Sayısal Haberleşme

FSK, ASK, QAM ve M'li modülasyonun vektör uzayı gösterimi

Orijinal not: Ders-12.pdf

UYARI Bu belge, el yazısı ders notlarından Claude (Anthropic) yardımıyla dijitale aktarılmıştır. Yardımcı kaynak olarak hazırlanmıştır; hata veya eksik içerebilir. Lütfen orijinal notları ve ders kaynaklarını esas alın.

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orj. s. 1

M-FSK

Ortogonal modülasyon

Frekans kaydırmalı Anahtarlama (FKA)

Frequency Shift keying (FSK)

$$s_i(t) = \sqrt{\frac{2E}{T}}\cos(2\pi f_i t), \quad 0 \le t \le T$$

$$i = 1, \ldots, M = 2^k$$

$$f_i = f_c + (i-1)\Delta f$$

$$\Delta f = \begin{cases} \frac{1}{2T} & \text{Faz tahminli demod.} \\ \frac{1}{T} & \text{" tahminsiz "} \end{cases}$$

$$\int_0^T s_i(t)\, s_j(t)\, dt = \begin{cases} 0 & i \neq j \\ E & i = j \end{cases}$$

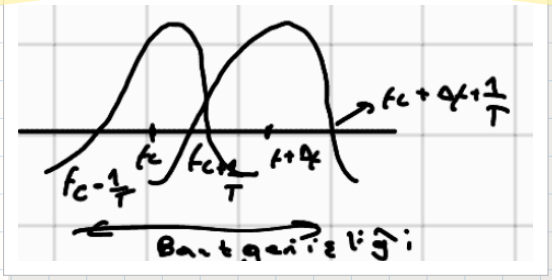
$$\forall_i \quad E_i = E$$

$$M \uparrow \rightarrow \text{BG} \uparrow \quad (\text{BGV} \downarrow) \quad \times \quad \underset{\text{performans iyileşir}}{P_b \downarrow} \quad \Rightarrow \quad \underset{\text{(Power efficient)}}{\text{güç verimli}}$$

$$\text{BGV} = \frac{R_b \text{ bit/sn}}{\text{BG Hz}}$$

Özel durum

$M = 2^1 \Rightarrow k = 1$ bit $\Rightarrow$ ikili FRKA (iFRKA)
Binary FSK (BFSK)



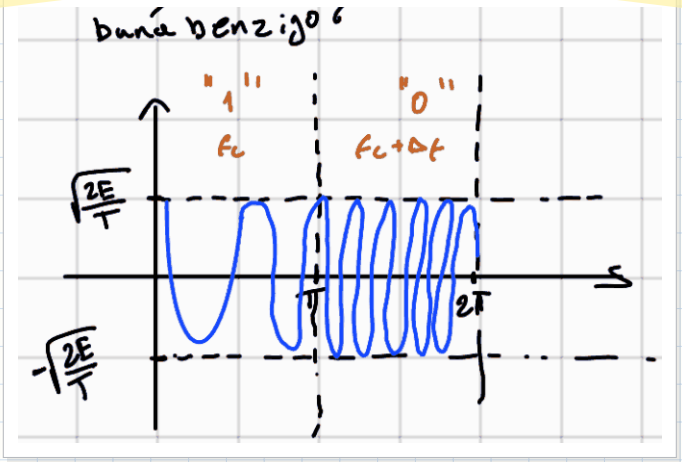
orijinal çizim, s. 1

$$\left.\begin{array}{l} \text{"1"} \rightarrow s_1(t) = \sqrt{\frac{2E}{T}}\cos(2\pi f_c t)\,,\ 0 \le t \le T \\ \text{"0"} \rightarrow s_2(t) = \sqrt{\frac{2E}{T}}\cos(2\pi(f_c + \Delta f)t)\,,\ 0 \le t \le T \end{array}\right\} \int_0^T s_1(t)\,s_2(t)\,dt = 0$$

$$\downarrow \rho = 0$$

$$P_b = Q\left(\sqrt{\frac{E_b}{N_0}}\right),\ E_b = E$$

Unipolar signaling × carrier



orijinal çizim, s. 1

$$\xrightarrow{\text{performansı daha iyi}} \text{BPSK}: P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

$$\text{BFSK}: P_b = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

↑ Bant genişliği daha fazla

M arttıkça FSK'ın PSK'dan daha iyi bir performansa sahip olur

PSK'nın zarfı sabit değil, devreleri doymaya sokuyor.

Sayfa 174
Şekil 4.5

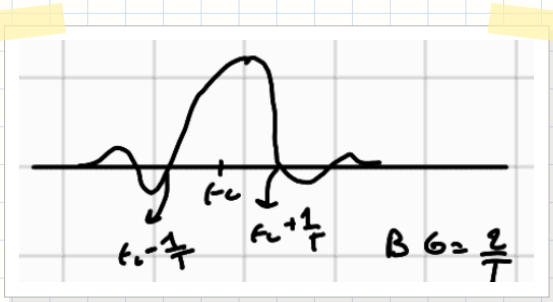
orj. s. 2

Ortogonal modülasyon

Birbirine ortogonal işaret

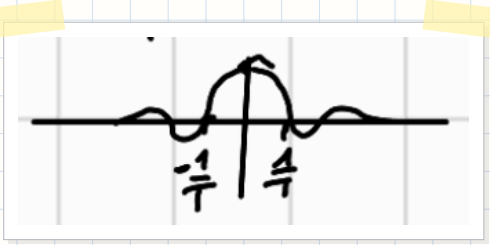
$$\left.\begin{aligned} s_1(t) &= \sqrt{\tfrac{2\mathcal{E}}{T}}\cos(2\pi f_c t)\,,\ 0 \le t \le T \\ s_2(t) &= \sqrt{\tfrac{2\mathcal{E}}{T}}\sin(2\pi f_c t)\,,\ 0 \le t \le T \end{aligned}\right\}\ \rho = 0$$

Bant genişliği: FSK'dan ? *tasarruf*

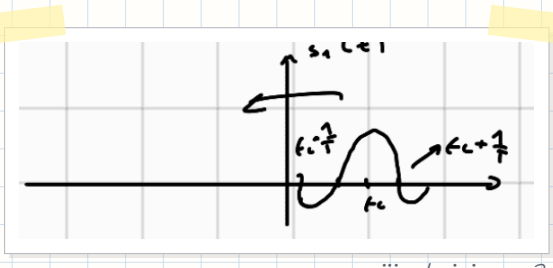


orijinal çizim, s. 2

$$\begin{aligned} s_1(t) &= \sqrt{\tfrac{2\mathcal{E}}{T}}\cos(2\pi f_c t)\,,\ 0 \le t \le T \\ &= \sqrt{\tfrac{2\mathcal{E}}{T}}\,\Pi\left(\frac{t - T/2}{T}\right)\cos(2\pi f_c t) \end{aligned}$$



orijinal çizim, s. 2



orijinal çizim, s. 2

Genlik Kaydırmalı Anahtarlama

Amplitude Shift Keying (ASK) / PAM

$$s_i(t) = \sqrt{\tfrac{2}{T}}\,A_i\cos(2\pi f_c t)\,,\ 0 \le t \le T\,,\ i = 1, \ldots, M = 2^k$$

$$BG = \frac{2}{T} \text{ Hz}$$

$M \uparrow \Rightarrow$ BG aynı (BGV $\uparrow$) $\times$ $P_b \uparrow$ $\Rightarrow$ BG verimli teknik

Örnek

$k = 2 \qquad M = 2^2 = 4$

		1. Teknik		2. Teknik
0	0	$\rightarrow A_1 = A$		$A_1 = A$
0	1	$\rightarrow A_2 = 2A$	$\Rightarrow$	$A_2 = -A$
1	0	$\rightarrow A_3 = 3A$		$A_3 = 2A$
1	1	$\rightarrow A_4 = 4A$		$A_4 = -2A$

$$\longrightarrow E_i = \underbrace{\frac{2}{T} A_i^2 \frac{1}{2}}_{\text{? güç}} \times \underset{\text{zaman}}{T} = A_i^2 \text{ joule}$$

$$E_i = A_i^2$$

$$E_{ort} = \frac{30A^2}{4} = 7{,}5A^2 \qquad E_{ort} = \frac{10A^2}{4} = 2{,}5A^2$$

⚠ Dikkat!

Sınav

Daha az enerjiyle aynı performans elde ederiz

Özel durum:

$M = 2 \rightarrow$ ikili GKA (iGKA)
Binary ASK (BASK)

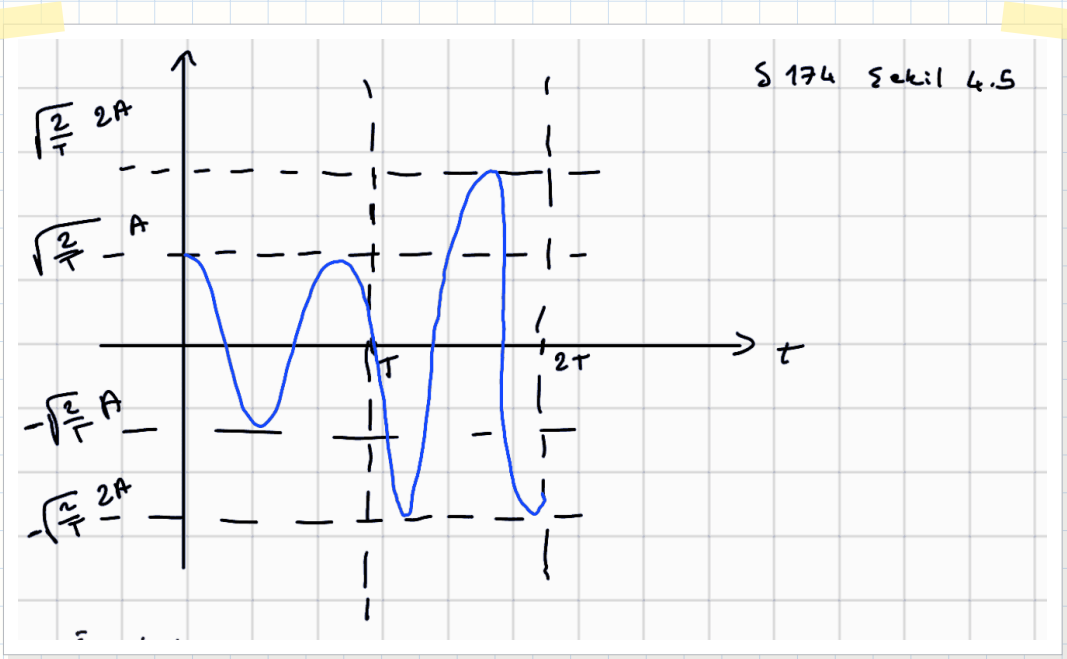
$$\left.\begin{aligned} s_1(t) &= \sqrt{\tfrac{2}{T}} A \cos(2\pi f_c t), \quad 0 \le t \le T \\ s_2(t) &= -\sqrt{\tfrac{2}{T}} A \cos(2\pi f_c t), \quad 0 \le t \le T \end{aligned}\right\} \text{ Binary ASK = Binary PSK}$$

$\nwarrow A_2 = -A$

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right), \quad E_b = A^2$$

orj. s. 3

$$M = 2 \ \{ A_1 = A, \ A_2 = 2A$$



orijinal çizim, s. 3

Özel durum

On off keying (OOK) / Var-yok anahtarlama (VYA)

$$s_1(t) = \sqrt{\frac{2}{T}}\,A\cos(2\pi f_c t)\,,\ 0 \le t \le T$$

$$s_2(t) = 0\,,\ 0 \le t \le T \longrightarrow \sqrt{\frac{2A^2}{T}} = \sqrt{\frac{2E}{T}}\,,\ E = A^2$$

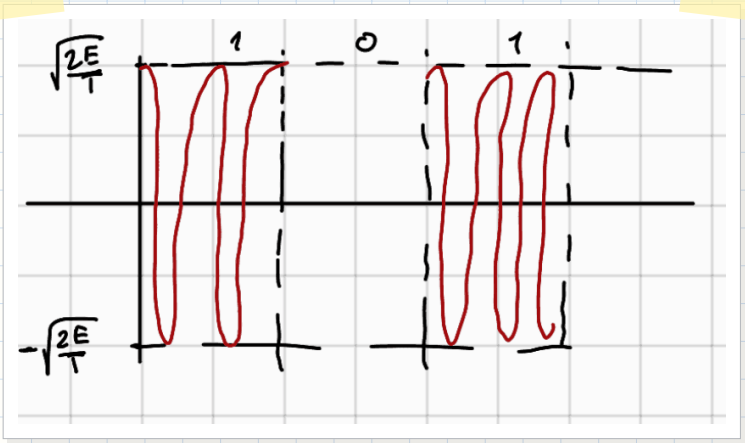
Unipolar signaling × carrier

$$\int_0^T s_1(t)\,s_2(t)\,dt = 0 \Rightarrow \rho = 0 \Rightarrow P_b = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

? BPSK'ın performansı daha iyi

Bant genişlikleri aynı

? BPSK'ta iki kaynak gerekiyor, burada ise tek kaynak yeter.



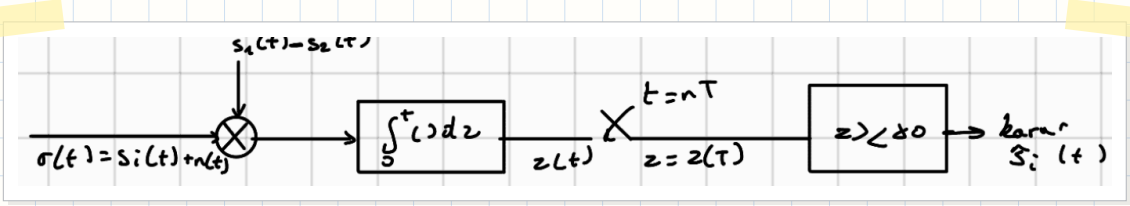
orijinal çizim, s. 3

$$\rightarrow E_b = \frac{A^2}{2}$$

Zarf daha kesik (Sadece FSK'da sabit)

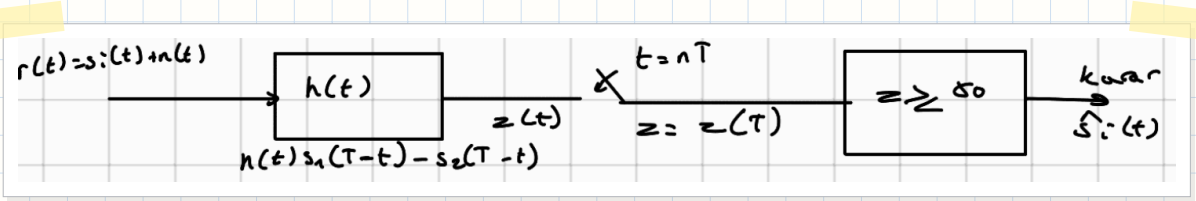
– İFKA, İFRKA, İGKA, VYA

Alıcı yapısı temel band haberleşmesi ile aynı,



orijinal çizim, s. 3

orj. s. 4



orijinal çizim, s. 4

İFKA (BPSK), İGKA (BASK)

$$h(t) = s_1(T-t) - s_2(T-t) = 2s_1(T-t) = s_1(T-t)$$

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

İFRKA (BFSK)

$$h(t) = s_1(T-t) - s_2(T-t)$$

$$P_b = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

VYA (OOK)

$$h(t) = s_1(T-t) - s_2(T-t) = s_1(T-t)$$

$$P_b = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

$$P_b = Q\left(\frac{a_1 - a_2}{2\sigma_0}\right) = Q\left(\sqrt{\frac{E_d}{2N_0}}\right) = Q\left(\sqrt{\frac{E_b}{N_0}(1-\rho)}\right) \checkmark$$

En mükemmel haberleşme beyin sinyalleri.

Genlik-Faz Kaydırmalı Anahtarlama

Amplitude-Phase Shift Keying (ASK-PSK)

= Quadrature Amplitude Modulation (QAM)
Dörtlü Genlik Modülasyon (DGM)

$$s_i(t) = \sqrt{\frac{2}{T}} \underbrace{A_i}_{a} \cos\left(2\pi f_c t + \overbrace{\phi_i}^{b}\right), \quad 0 \le t \le T, \quad i = 1, \ldots, M = 2^k$$

$$\phi_i = \frac{2\pi(i-1)}{M}$$

$$= \sqrt{\frac{2}{T}} A_i \cos\phi_i \cos(2\pi f_c t) - \sqrt{\frac{2}{T}} A_i \sin\phi_i \sin(2\pi f_c t)$$

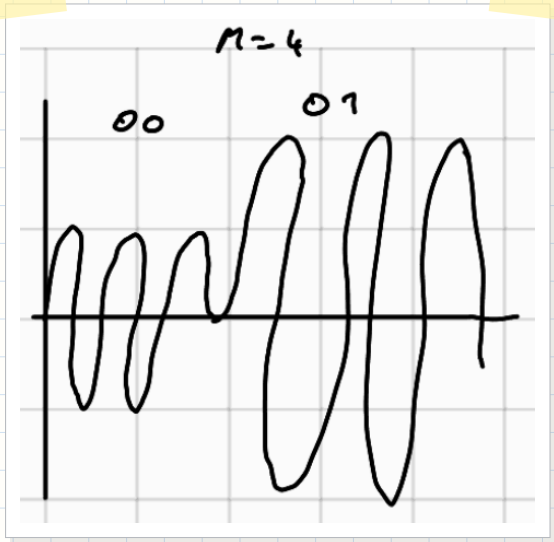
ADSL ?Modemlerde önce kullanılıyordu

$M \uparrow \Rightarrow$ BG aynı (BGV $\uparrow$) $\times$ $P_b \uparrow \Rightarrow$ BG verimli teknik

BGV Genlik K.A, PSK, QAM

M en az ?okunamadı olmalı PSK, ve ASK'dan daha iyi P_b'ye sahiptir

orj. s. 5



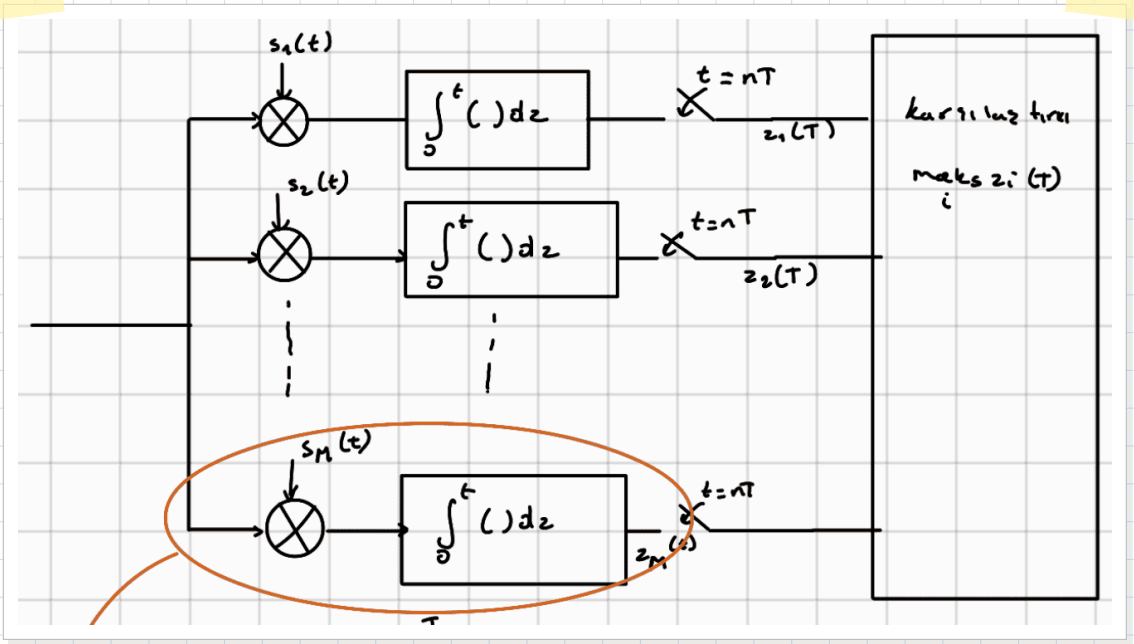
orijinal çizim, s. 5

Sınır 4 kbit bakır kablolu
QAM modem hızı 33,6 kbit/sn
Trellis Coded Modulation (TCM) hem kaynak hem kanal kodlama
Modem hızı 56 kbit/sn

ADSL (OFTM'in muadili: Discrete Multitone Modulation)
kablosuz kablolu

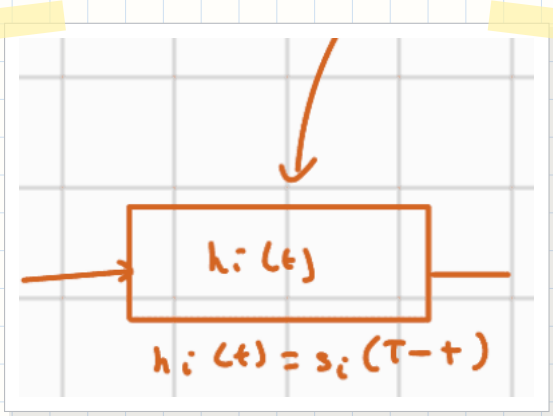
M'li modülasyon teknikleri: Alım

(AWGN kanalda) optimum alıcı



orijinal çizim, s. 5

$$z_i(T) = \int_0^T r(t)\, s_i(t)\, dt \qquad i = 1, \ldots, M$$



orijinal çizim, s. 5

orj. s. 6

M'li modülasyon işaretlerini vektör uzayında temsil etmek

M-PSK

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos(2\pi f_c t + \phi_i)\,, \quad 0 \le t \le T\,, \quad i = 1, \ldots, M = 2^k$$

$$= \sqrt{\frac{2E}{T}} \cos\phi_i \cos(2\pi f_c t) - \sqrt{\frac{2E}{T}} \sin\phi_i \sin(2\pi f_c t)$$

Tanım:

Temel fonksiyonlar (Basis function)

$$\left.\begin{aligned}\psi_1(t) &= \sqrt{\tfrac{2}{T}}\cos(2\pi f_c t)\,,\ 0 \le t \le T\\ \psi_2(t) &= -\sqrt{\tfrac{2}{T}}\sin(2\pi f_c t)\,,\ 0 \le t \le T\end{aligned}\right\} \qquad \int_0^T \psi_1(t)\,\psi_2(t)\,dt = 0$$

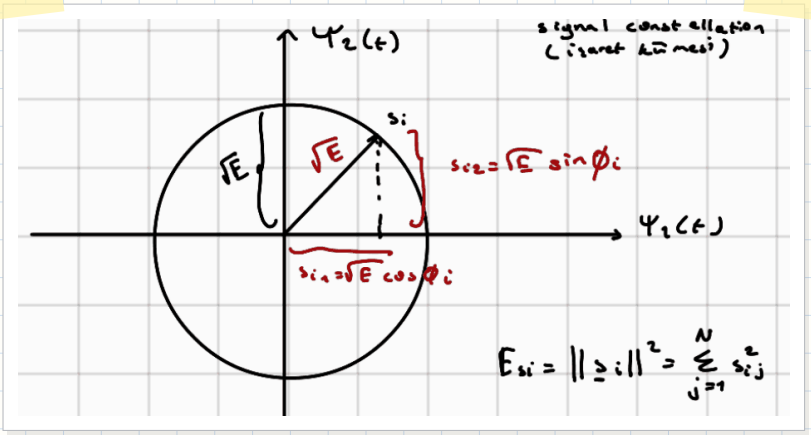
$$E_{\psi_1} = E_{\psi_2} = \frac{2}{T}\cdot\frac{1}{2}T = 1$$

$\psi_1(t)$ ve $\psi_2(t)$ ortonormal işaretler

$$s_i(t) = \underbrace{\sqrt{E}\cos\phi_i}_{s_{i1}}\,\psi_1(t) + \underbrace{\sqrt{E}\sin\phi_i}_{s_{i2}}\,\psi_2(t) \quad \checkmark$$

M tane işaret ψ_1 ve ψ_2 dediğimiz ortonormal işaretlerin lineer kombinasyonu şeklinde yazılabilir. Vektör olarak temsil edilir

$$\underset{\text{vektör}}{\vec{s}_i} = \underbrace{[\,s_{i1}\ \ s_{i2}\,]}_{\substack{\text{uzayın eksenleri}\\ \psi_1 \text{ ve } \psi_2 \text{ iken}}} = \left[\sqrt{E}\cos\phi_i\ \ \sqrt{E}\sin\phi_i\right],\ i = 1,\ldots,M = 2^k$$



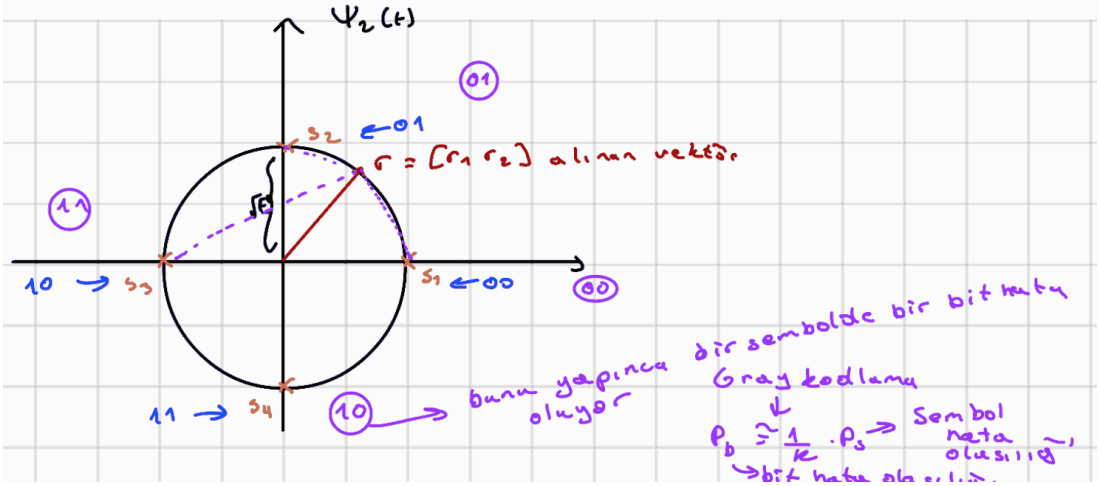
orijinal çizim, s. 6

bu örnek için $E_{s_i} = \|s_i\|^2 = \sum_{j=1}^{N=2} s_{ij}^2 = s_{i1}^2 + s_{i2}^2 = E$ $\|s_i\|$ uzunluk

$$M = 4 \quad (k = 2)$$

i	$\phi_i = \frac{2\pi(i-1)}{M}$	
1	0	$\rightarrow s_1 = [\sqrt{E}\ \ 0]$
2	$\pi/2$	$\rightarrow s_2 = [0\ \ \sqrt{E}]$
3	π	$\rightarrow s_3 = [-\sqrt{E}\ \ 0]$
4	$3\pi/2$	$\rightarrow s_4 = [0\ \ -\sqrt{E}]$

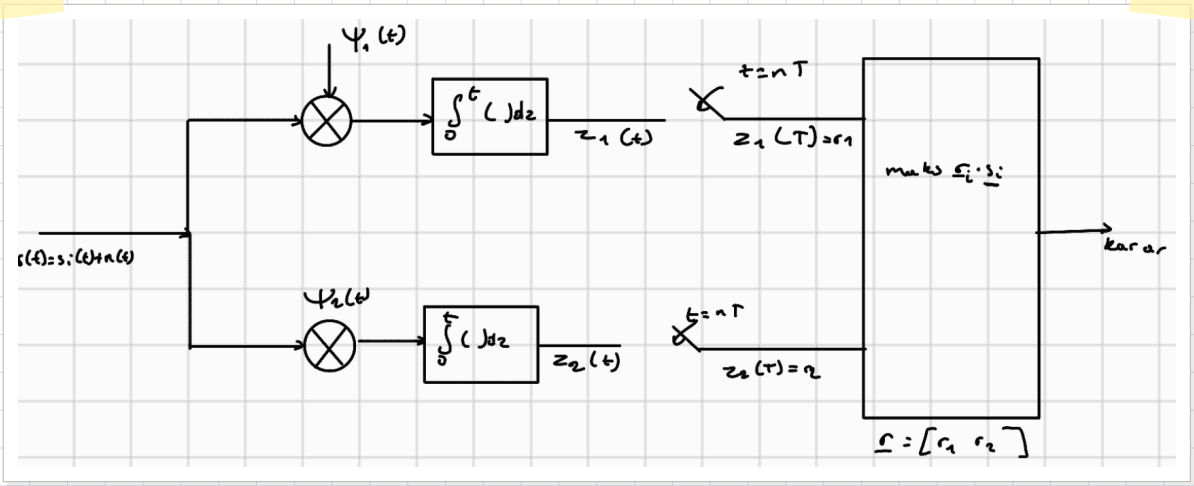
orj. s. 7



orijinal çizim, s. 7

r hangisine daha yakınsa ? o olarak karar veriyor

bir sembollük hata 2 bitlik hataya karşılık geliyor
s_2 gönderince en çok s_3 ve s_1'in lehine hata yapmaktır.



orijinal çizim, s. 7

$$x = [x_1 \; x_2 \; \cdots \; x_N] \qquad x \cdot y = \sum_{i=1}^{N} x_i y_i$$

$$y = [y_1 \; y_2 \; \cdots \; y_N] \qquad \uparrow \text{ iç çarpım (inner product)}$$

FSK'ya uygulanamaz