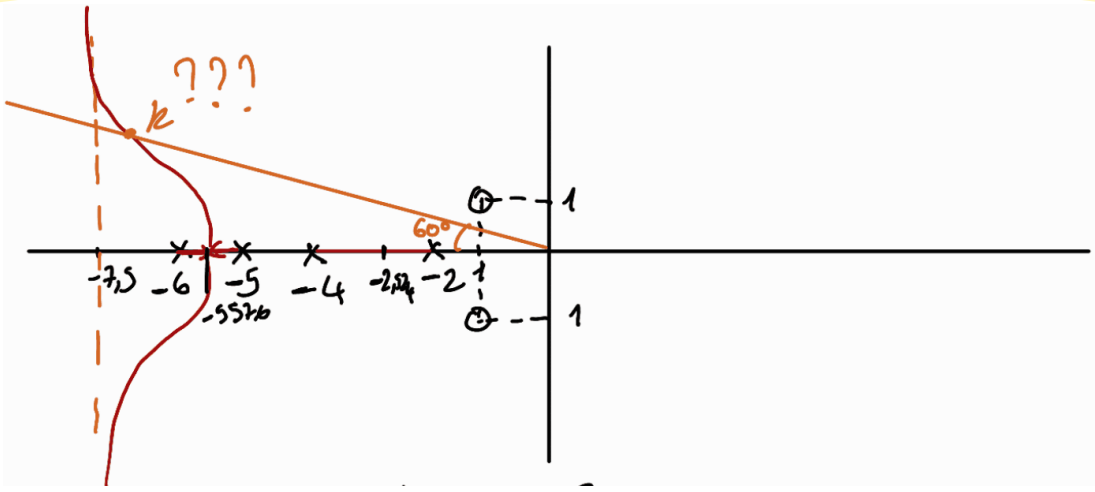
orijinal çizim, s. 11

Dominant pole = −3 + 2j
steady state error = 10%

orj. s. 12

> **Şekil:** Basılı soru 27 (Şekil P8.3, Bölüm 8.7): Birim geri beslemeli sistem, $G(s) = \dfrac{K(s^2 - 2s + 2)}{(s + 2)(s + 4)(s + 5)(s + 6)}$. İstenenler: a) kök yer eğrisini çiz, b) asimptotları bul, c) sistemi kararlı yapan K aralığını bul, d) kırılma noktalarını bul (üstü el ile çizilmiş), e) kapalı çevrim basamak cevabında %25 aşım veren K değerini bul. Basılı metin dijitalleştirilmedi; $G(s)$'in yanına el yazısıyla aşağıdaki çarpanlar eklenmiş.
> *(orijinal notta, s. 12)*

$$(1 + j)(1 - j)$$



orijinal çizim, s. 12

$$n - m = 4 - 2 = 2$$

$$\sigma_c = \frac{\dfrac{\Sigma \text{ r.p. poles} - \Sigma \text{ r.p. zeroes}}{2}}{2} = \frac{-6 - 5 - 4 - 2 \cancel{+1+1}}{2} = \frac{-15}{2} = -7{,}5$$

$$\phi = \frac{(2i+1)}{2} = \pm 90$$

orj. s. 13

$$\Delta(s) = 1 + \frac{K(s^2 - 2s + 2)}{(s+2)(s+4)(s+5)(s+6)}$$