Sayısal Haberleşme

Matched filter, SAK ve 1. Nyquist kriteri

Orijinal not: Ders-10.pdf

UYARI Bu belge, el yazısı ders notlarından Claude (Anthropic) yardımıyla dijitale aktarılmıştır. Yardımcı kaynak olarak hazırlanmıştır; hata veya eksik içerebilir. Lütfen orijinal notları ve ders kaynaklarını esas alın.

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orj. s. 1

genel

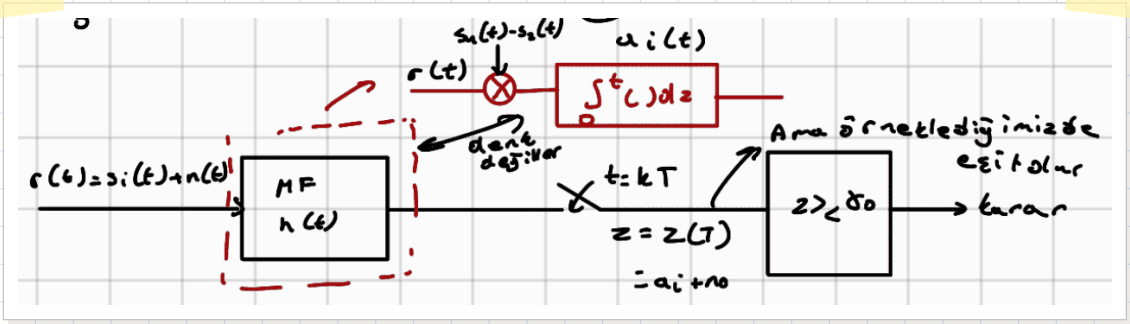
$$\gamma_0 = \frac{\sigma_0^2}{a_1 - a_2} \ln \frac{P(s_2)}{P(s_1)} + \frac{a_1 + a_2}{2} \rightarrow$$

$$P_b = \left[1 - Q\left(\frac{\gamma_0 - a_1}{\sigma_0}\right)\right] P(s_1) + Q\left(\frac{\gamma_0 - a_2}{\sigma_0}\right) P(s_2)$$

$$\downarrow \quad P(s_1) = P(s_2) = \frac{1}{2}$$

$$\gamma_0 = \frac{a_1 + a_2}{2} \rightarrow P_b = Q\left(\frac{a_1 - a_2}{2\sigma_0}\right) = Q\left(\sqrt{\frac{E_d}{2N_0}}\right) = Q\left(\sqrt{\frac{E_b}{N_0}(1-\rho)}\right)$$

$$a_i = \int_0^T s_i(t)\left[s_1(t) - s_2(t)\right] dt = \underbrace{s_i(t) \circledast h(t)}_{a_i(t)}\Big|_{t=T}$$

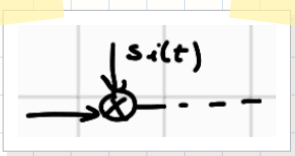


orijinal çizim, s. 1

$$h(t) = s_1(T - t) - s_2(T - t)$$

! Formüllerin nasıl çıktığını öğren

Soruda $h(t)$ $s_1(t)$'yi karşılıyor dersek $h(t) = s_1(T - t)$ olur integral devresinde



orijinal çizim, s. 1

$$\sigma_0^2 = \frac{N_0}{2} E_h$$

$$E_d = \int_0^T [s_1(t) - s_2(t)]^2 dt \qquad E_b = \sum_{i=1}^{2} P(s_i) E_{s_i}$$

$$\rho = \frac{1}{E_b} \int_0^T s_1(t) s_2(t) dt$$

Örnek (Unipolar signaling)

"1" $\rightarrow s_1(t) = A,\ 0 \le t \le T$

"0" $\rightarrow s_2(t) = 0,\ 0 \le t \le T$

$P(s_1) = \frac{1}{4},\ P(s_2) = \frac{3}{4}$

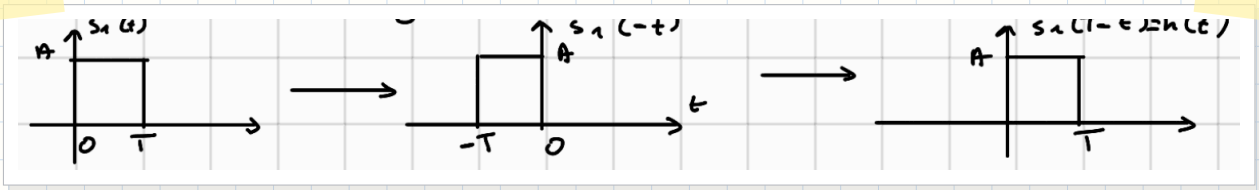
$A = 1\ V,\ T = 10^{-3}\ sn,\ \frac{N_0}{2} = 10^{-3}\ \frac{Watt}{Hz}$

$\gamma_0 = ?\quad P_b = ?$

$$a_1 = A^2 T = 1^2 \cdot 10^{-3} = 10^{-3}$$

$$a_2 = 0$$

$$h(t) = s_1(T-t) \underbrace{- s_2(T-t)}_{0} = s_1(T-t)$$



orijinal çizim, s. 1

orj. s. 2

$$E_h = \int_0^T h^2(t) dt = \int_0^T A^2 dt = A^2 T = 10^{-3}$$

$$\sigma_0^2 = \frac{N_0}{2} \cdot E_h = 10^{-3} \cdot 10^{-3} = 10^{-6}$$

$$\gamma_0 = \frac{\sigma_0^2}{a_1 - a_2} \ln \frac{P(s_2)}{P(s_1)} + \frac{a_1 + a_2}{2}$$

$$= \frac{10^{-6}}{10^{-3}} \ln \frac{3/4}{1/4} + \frac{10^{-3} + 0}{2} = 1{,}5986 \cdot 10^{-3} \simeq 1{,}6 \cdot 10^{-3}$$

$$P_b = \left[1 - Q\left(\frac{\overset{1{,}6\cdot10^{-3}}{\gamma_0} - \overset{10^{-3}}{a_1}}{\underset{10^{-3}}{\sigma_0}}\right)\right] \underset{\frac{1}{4}}{P(s_1)}$$

$$+ Q\left(\frac{\overset{1{,}6\cdot10^{-3}}{\gamma_0} - \overset{0}{a_2}}{\underset{10^{-3}}{\sigma_0}}\right) \underset{\frac{3}{4}}{P(s_2)}$$

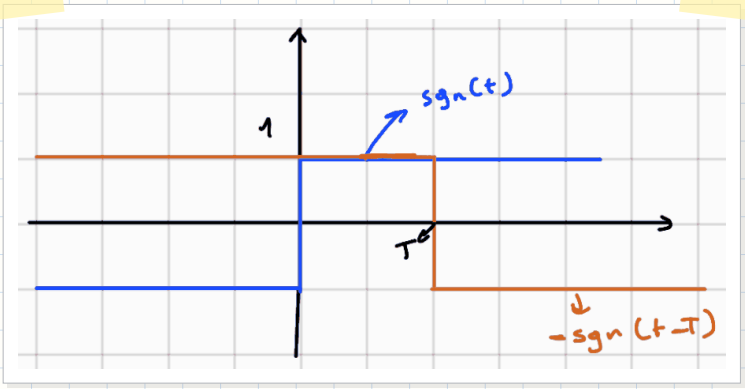
$$P_b = \left[1 - \underbrace{Q(0{,}6)}_{2{,}7426\cdot10^{-1}}\right]\frac{1}{4} + \underbrace{Q(1{,}6)}_{5{,}4799\cdot10^{-2}}\cdot\frac{3}{4}$$
$$= 0{,}2225$$

Örnek

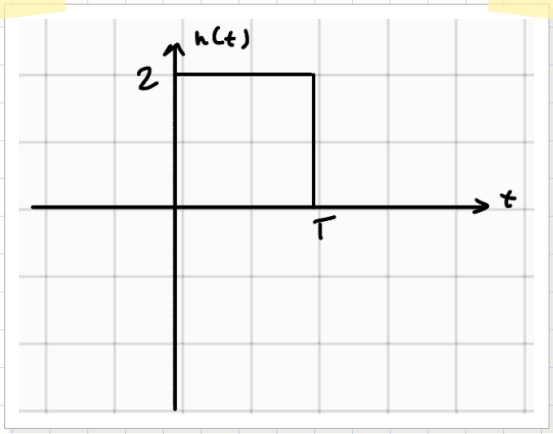
Transfer function of a MF is $H(f) = \dfrac{2 - 2e^{-j2\pi fT}}{j2\pi f}$

a) Find impulse response of MF and sketch it.
b) " the pulse $s(t)$ matched by this MF an sketch it.
c) If this $s(t)$ is applied to the input of MF, find the output signal of the MF and sketch it. Find the value at the sample instant.
d) If this $s(t)$ is applied to the input of the corelator, find the output signal of the correlator and sketch it. Find the value at the simple instant.

a) $H(f) = \dfrac{2}{j2\pi f} - \dfrac{2e^{-j2\pi fT}}{j2\pi f} \xrightarrow{\mathcal{F}^{-1}} h(t) = \text{sgn}(t) - \text{sgn}(t-T)$



orijinal çizim, s. 2

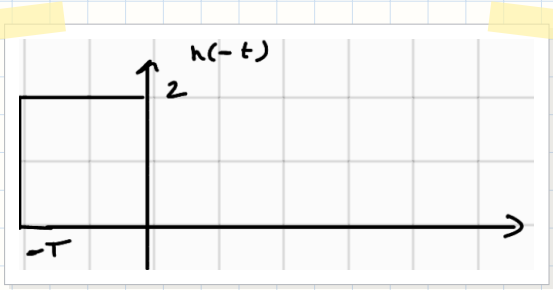


orijinal çizim, s. 2

b) $h(t) = s(\underbrace{T-t}_{t'}) \rightarrow h(T-t') = s(t')$

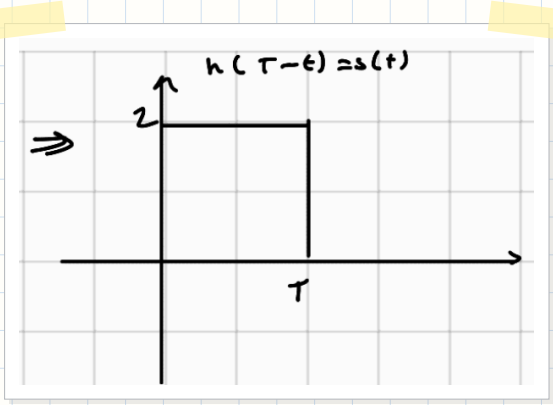
$t' = T - t \Longrightarrow \boxed{s(t) = h(T-t)}$

$t = T - t'$

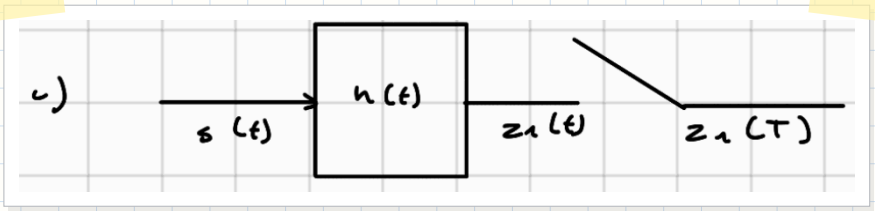


orijinal çizim, s. 2

orj. s. 3



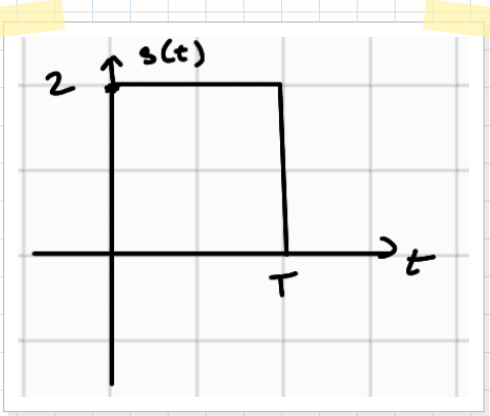
orijinal çizim, s. 3



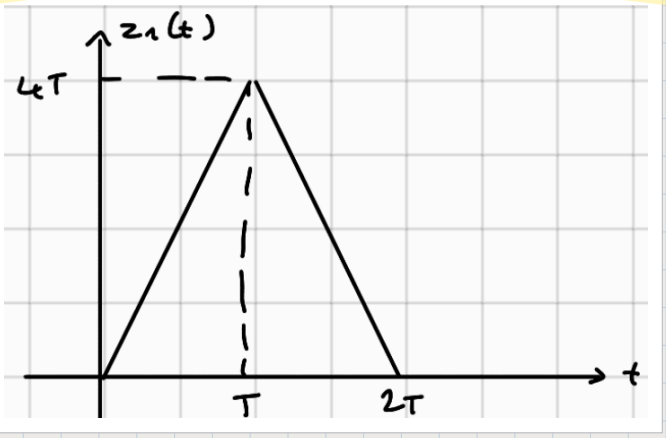
orijinal çizim, s. 3

$$z_1(t) = s_1(t) \circledast h(t) = s(t) \circledast s(t)$$

$$A\,\Pi\left(\frac{t}{T}\right) \circledast A\,\Pi\left(\frac{t}{T}\right) = A^2 T\,\Lambda\left(\frac{t}{T}\right)$$

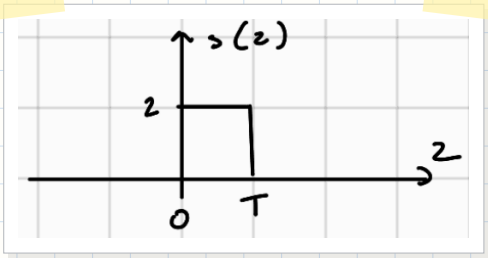


orijinal çizim, s. 3

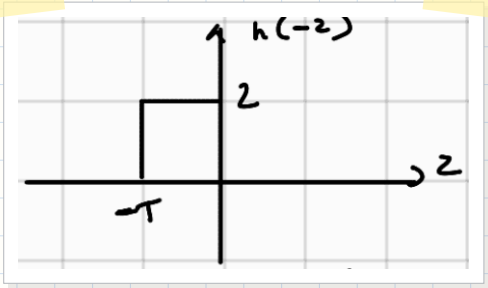


orijinal çizim, s. 3

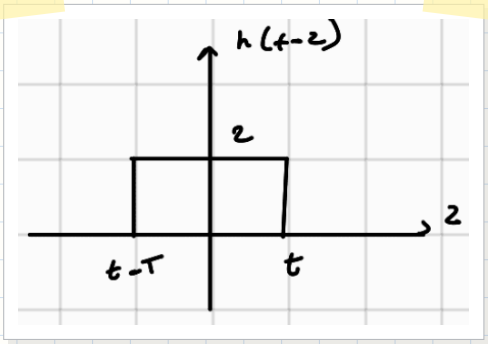
$$z_1(T) = 4T$$



orijinal çizim, s. 3



orijinal çizim, s. 3



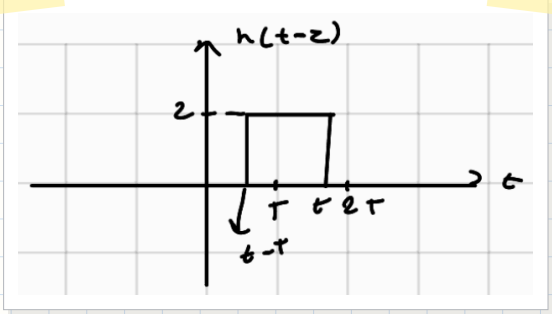
orijinal çizim, s. 3

$$z_1(t) = s(t) \circledast h(t) = \int_{-\infty}^{\infty} s(z)\, h(t - z)\, dz$$

$$t < 0: \quad z_1(t) = 0$$

$$0 < t < T: \quad z_1(t) = \int_0^t 2 \cdot 2\, dt = 4t$$

orj. s. 4

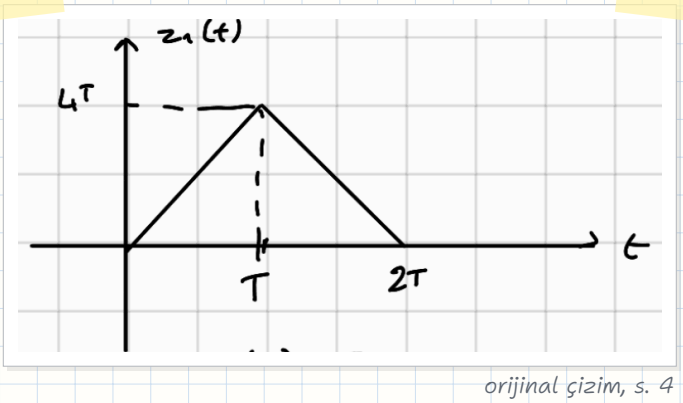


orijinal çizim, s. 4

$$T < t < 2T: \quad z_1(t) = \int_{t-T}^{T} 2 \cdot 2\, dt =$$

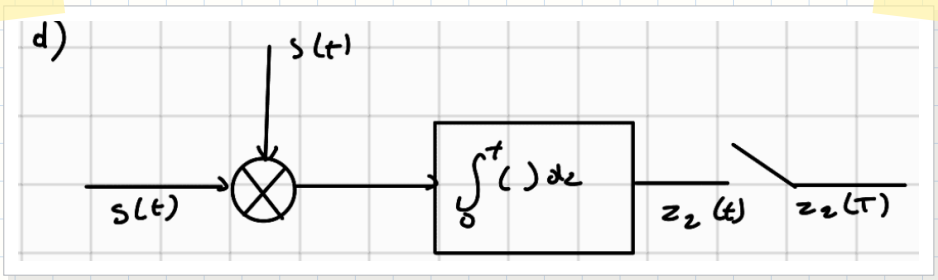
$$t > 2T: \quad z_1(t) = 0$$

$$z_1(t) = \begin{cases} 0, & t < 0 \\ 4t, & 0 < t < T \\ 4(2T - t), & T < t < 2T \\ 0, & t > 2T \end{cases}$$



orijinal çizim, s. 4

$$z_1(T) = 4T$$



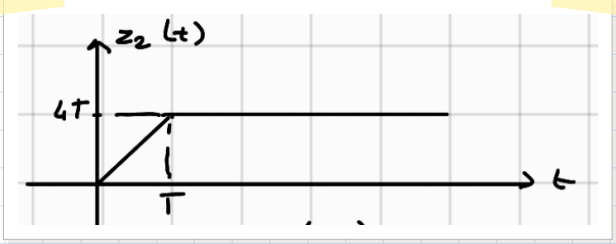
orijinal çizim, s. 4

$$z_2(t) = \int_0^t s(z)\,s(z)\,dz$$

$$= \int_0^t s^2(z)\,dz = 4t \qquad t < T$$

$$\int_0^T s^2(z)\,dz = 4T \qquad t > T$$

$$z_2(t) = \begin{cases} 4t, & t < T \\ 4T, & t > T \end{cases}$$



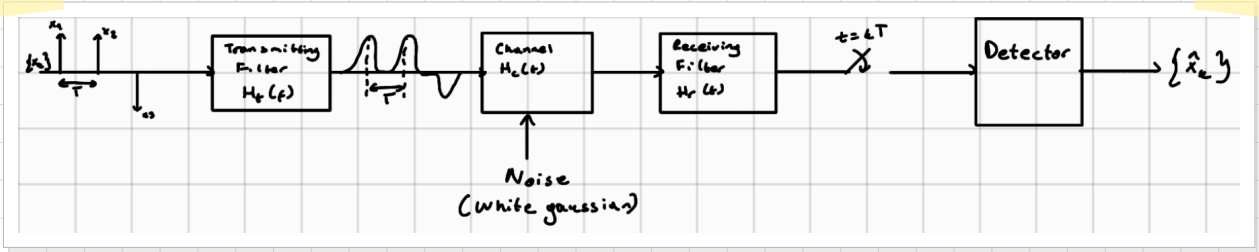
orijinal çizim, s. 4

$$z_2(T) = 4T$$

Semboller arası karışım (SAK)

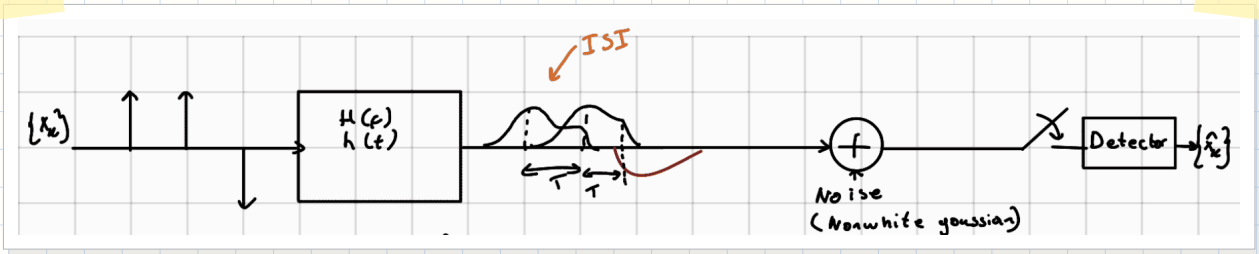
(Inter symbol Interference - ISI)

(Pulse shaping - Darbe şekillendirme)



orijinal çizim, s. 4

orj. s. 5



orijinal çizim, s. 5

$$H(f) = H_t(f)\,H_c(f)\,H_r(f)$$

Sıfır SAK için 1. Nyquist kriteri

(1st Nyquist criterion for zero ISI)

$$h(t) = \begin{cases} 1, & t = 0 \\ 0, & t = \pm nT \end{cases} \Rightarrow h(nT) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

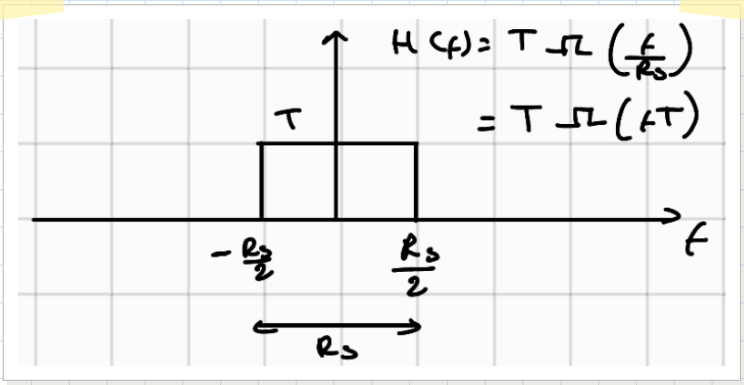
$n = 1,2,3,\ldots$

$T = \dfrac{1}{R_s}$ ↑ sembol hızı

Hız $= R_s \dfrac{\text{sembol}}{\text{sn}}$

$\Rightarrow BG = \dfrac{R_s}{2}$ Hz

$$\xrightarrow{\mathcal{F}^{-1}} \quad h(t) = \text{sinc}\left(\frac{t}{T}\right)$$

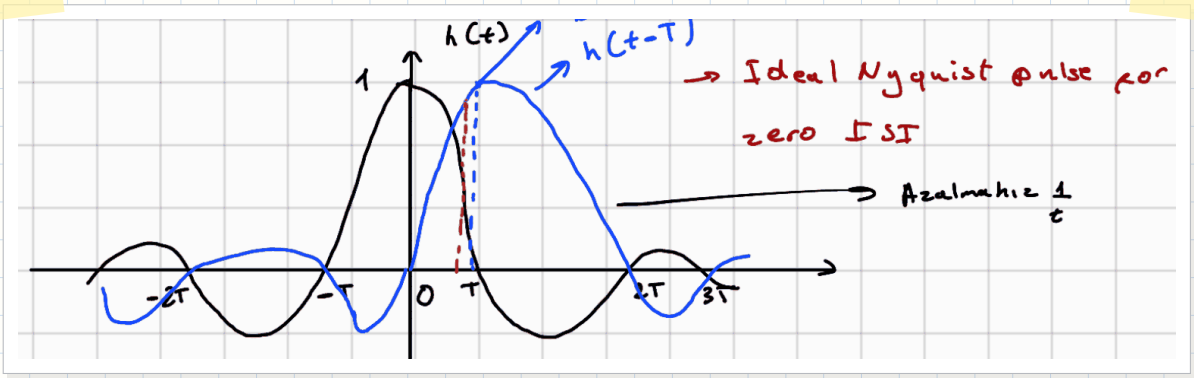


orijinal çizim, s. 5

$$A\tau\,\text{sinc}(t\tau) \overset{\mathcal{F}}{\longleftrightarrow} A\Pi\left(\frac{f}{\tau}\right)$$

← ideal Nyquist filter for zero ISI

mavinin katkısı 1, siyahın 0 semboller arası karışım 0



orijinal çizim, s. 5

Senkron hatasını yaptığımızı düşünelim sembol arası karışım olabilir.

$$\frac{t}{T} = k \in \mathbb{Z} - \{0\} \Rightarrow t = kT$$

Raised cosine (RC) darbesi

Daha yüksek azalma hızına sahiptir senkronizasyon hatası azalır

$$BG = k\frac{R_s}{2} \qquad 1 \le k \le 2$$

1. Nyquist kriterinin diğer ifadesi

$$\underset{\substack{\text{sampled}\\ h(t)}}{h_s(t)} = h(t) \underset{\text{? darbe katarı,}}{s_T(t)} = h(t)\sum_k \delta(t-kT)$$

$$= \sum_k h(kT)\,\delta(t-kT) = \delta(t)$$

orj. s. 6

$$\mathcal{F}\left\{h(t)\sum_k \delta(t-kT)\right\} = \mathcal{F}\{\delta(t)\}$$

$$H(f) \circledast \frac{1}{T}\sum_k \delta\left(f-\frac{k}{T}\right) = 1$$

$$\frac{1}{T}\sum_k H\left(f-\frac{k}{T}\right) = 1 \Rightarrow \boxed{\sum_k H\left(f-\frac{k}{T}\right) = T}$$