Sayısal Haberleşme

Düz tepeli örnekleme, PAM/PWM/PPM, TDM ve PCM

Orijinal not: Ders-5.pdf

UYARI Bu belge, el yazısı ders notlarından Claude (Anthropic) yardımıyla dijitale aktarılmıştır. Yardımcı kaynak olarak hazırlanmıştır; hata veya eksik içerebilir. Lütfen orijinal notları ve ders kaynaklarını esas alın.

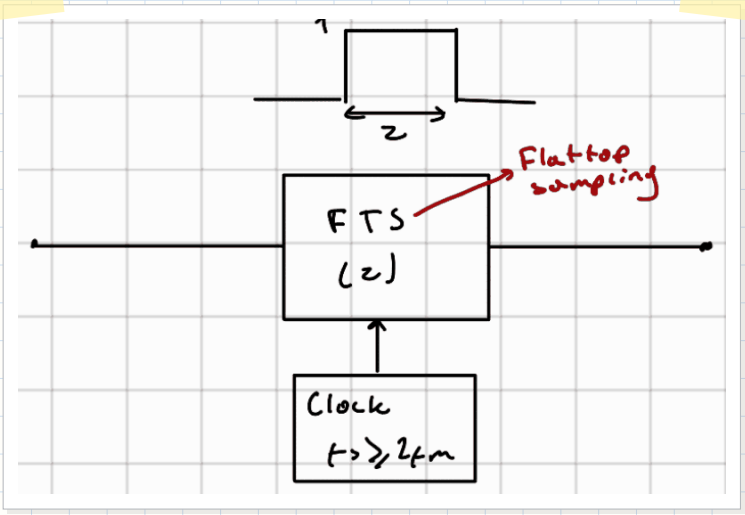
NOTICE *This document was transcribed from handwritten lecture notes with the help of Claude (Anthropic). It is meant as a supplementary resource and may contain errors or omissions. Always check against the original notes and official course material.*

Not: Orijinal dosyadaki basılı sayfalar (s. 9) dijitalleştirilmedi; yalnızca el yazısı sayfalar aktarıldı.

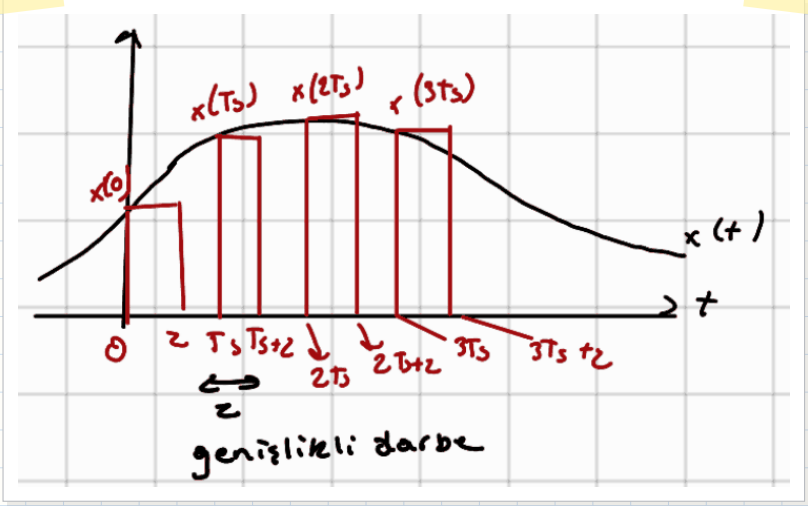
orj. s. 1

Sample and Hold operation (örnekle ve tut işlemi)

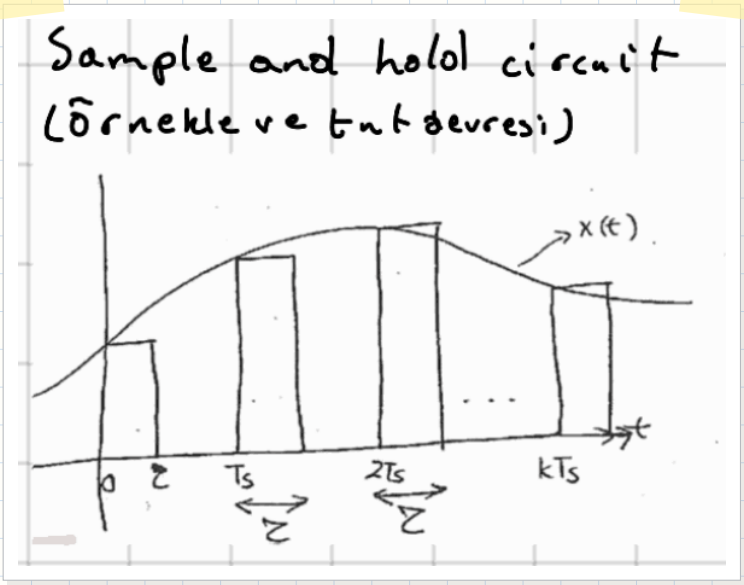
Düz Tepeli örnekleme (Flat top sampling) (FTS)



orijinal çizim, s. 1



orijinal çizim, s. 1



orijinal çizim, s. 1

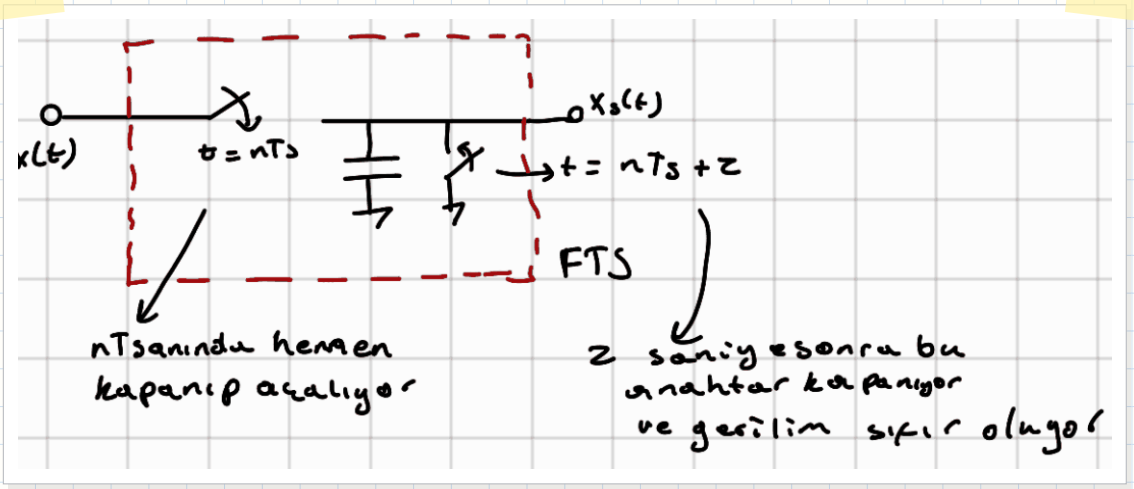
$x_s(t)$: Darbe Genlik mod (DGM)
Pulse amplitude mod

Burda darbeler düz tepeli

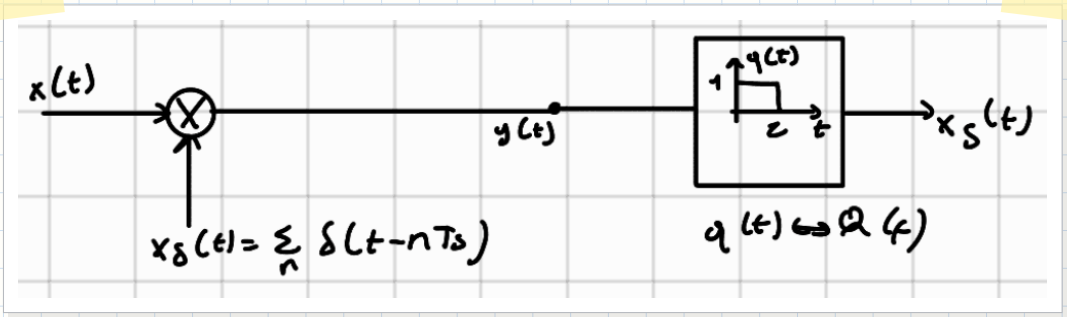
bu discrete time analog modulation zaman ayrık ama genlik değil

Discrete time analog modulation

Sample/ Hold (Dump) circuit



orijinal çizim, s. 1



orijinal çizim, s. 1

$$g(t) = x(t)\,x_\delta(t) = \sum_n x(nT_s)\,\underline{\delta(t-nT_s)}$$

$$x_s(t) = q(t) \circledast g(t) = \sum_n x(nT_s)\,\underline{q(t-nT_s)}$$

$$x_s(t) = q(t) \circledast \left[x(t)\,x_\delta(t)\right] = q(t) \circledast \left[\sum_n x(nT_s)\,\delta(t-nT_s)\right]$$

$$= \sum_n x(nT_s)\,q(t-nT_s)$$

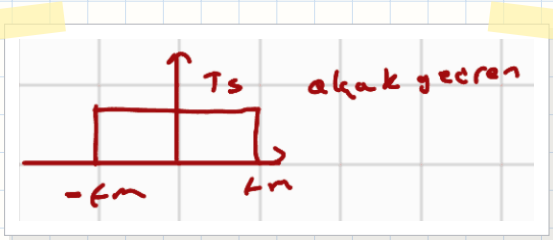
orj. s. 2

$$X_s(f) = Q(f)\left[X(f) \circledast X_\delta(f)\right] = Q(f)\left[X(f) \circledast f_s \sum_n \delta(f-nf_s)\right]$$

$$X_s(f) = Q(f)\left[f_s \sum_n X(f-nf_s)\right]$$

$$\hookrightarrow\ \tau\,\mathrm{sinc}(f\tau)\,e^{-j2\pi f\tau/2}$$

$$X_s(f) = \underbrace{f_s\,Q(f)\,X(f)}_{!!} + Q(f)\cdot f_s \sum_{n\neq 0} X(f-nf_s)$$

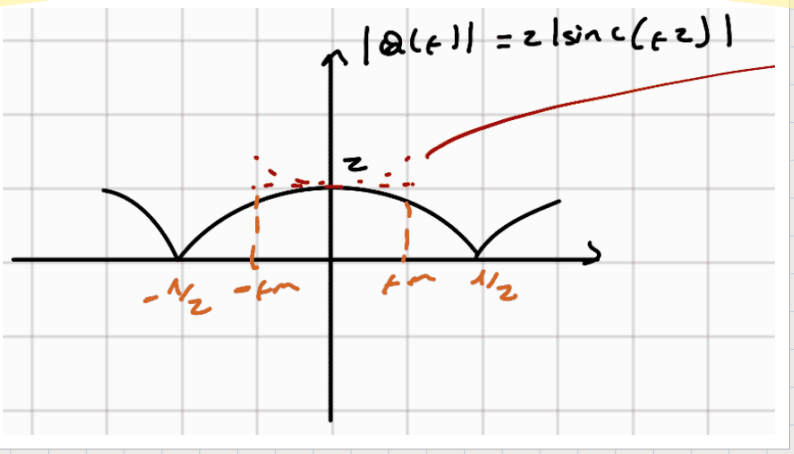


orijinal çizim, s. 2

Equalizing LPF (Dengeleyici AGF)

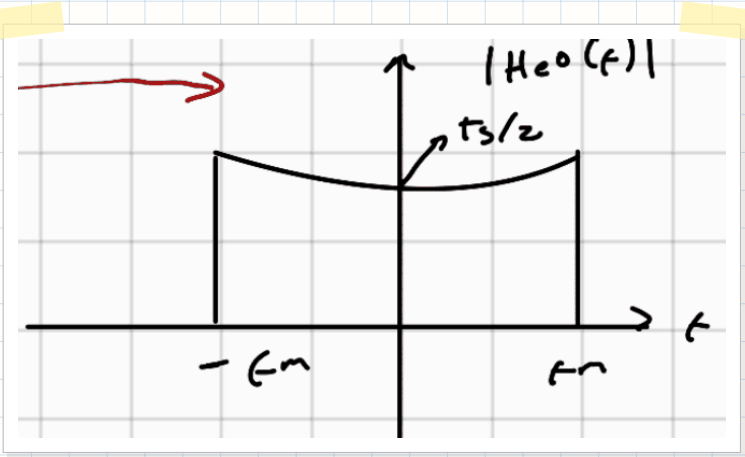
$$x_s(t) \longrightarrow \boxed{H_{eq}(f) = \frac{1}{Q(f)}} \longrightarrow \boxed{\begin{array}{l}\text{Ideal LPF}\\ \text{Gain} = T_s\\ \text{BW} = f_m\end{array}} \longrightarrow x(t)$$

$$H_{eq}(f) = \begin{cases} \dfrac{T_s}{Q(f)} & |f| \le f_m \\ 0 & \text{else} \end{cases}$$



orijinal çizim, s. 2

$$H_{eq}(f) = \begin{cases} \dfrac{T_s}{Q(f)} & |f| \le f_m \\ 0 & \text{else} \end{cases}$$



orijinal çizim, s. 2

$$H_{eq}(f) = \frac{T_s}{Q(f)} = \frac{T_s}{\tau\ \mathrm{sinc}(f\tau)}\ \underline{e^{j2\pi f\tau/2}}, \quad |f| \le f_m$$

↪ advance in time → $h_{eq}(t)$ is not causal

For this reason, $H'_{eq}(f) = H_{eq}(f)\, e^{-j2\pi f T_x}$ is chosen

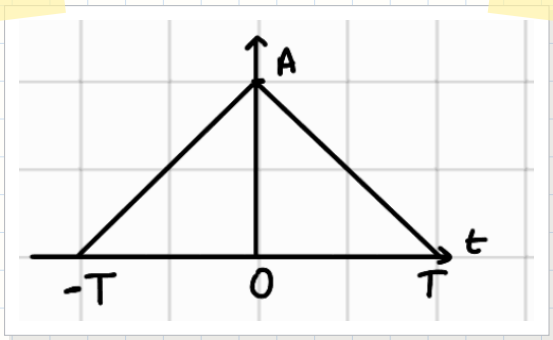
↓

$$h'_{eq}(t) = h_{eq}(t - T_x) \qquad T_x = \frac{\tau}{2} + T_0$$

$$x_s(t) \longrightarrow \boxed{H'_{eq}(f)} \longrightarrow x(t - T_0)$$

orj. s. 3

Reminder

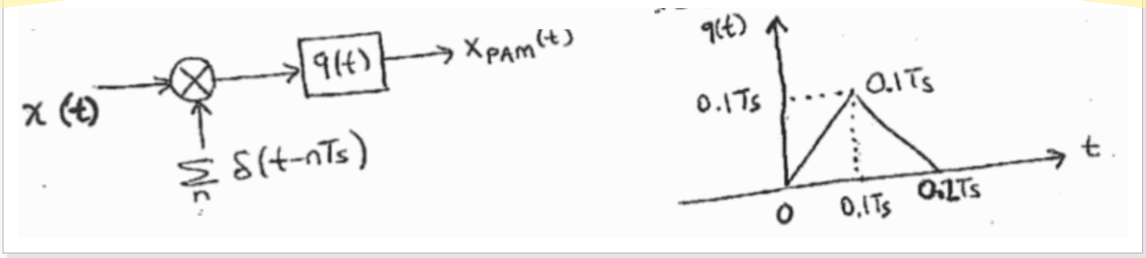


orijinal çizim, s. 3

$$A\,\Lambda\left(\frac{t}{T}\right) \overset{\mathcal{F}}{\longleftrightarrow} A\,T\,\mathrm{sinc}^2(fT)$$

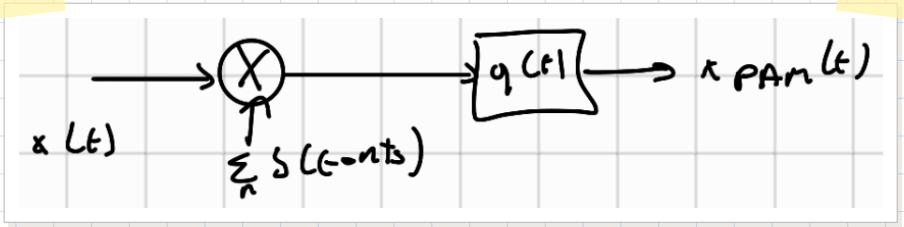
Örnek (2004 Midterm Ex.Q.)

A PAM signal is produced as shown in the figure below. a) If the bandwidth of $x(t)$ is 1 kHz, what is the maximum value of T_s to obtain $x(t)$ without distortion at the receiver? b) For $T_s = T_s(\max)$, sketch PAM demodulator. Find the transfer function of demodulator.



orijinal çizim, s. 3

Çözüm:



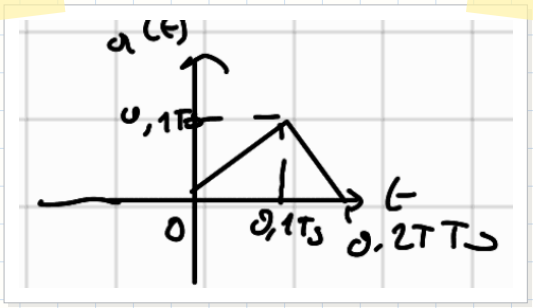
orijinal çizim, s. 3

a) $BW\{x(t)\} = 1\ kHz = f_m$

$f_s \geq 2f_m = 2\ kHz$

$T_s \leq \frac{1}{2f_m} = 0{,}5\ msec$

$T_s(max) = 0{,}5\ msec$



orijinal çizim, s. 3

b)

$$x_{pam}(t) \longrightarrow \boxed{H_{eq}(f) = \frac{1}{Q(f)}} \longrightarrow \boxed{\begin{array}{c}\text{Ideal}\\ \text{LPF}\\ BW = f_m\end{array}} \longrightarrow x(t)$$

$$H_{eq}(f) = \begin{cases} \dfrac{T_s}{Q(f)}, & |f| \leq f_m \\ 0, & |f| > f_m \end{cases}$$

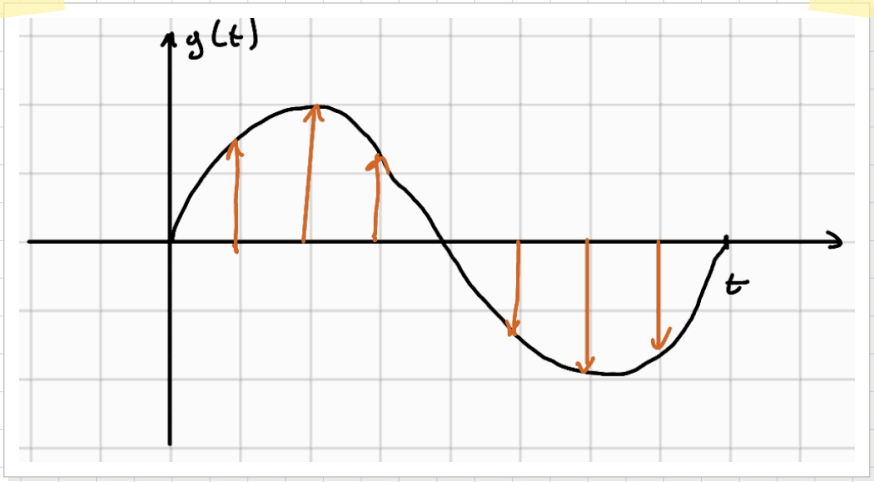
$$Q(f) = 0{,}1T_s\left(0{,}1T_s\,\text{sinc}^2(0{,}1T_s f)\,e^{-j2\pi f\,0{,}1T_s}\right)$$

$$= 0{,}01\,T_s^2\,\text{sinc}^2(0{,}1T_s f)\,e^{-j2\pi f\,0{,}1T_s}$$

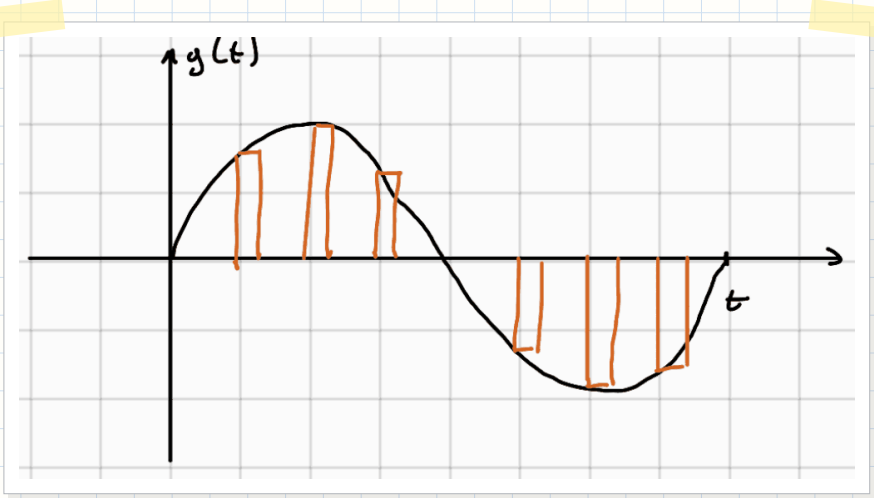
$$H_{eq}(f) = \frac{T_s\, e^{j2\pi f 0{,}1T_s}}{0{,}01\, T_s^2\, \mathrm{sinc}^2(0{,}1T_s f)}$$

$$= \begin{cases} \dfrac{2\cdot 10^5\, e^{j2\pi f 5\cdot 10^{-5}}}{\mathrm{sinc}^2(5\cdot 10^{-5} f)}, & |f| \le 1\ \text{kHz} \quad \text{iken} \\ 0, & \text{else} \end{cases}$$

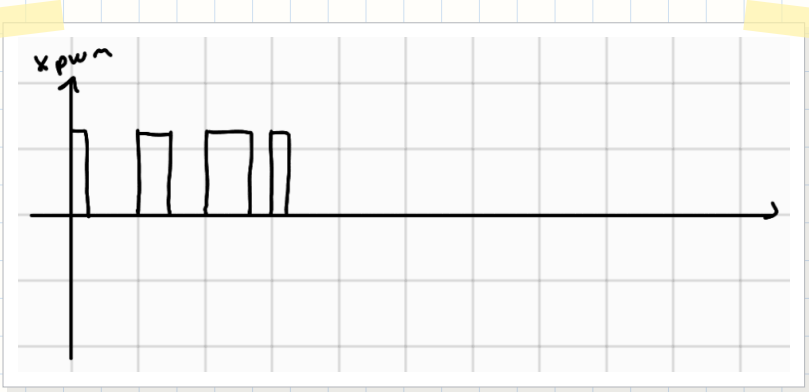
orj. s. 4



orijinal çizim, s. 4



orijinal çizim, s. 4



orijinal çizim, s. 4



orijinal çizim, s. 4

$x_{PPM}(t)$: Pulse Width modulation[1]

orj. s. 5

> Şekil: Bu sayfa basılı materyal içeriyor (ders kitabından şekil); dijitalleştirilmedi. Dört grafik üst üste: $g(t)$ (sinüs benzeri eğri, örnekleme okları), $x_{PAM}(t)$, $x_{PWM}(t)$ (basılı not: Pulse locations are the same but their widths change.) ve $x_{PPM}(t)$ (basılı not: Pulse widths are the same but their locations change.). El yazısı eklemeler: grafiklerin sağında PAM, PWM, PPM etiketleri ve altta aşağıdaki iki açıklama.
> *(orijinal notta, s. 5)*

PWM: Pulse Width Modulation (Darbe Genişlik Modülasyonu)

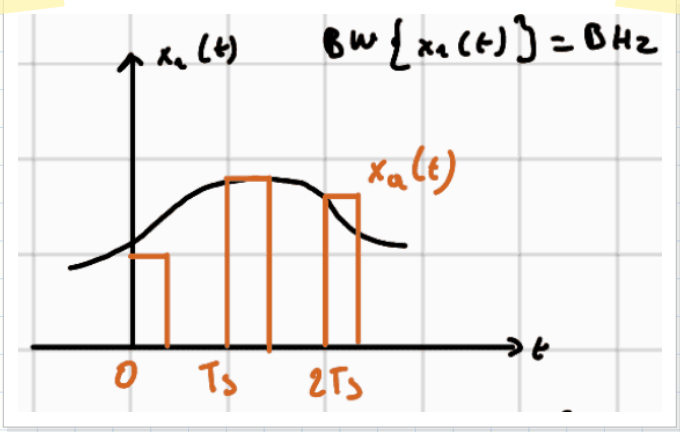
PPM: Pulse Position Modulation (Darbe Pozisyon Modülasyonu)

orj. s. 6

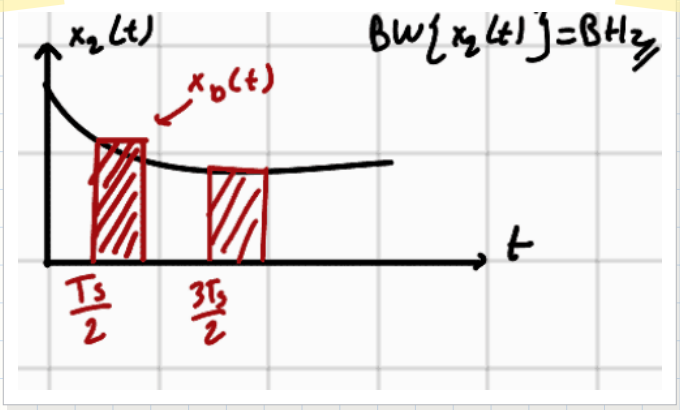
Time division multiplexing (TDM) Zaman bölmeli çoklama (ZBÇ)

- Multiple PAM signals are transmitted without overlapping.
- At the receiver, PAM signals are separated by synchronization

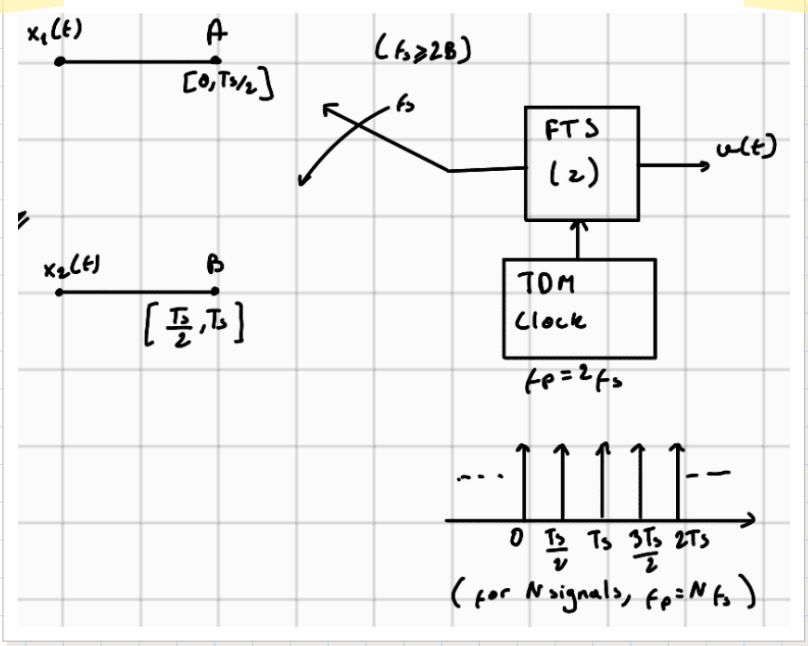
[1] Claude notu: muhtemelen Pulse Position modulation olmalı.



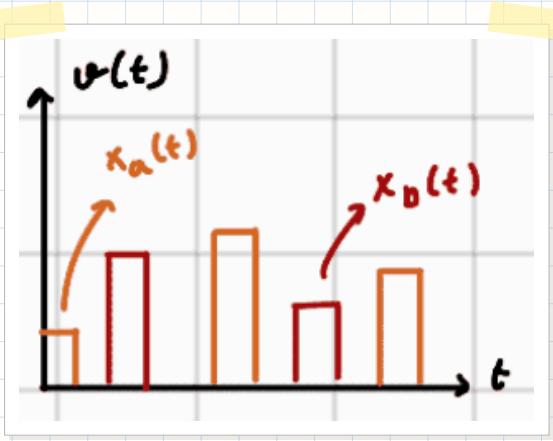
orijinal çizim, s. 6



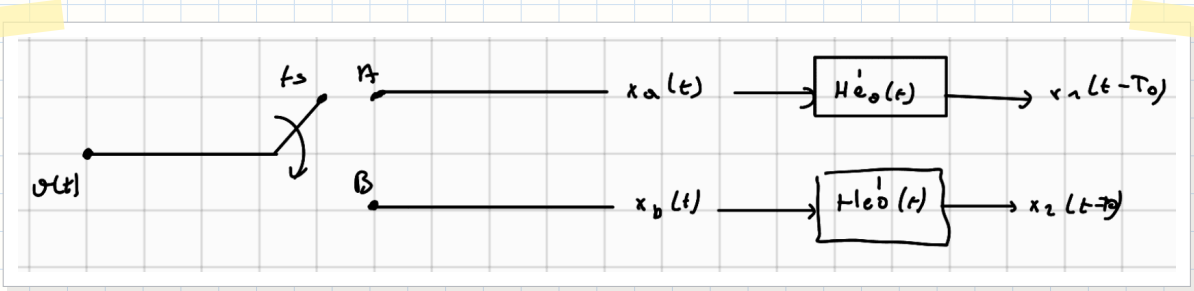
orijinal çizim, s. 6



orijinal çizim, s. 6



orijinal çizim, s. 6



orijinal çizim, s. 6

FDM x TDM

Advantages: 1) Easier to implement 2) Less crosstalk than FDM

Disadvantage: 1) Strict synchronization

Çok kaynaklı (Multi-source) → multiplexing (çoklama)

Çok kullanıcılı (Multi user) → Multiple Access (çoklu erişim)

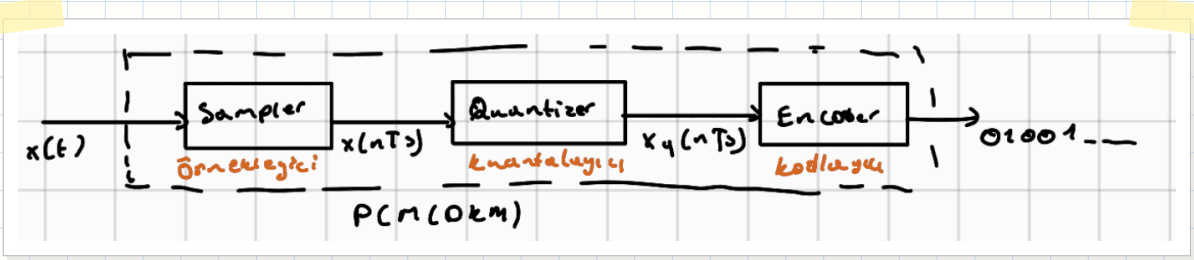
↪ TDMA (2.nesil, avrupada GSM), FDMA (1.nesil)

3.nesil: Code Division multiplexing

4.nesil: OFDM

orj. s. 7

Pulse code Modulation (Darbe kod modülasyonu - DKM)



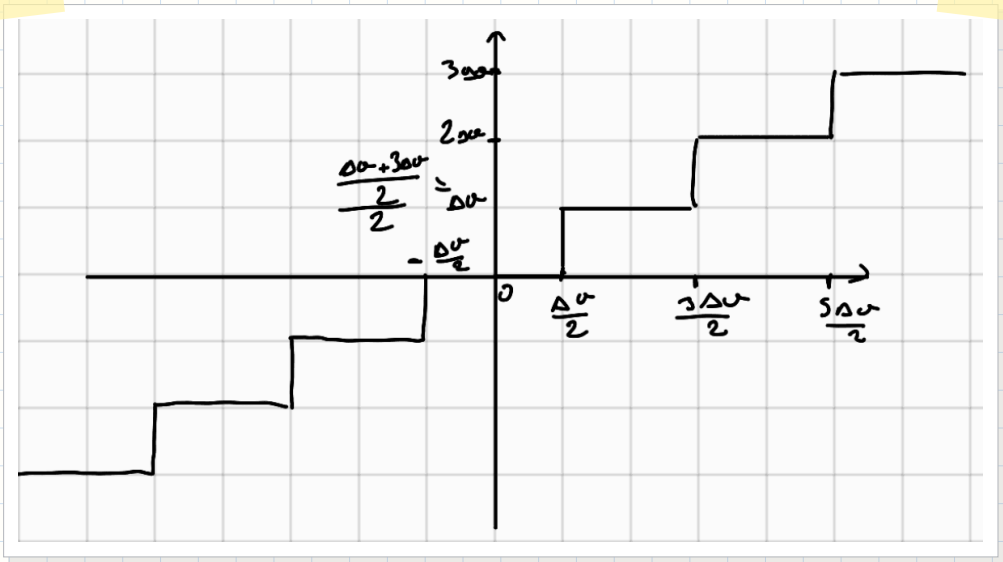
orijinal çizim, s. 7

x(t) analog signal

$-x_p$ den $+x_p$ ye kadar

x_p = Maks $|x(t)|$ peak value (Tepe değer)

kuantalayıcı adımı (Quantizer step) = $\Delta v = \frac{2x_p}{L}$ ⟶ kuantalayıcıdaki seviye sayısı (# of levels in quantizer)

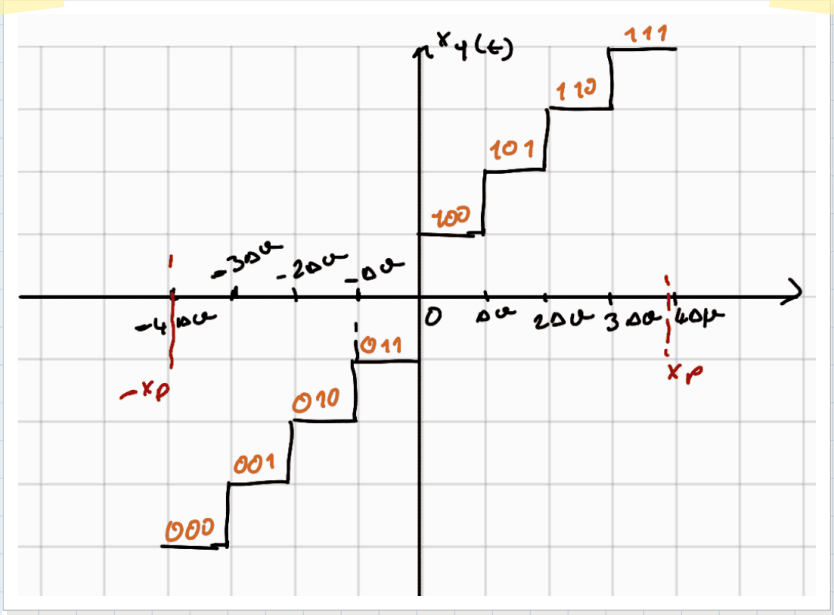


orijinal çizim, s. 7

Assume $\Delta v = 1$ V

$x(t)$		$x_q(t)$
1,07	⟶	1 V
1,40 V	⟶	1 V
0,55 V	⟶	1 V
2,25 V	⟶	2 V
−3,1 V	⟶	−3 V

orj. s. 8



orijinal çizim, s. 8

8 seviyeyi kodluyoruz

$L = 8$ *(Mid rise quantizer)*

Assume $\Delta v = 1$ V

$x(t)$	$x_q(t)$	
1,03 V	1,5 V	$+\frac{\Delta v}{2}$ hata
0,55 V	0,5 V	
−3,1 V	−3,5 V	$-\frac{\Delta v}{2}$ hata
1,01 V $\longrightarrow$	1,5 V	

* Each sample is approximated with a quantized value, the approximation will result in an error less than $\Delta v/2$ in the positive direction or $-\Delta v/2$ in the negative ? *direction*

Quantization error : $(-\Delta v/2,\ \Delta v/2)$

$$\underbrace{x_q(t)}_{\text{kuantalanmış işaret}} = \underbrace{x(t)}_{\text{Analog işaret}} + \overbrace{q(t)}^{\substack{\text{kuantalama gürültüsü} \\ \text{(Quantization noise)}}}$$

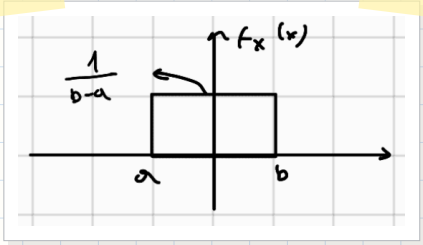
q is uniform in $[-\Delta v/2,\ \Delta v/2]$

Average quantization noise power = $N_q = E[q^2]$

(ort. kuantalama gürültüsü gücü)

Reminder

$x \sim U[a, b]$ düzgün dağ.



orijinal çizim, s. 8

$$\mu_x = \frac{a+b}{2} \qquad \sigma_x^2 = \frac{(b-a)^2}{12}$$

$$\sigma_q^2 = E\left[(q-\mu_q)^2\right] = E[q^2] = N_q$$

$$N_q = \frac{\left[\frac{\Delta v}{2} - \left(-\frac{\Delta v}{2}\right)\right]^2}{12} = \frac{\Delta v^2}{12}$$

$$\Delta v = \frac{2x_p}{L} \Longrightarrow N_q = \frac{\Delta v^2}{12} = \frac{x_p^2}{3L^2}$$

İşaretin gücü = $S_0 = \langle x^2(t) \rangle = P_x$

Gürültünün gücü = $N_0 = N_q$

İşaretin gürültüye oranı (İGO)

Signal to noise Ratio (SNR)

signal to Quantization noise Ratio (SQNR) ↓

$$SNR = \frac{S_0}{N_0} = \frac{P_x}{x_p^2} \cdot 3L^2$$

← önemli