Sayısal Haberleşme

RC darbe, kontrollü SAK, sayısal taşıyıcılı modülasyon (PSK/FSK)

Orijinal not: Ders-11.pdf

UYARI Bu belge, el yazısı ders notlarından Claude (Anthropic) yardımıyla dijitale aktarılmıştır. Yardımcı kaynak olarak hazırlanmıştır; hata veya eksik içerebilir. Lütfen orijinal notları ve ders kaynaklarını esas alın.

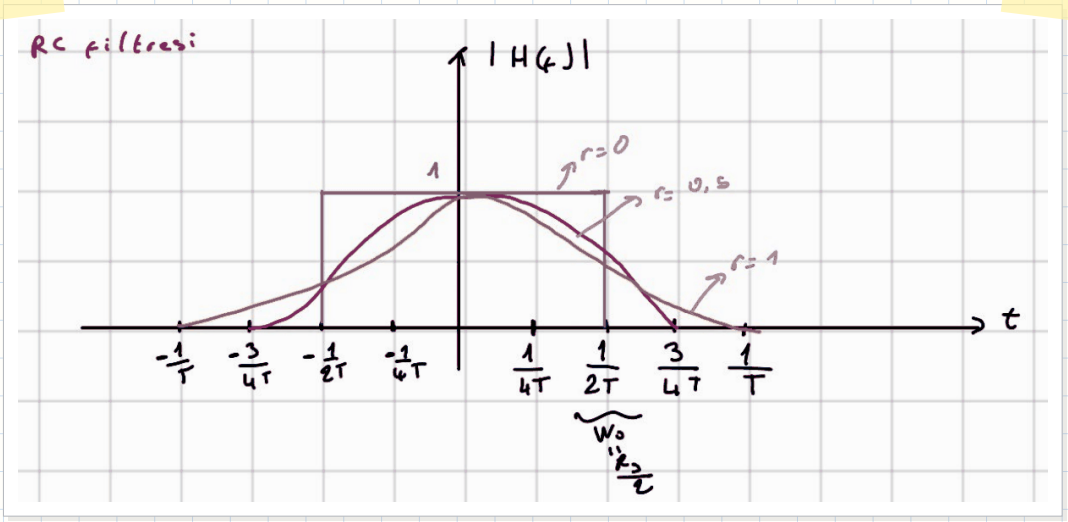
NOTICE *This document was transcribed from handwritten lecture notes with the help of Claude (Anthropic). It is meant as a supplementary resource and may contain errors or omissions. Always check against the original notes and official course material.*

orj. s. 1

Raised Cosine (RC) Pulse

Yükseltilmiş kosinüs darbesi

$$H(f) = \begin{cases} 1, & |f| < 2W_0 - W \\ \cos^2\left(\frac{\pi}{4}\frac{|f| + W - 2W_0}{W - W_0}\right), & 2W_0 - W < |f| < |W| \\ 0, & |f| > W \end{cases}$$



orijinal çizim, s. 1

$W_0 = \frac{R_s}{2} = \frac{1}{2T}$ min Nyquist BW for rectangular spectrum

$W - W_0$ = Aşırı BG (Excess BW)

$$r = \frac{W - W_0}{W_0} \qquad \text{Azalma faktörü (roll-off factor)}$$

$$0 \le r \le 1$$

$$\rightarrow \; W = W_0(1 + r)$$

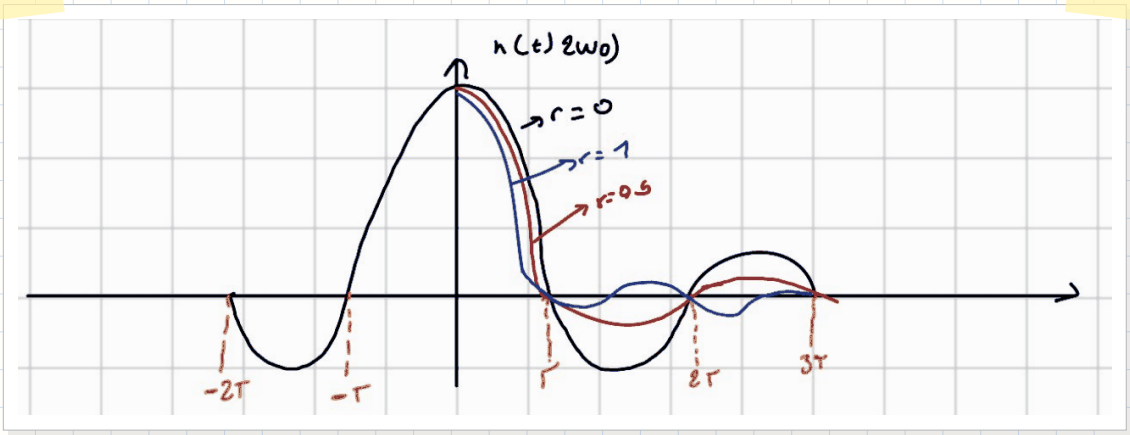
$$W = \frac{R_s}{2}(1 + r) \text{ Hz}$$

→ Temel band haberleşmede RC darbesinin BG
W → Mutlak BG (Absolute BW)

→ Band geçiren Haberleşme

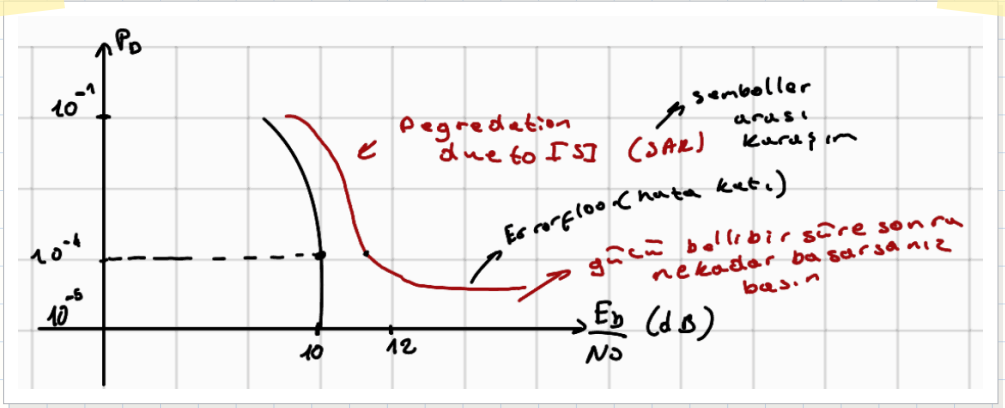
$$W_{ÇYB} = R_s(1 + r) \text{ Hz} \qquad \text{(Taşıyıcılı haberleşme)}$$

$$h(t) = 2W_0 \,\text{sinc}(2W_0 t)\frac{\cos[2\pi(W - W_0)t]}{1 - [4(W - W_0)t]^2} \qquad \text{azalma hızı } \frac{1}{t^3}$$



orijinal çizim, s. 1

orj. s. 2



orijinal çizim, s. 2

Çözüm → equalization

$$H(f) = H_t(f)\,\underbrace{H_c(f)}_{\text{channel}}\,\underbrace{H_r(f)}_{H_e(f)\,H_r'(f)} \quad \Rightarrow \quad \underbrace{H(f)}_{\text{RC}} = \underbrace{H_t(f)}_{\text{Root RC (RRC)}}\,\underbrace{H_r'(f)}_{\text{RRC}}$$

$H_e(f) = \frac{1}{H_c(f)}$

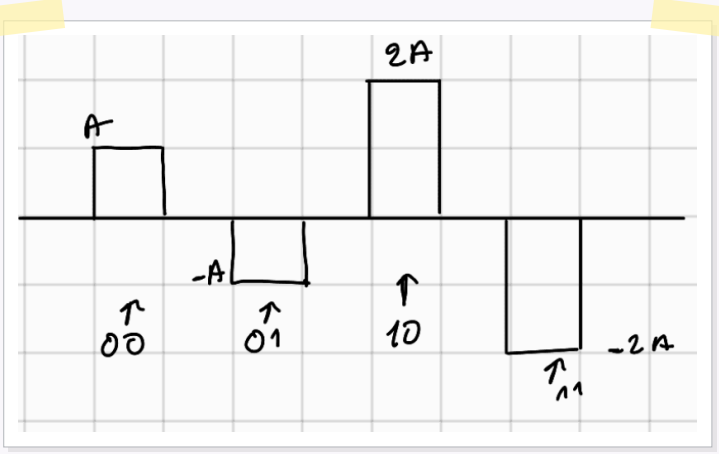
$h_r'(t) = h_t(T - t)$ Karşılayıcı filtre

Örnek 3.3

(s.143, Ex 3.3)

> *Şekil:* Basılı kitap kesiti ("Example 3.3 Bandwidth Requirements", (a) ve (b) şıkları) basılı materyal içeriyor; dijitalleştirilmedi.
> *(orijinal notta, s. 2)*

4 level PAM
$\mathcal{R} = 2400$ bits/s
RC 100% excess BW ($r = 1$)



orijinal çizim, s. 2

$$M = 2^k = 4 \Rightarrow k = 2 \text{ bits/symbol (pulse)}$$

$$\underset{\text{sembol hızı}}{\mathcal{R}_s} = \frac{\mathcal{R}_b \text{ bit/s}}{k \text{ bits/sembol}} = \frac{2400 \text{ bit/sn}}{2 \text{ bit/sn}^{1}} = 1200 \ \frac{\text{sembol}}{\text{sn}} \text{ (darbe)}$$

$$W = \frac{\mathcal{R}_s}{2}(1 + r) = \frac{1200}{2}(1 + 1) = 1200 \text{ Hz}$$

b) $W_{DSB} = \mathcal{R}_s(1 + r) = 2400$ Hz

orj. s. 3

Kontrollü SAK ile işaretleşme

(Signaling with controlled ISI)

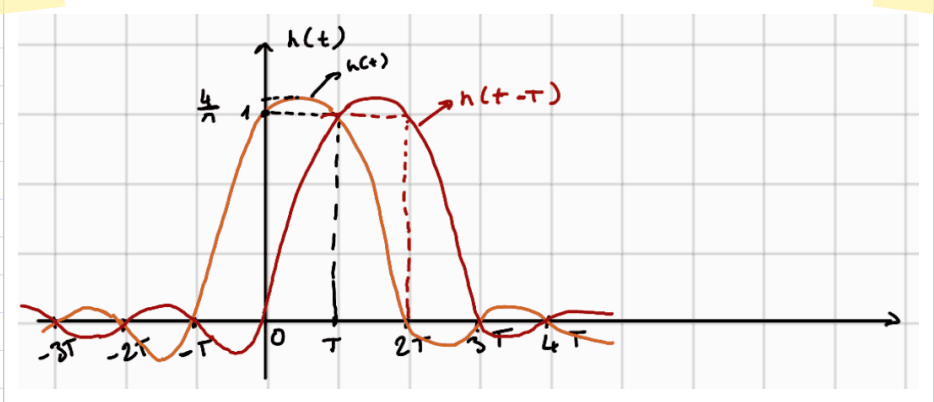
[1] Claude notu: muhtemelen 2 bit/sembol olmalı.

(Kısmi cevaplı işaretleşme – Partial response signaling)

Sıfır SAK için 2. Nyquist kriteri:

$$h(nT) = \begin{cases} 1, & n = 0,1 \\ 0, & \text{Diğer} \end{cases}$$

Çift kutuplu
"1" → $h(t)$
"0" → $-h(t)$



orijinal çizim, s. 3

Her iki darbe + ⇒ Örnek = 2
" " " − ⇒ " = −2
Darbeler zıt işaretli ⇒ " = 0
Eğer örnek > 0 ⇒ Mevcut bit ve önceki bit "1"
örnek < 0 ⇒ " " " " "0"
" = 0 ⇒ Mevcut bit önceki bitin tümleyenidir

İletilen dizi:	1	1	0	1	1	0	0	0	1	0	1	1	1
Örnekler:	1	2	0	0	2	0	−2	−2	0	0	0	2	2
Tespit edilen dizi:	1	1	0	1	1	0	0	0	1	0	1	1	1

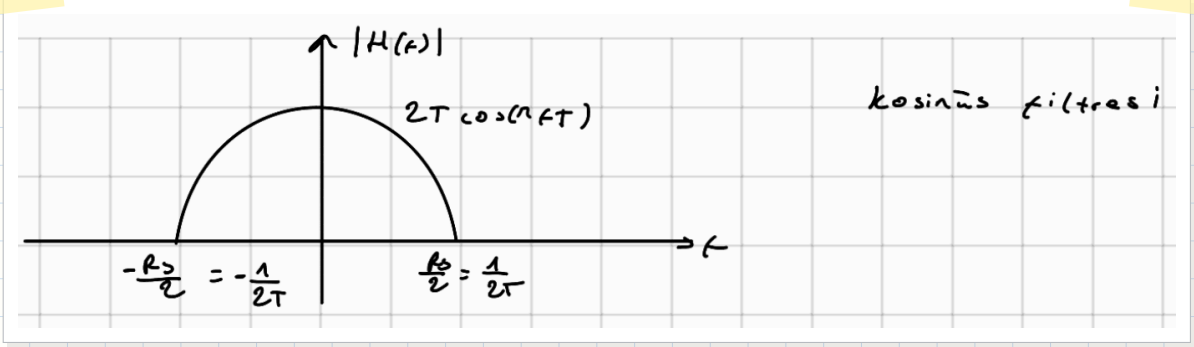
Duo binary darbe

$$h(t) = \frac{\sin(\pi R_s t)}{\pi R_s t\,(1 - R_s t)} = \frac{\sin(\pi t/T)}{\frac{\pi t}{T}\left(1 - \frac{t}{T}\right)} = \underbrace{\operatorname{sinc}\left(\frac{t}{T}\right) + \operatorname{sinc}\left(\frac{t-T}{T}\right)}_{\text{? süresi: } 3T}$$

→ Azalma hızı $\frac{1}{t^2}$

$$h(t) \xrightarrow{\mathcal{F}} H(f) = \frac{2}{R_s}\cos\left(\frac{\pi f}{R_s}\right)\Pi\left(\frac{f}{R_s}\right)e^{-j\pi f/R_s}$$
$$= 2T\cos(\pi f T)\,\Pi(fT)\,e^{-j\pi fT}$$

orj. s. 4



orijinal çizim, s. 4

RC darbesinden daha az BG'ye sahip
bu filtre $\frac{1}{t^2}$ ile azalmakta senkronizasyon hataları
daha fazla RC'de $\frac{1}{t^3}$ ile azalmakta

Örnek 3.8
S.164 Pr 3.8

> **Şekil:** Basılı kitap problemi (3.8, (a) ve (b) şıkları) basılı materyal içeriyor; dijitalleştirilmedi.
> *(orijinal notta, s. 4)*

a) R = 10 Mbit/s
16 level PAM w/o ISI

$$M = 2^k = 16 \rightarrow k = 4 \text{ bits/symbol (pulse)}$$

$$R_s = \frac{R}{k} = \frac{10 \text{ Mbits/s}}{4 \text{ bits/symbol}} = 2{,}5 \text{ M } \frac{\text{symbols}}{\text{s}}$$

$$W = \frac{R_s}{2} = 1{,}25 \text{ MHz}$$

b) W = 1,375 MHz

$$\underset{1{,}375}{W} = \underset{1{,}25}{\frac{R_s}{2}}(1+r) \rightarrow r = 0{,}1$$

Örnek 3.9
Pr. 3.9

$Q_{dist} \leq \pm 0{,}1\%$ of peak to peak signal voltage
f_s = 8000 samples/s
M = 32 PAM

> **Şekil:** Basılı kitap problemi (3.9) basılı materyal içeriyor; dijitalleştirilmedi.
> *(orijinal notta, s. 4)*

Min theo BW w/o ISI

$$\Delta v = \frac{2x_p}{L} \rightarrow \frac{\Delta v}{2} = \frac{x_p}{L} \leq \frac{0{,}1}{100} 2x_p \rightarrow L \geq \frac{100}{0{,}2} = 500$$

$$\underbrace{2^n}_{512} \geq L > 500 \Rightarrow 2^n = 512 \Rightarrow n = 9 \text{ bits/sample} \qquad BG = 36 \text{ kHz}$$

$$\left.\begin{aligned} R_b = n f_s = 9 \frac{\text{bits}}{\text{sample}} \times 8000 \frac{\text{samples}}{\text{s}} = 72 \text{ k bits/s} \\ M = 32 = 2^5 \Rightarrow k = 5 \text{ bits/symbol (pulse)} \end{aligned}\right\} \text{5 kat BG düştü}$$

orj. s. 5

$$R_s = \frac{R_b}{k} = \frac{72 \text{ k bits/s}}{5 \text{ bits/symbol}} = 14\,400 \frac{\text{symbols}}{\text{s}} \rightarrow W = \frac{R_s}{2} = 7200 \text{ Hz}$$

↑ Bit hata olasılığı daha kötü

Örnek

In a CD recording system, 2 voice signals are sampled with a sampling freq. of 44,1 kHz, multiplexed an encoded with 16 bits.
a) What is the bitrate?
b) How many bits are needed for a recording of 50 minutes?

$$\text{a) } R_b = \overset{\text{sinyal sayısı}}{2} \cdot n \cdot f_s = 2 \cdot \frac{16 \text{ bits}}{\text{sample}} \cdot \frac{44{,}1 \text{ ksample}}{\text{sec}} = 1\,411\,200 \frac{\text{bits}}{\text{s}}$$

$$\text{b) \# of bits} = 1\,411\,200 \frac{\text{bits}}{\text{s}} \times 50 \times 60 \text{ s} = 4\,233\,600\,000 \text{ bits}$$

8'e bölelim: 529,2 M byte
sonra 1000'e

Örnek 3.9

Şekil: Basılı kitap problemi (3.9) basılı materyal içeriyor; dijitalleştirilmedi.
(orijinal notta, s. 5)

Min theo BW w/o ISI

$$\Delta v = \frac{2x_p}{L} \rightarrow \frac{\Delta v}{2} = \frac{x_p}{L} \leq \frac{0{,}1}{100} 2x_p \rightarrow L \geq \frac{100}{0{,}2} = 500$$

$$\underbrace{2^n}_{512} \geq L > 500 \Rightarrow 2^n = 512 \Rightarrow n = 9 \text{ bits/sample}$$

$$\left.\begin{aligned} R_b = n f_s = 9 \frac{\text{bits}}{\text{sample}} \times 8000 \frac{\text{samples}}{\text{s}} = 72 \text{ k bits/s} \qquad BG = 36 \text{ kHz} \\ M = 32 = 2^5 \Rightarrow k = 5 \text{ bits/symbol (pulse)} \end{aligned}\right\} \text{5 kat BG düştü}$$

$$R_s = \frac{R_b}{k} = \frac{72 \text{ k bits/s}}{5 \text{ bits/symbol}} = 14\,400 \frac{\text{symbols}}{\text{s}} \rightarrow W = \frac{R_s}{2} = 7200 \text{ Hz}$$

↑ Bit hata olasılığı daha kötü

c) $r = 0{,}25 \Rightarrow$ Bipolar signaling BG?

$$R_b = 72 \text{ k bits/s} \rightarrow W = \frac{R_b}{2}(1 + r) = \frac{72 \text{ k}}{2}(1 + 0{,}25) = 45 \text{ kHz}$$

$r = 0{,}25 \rightarrow M = 32$ PAM BG ? $\qquad R_s = 14\,400$ symbols/s $\rightarrow W = \frac{R_s}{2}(1+r) = \frac{14\,400\,(1 + 0{,}25)}{2} = 9 \text{ kHz}$ //

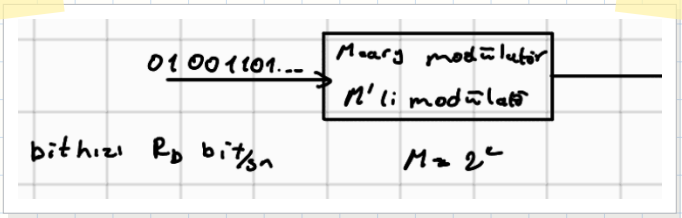
orj. s. 6

Sayısal Taşıyıcılı Modülasyon

(Carrier Mod / Bandpass Mod)

Faz Kaydırmalı Anahtarlama (FKA) Mod

Phase Shift Keying (PSK) Mod



orijinal çizim, s. 6

1 sembol

$$s_i(t) \;;\; i = 1, \ldots, M = 2^k \;;\; 0 \le t \le T$$

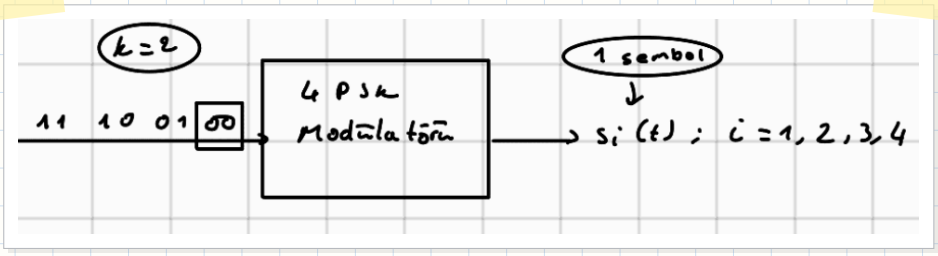
$$R_s = \frac{R_b}{k} = \frac{1 \text{ sembol}}{T \text{ sn}} = \frac{k \text{ bit}}{T \text{ sn}}$$

$$\text{PSK:} \quad s_i(t) = \sqrt{\frac{2E}{T}} \cos(2\pi f_c t + \phi_i) \;;\; 0 \le t \le T$$

$$\phi_i = \frac{2\pi(i-1)}{M} \qquad i = 1, \ldots, M = 2^k$$

Örnek

$M = 4 = 2^k \Rightarrow k = 2$ bit/sembol



orijinal çizim, s. 6

$$00 \rightarrow s_1(t) = \sqrt{\frac{2E}{T}} \cos(2\pi f_c t + 0) \;;\; 0 \le t \le T$$

$$01 \rightarrow s_2(t) = \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + \frac{\pi}{2}\right) \;;\; 0 \le t \le T$$

$$10 \rightarrow s_3(t) = \sqrt{\frac{2E}{T}} \cos(2\pi f_c t + \pi) \;;\; ''$$

$$11 \rightarrow s_4(t) = \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + \frac{3\pi}{2}\right) \;;\; ''$$

Enerji:

$$E_{s_i} = \underbrace{\frac{2E}{T}\frac{1}{2}}_{\frac{A^2}{2}} T = E$$

$$\forall\, E_{s_i} = E$$

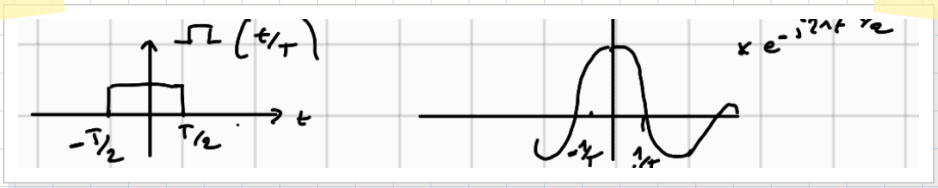
$$E_{s_i} = \int_0^T s_i^2(t)\,dt = \frac{2E}{T}\int_0^T \underbrace{\cos^2(2\pi f_c t + \phi_i)\,dt}_{\frac{1}{2}\left[1+\cos(4\pi f_c t)\right]dt}$$

$$= \frac{2E}{T}\frac{1}{2}\int_0^T dt = E$$

$$\int_0^T \cos(4\pi f_c t)\,dt \approx 0$$

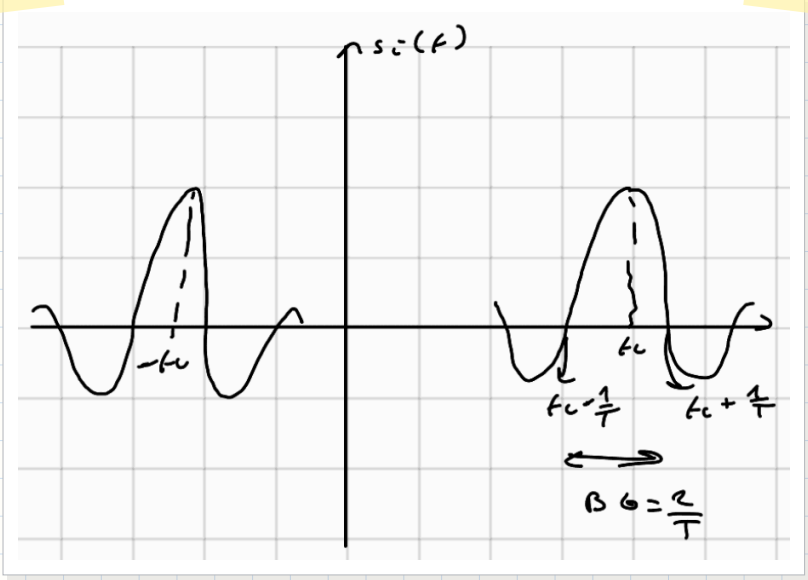
BG

$$s_i(t) = \sqrt{\frac{2E}{T}}\,\Pi\left(\frac{t - T/2}{T}\right)\cos(2\pi f_c t + \phi_i)$$



orijinal çizim, s. 6

orj. s. 7



orijinal çizim, s. 7

M arttığında BG aynı kalıyor. Bu önemli bir avantaj

Bant genişliği verimli teknik

$$\underset{\text{BG verimliliği}}{\text{BGV}} = \frac{R_b \text{ bit/sn}}{W \text{ Hz}} \quad \frac{\text{bit/sn}}{\text{Hz}}$$

(W: BG)

↗ bir Hz bant genişliğinde bir saniyede gönderilen bit

MPSK: $M \uparrow \Rightarrow$ BG aynı $\rightarrow$ BGV↑ BGV verimli teknik (BW efficient mod technique) $\times P_b \uparrow$ (performans kötüleşiyor)

Bant genişliğinden kazanıyoruz güçten kaybediyoruz

Özel durum

$k = 1$, $M = 2 \Rightarrow$ İkili FKA = İFKA
Binary PSK = BPSK

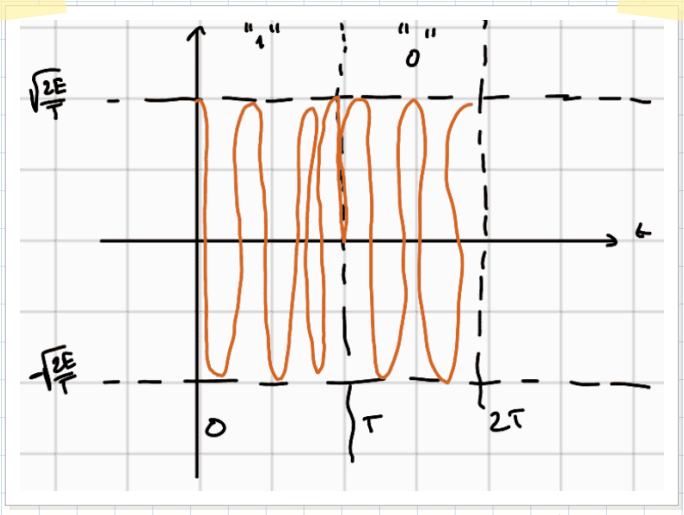
$$\text{"1"} \rightarrow s_1(t) = \sqrt{\frac{2E}{T}} \cos(2\pi f_c t + 0) \; ; \; 0 \le t \le T$$

$$\text{"0"} \rightarrow s_2(t) = \sqrt{\frac{2E}{T}} \cos(2\pi f_c t + \pi) \; ; \; 0 \le t \le T$$

$$= -\sqrt{\frac{2E}{T}} \cos(2\pi f_c t) \; ; \; 0 \le t \le T$$

Bipolar signaling × carrier $\rightarrow P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$ $E_b = E$

→ Sayfa 174
Şekil 4.5



orijinal çizim, s. 7

orj. s. 8

Ortogonal modülasyon

Frekans Kaydırmalı Anahtarlama (FRKA)

Frequency Shift Keying (FSK)

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos(2\pi f_i t)\ ;\ 0 \le t \le T\ ,\ i = 1,2,3,4$$

$$f_i = f_c + (i-1)\Delta f$$

$$\int_0^T s_i(t)\, s_j(t)\, dt = \begin{cases} 0, & i \neq j \\ E, & i = j \end{cases} \Rightarrow \underset{\text{(Dik)}}{\text{ortogonal}}$$

Ortogonallik şartı:

$$\Delta f = \begin{cases} \frac{1}{2T} & \text{Faz tahminli demod. (Coherent demod.)} \quad \frac{BG}{2} \\ \frac{1}{T} & \text{" tahminsiz " (Non coherent dmd.)} \quad BG \nearrow \end{cases}$$

→ Sayfa 202 Ex. 4.3

$$M \uparrow \Rightarrow BG \uparrow \Rightarrow BGV \downarrow \qquad \times P_b \downarrow$$

(Power efficient Mod.)