# Sayısal Haberleşme

## Temel band haberleşmesi, Matched Filter, Correlator

Orijinal not: Ders-8.pdf

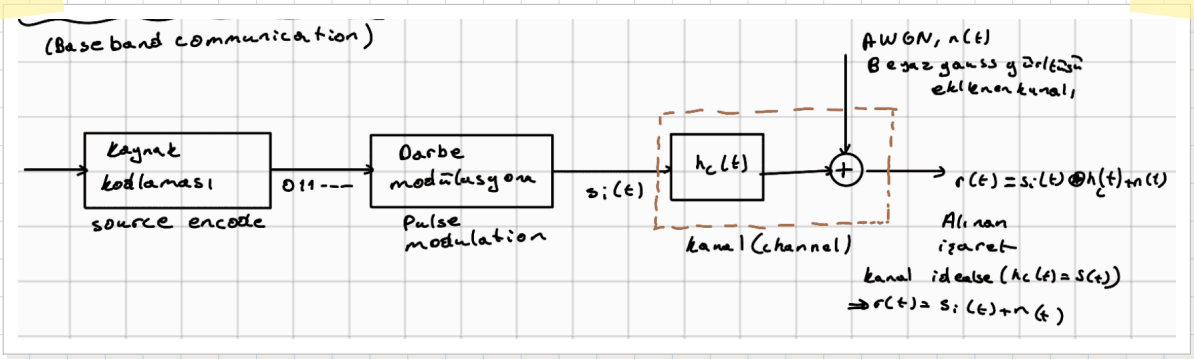
**UYARI** Bu belge, el yazısı ders notlarından Claude (Anthropic) yardımıyla dijitale aktarılmıştır. Yardımcı kaynak olarak hazırlanmıştır; hata veya eksik içerebilir. Lütfen orijinal notları ve ders kaynaklarını esas alın.

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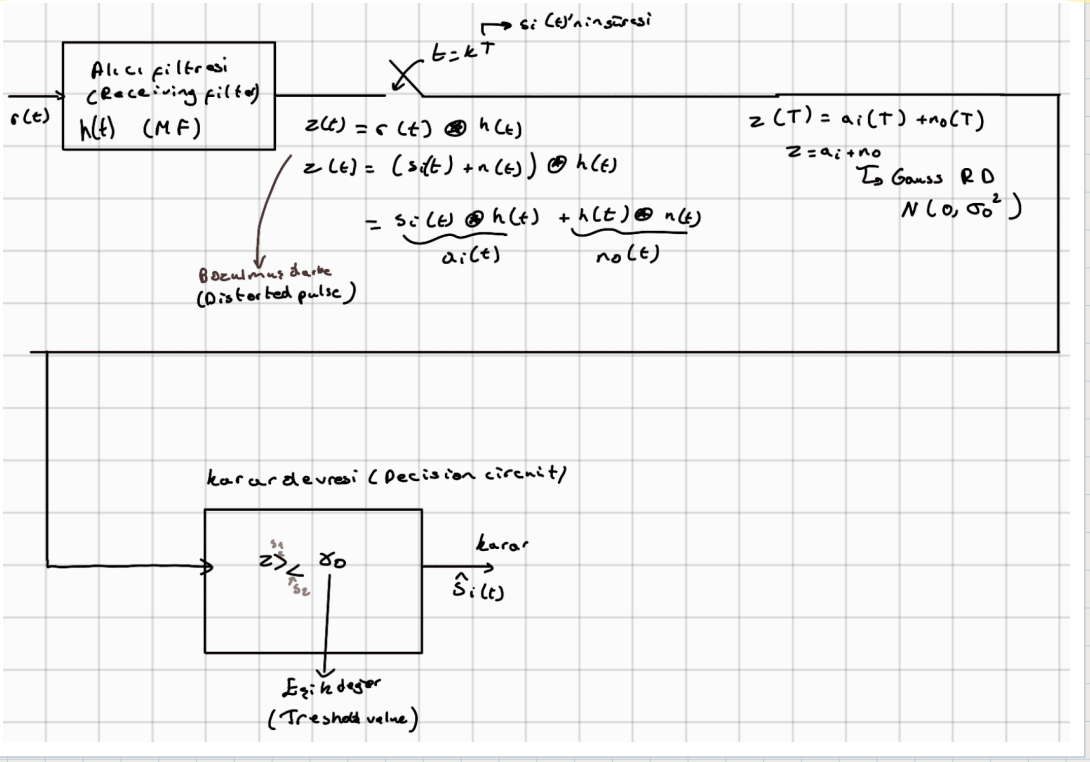
orj. s. 1

**Ders8**

## Temel Band haberleşmesi




orijinal çizim, s. 1




orijinal çizim, s. 1

$$z = a_i + n_0$$
$$\downarrow \qquad \hookrightarrow \sim N(0, \sigma_0^2)$$
$$\sim N(a_i, \sigma_0^2)$$

$$f(n_0) = \frac{1}{\sqrt{2\pi}\,\sigma_0} \exp\left\{-\frac{1}{2}\left(\frac{n_0}{\sigma_0}\right)^2\right\}$$

Eğer $s_1$ iletilirse

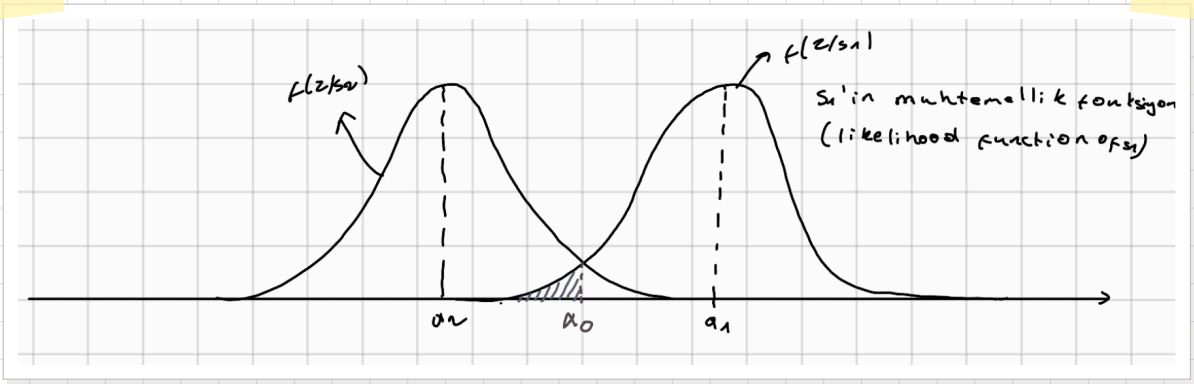
$$z = a_1 + n_0 \qquad \hookrightarrow \sim N(a_1, \sigma_0^2)$$
$$\Rightarrow f(z|s_1) = \frac{1}{\sqrt{2\pi}\,\sigma_0} \exp\left\{-\frac{(z-a_1)^2}{2\sigma_0^2}\right\}$$

orj. s. 2

Eğer $s_2$ iletilirse

$$z = a_2 + n_0 \qquad \hookrightarrow \sim N(a_2, \sigma_0^2)$$
$$\Rightarrow f(z|s_2) = \frac{1}{\sqrt{2\pi}}^{1} \exp\left\{-\frac{(z-a_2)^2}{2\sigma_0^2}\right\}$$

[1] Claude notu: payda muhtemelen $\sqrt{2\pi}\,\sigma_0$ olmalı.




orijinal çizim, s. 2

$s_1(t)$ iletilmiş olsun : Doğru karar $(z > \gamma_0)$

$$\text{Hata} : P(h|s_1) = P(z < \gamma_0) = \int_{-\infty}^{\gamma_0} f(z|s_1)\,dz$$

$s_2(t)$ iletilmiş olsun doğru karar: $(z < \gamma_0)$

$$\text{Hata} : P(h|s_2) = P(z > \gamma_0) = \int_{\gamma_0}^{\infty} f(z|s_2)\,dz$$

$$\underset{\text{Bit hata olasılığı}}{P_b} = \sum_{i=1}^{2} P(h|s_i)P(s_i) = P(h|s_1)P(s_1) + P(h|s_2)P(s_2)$$

Eşit olasılıklı işaretler: $P(s_1) = P(s_2) = \frac{1}{2}$
(Equally likely)

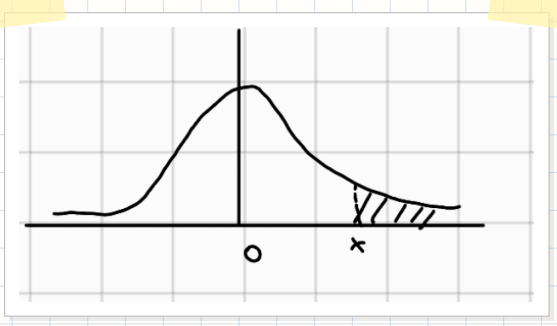
$$\Rightarrow \gamma_0 = \frac{a_1 + a_2}{2} \qquad \Rightarrow \text{Simetriden dolayı,}$$

$$P(h|s_1) = P(h|s_2)$$

$$\Rightarrow P_b = P(h|s_1) = P(h|s_2)$$

$$P_b = P(h|s_2) = \int_{\gamma_0}^{\infty} f(z|s_2)\,dz = \frac{1}{\sqrt{2\pi}\,\sigma_0} \int_{\frac{a_1+a_2}{2}}^{\infty} e^{-\frac{(z-a_2)^2}{2\sigma_0^2}}\,dz$$

$$Q(x) = P(X > x)$$
$$X \sim N(0,1)$$




orijinal çizim, s. 2

orj. s. 3

$$Q(x) = P(X > x) = \int_x^\infty f_X(x)\,dx$$
$$= \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2}\,dt, \quad x \geq 0$$

Table B.1, Page 1045

$$Q(0) = 0{,}5$$
$$Q(-x) = 1 - Q(x)$$

$$P_b = P(h|s_2) = \int_{\gamma_0}^\infty f(z|s_2)\,dz = \frac{1}{\sqrt{2\pi}\,\sigma_0} \int_{\frac{a_1+a_2}{2}}^\infty e^{-\frac{(z-a_2)^2}{2\sigma_0^2}}\,dz$$
$$= \frac{1}{\sqrt{2\pi}\,\sigma_0} \int_{\frac{a_1-a_2}{2\sigma_0}}^\infty e^{-\frac{t^2}{2}}\,\sigma_0\,dt = Q\left(\frac{a_1-a_2}{2\sigma_0}\right)$$

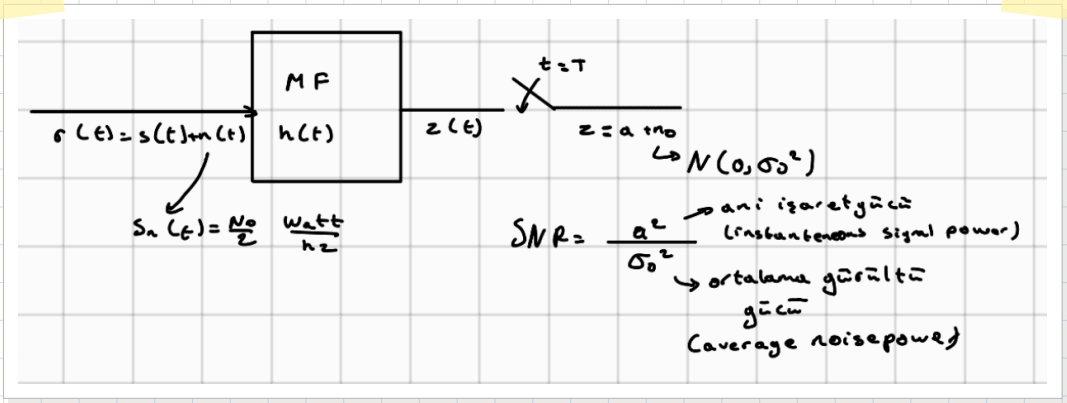
$$\frac{z-a_2}{\sigma_0} = t$$
$$z = \sigma_0 t + a_2$$
$$dz = \sigma_0\,dt$$
$$z = \frac{a_1+a_2}{2} \rightarrow t = \frac{\frac{a_1+a_2}{2} - a_2}{\sigma_0} = \frac{a_1-a_2}{2\sigma_0}$$
$$z = \infty \rightarrow t = \infty$$

$$\boxed{P_b = Q\left(\frac{a_1-a_2}{2\sigma_0}\right)}$$

bu ? oran ne kadar artarsa o kadar $P_b$ azalır

## Karşılayıcı Filtre (Matched Filter - MF)

- MF provides max SNR at its output for a given transmitted waveform




orijinal çizim, s. 3

$$SNR = \left(\frac{S}{N}\right)_{t=T} = \frac{a^2}{\underbrace{\sigma_0^2}_{N_0/2}} = \frac{2E_s}{N_0} \qquad \text{eğer } h(t) = s(T-t),\ 0 \le t \le T$$

$$E_s = \int_0^T |s(t)|^2\, dt$$

orj. s. 4

$$z(t) = r(t) \circledast h(t) = \underbrace{s(t) \circledast h(t)}_{a(t)} + \underbrace{n(t) \circledast h(t)}_{n_0(t)}$$

$$z(T) = a(T) + n_0(T)$$

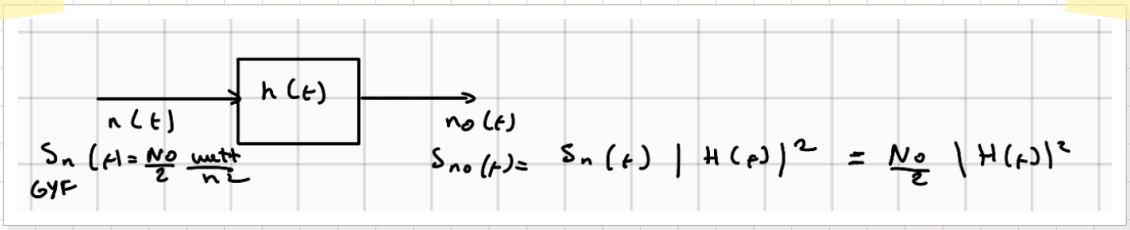
$$z = a + n_0$$

$$\hookrightarrow N(0, \sigma_0^2)$$

$$SNR = \left(\frac{S}{N}\right)_T = \frac{a^2}{\sigma_0^2}$$

$$a(t) = \mathcal{F}^{-1}\{A(f)\} = \int_{-\infty}^{\infty} S(f)H(f)e^{j2\pi fT}\, df$$

$$\rightarrow a = a(T)$$




orijinal çizim, s. 4

$$P_{n_0} = \sigma_0^2 = \int_{-\infty}^{\infty} S_{n_0}(f)\, df = \frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2\, df$$

$$SNR = \frac{a^2}{\sigma_0^2} = \frac{\left|\int_{-\infty}^{+\infty} S(f)H(f)e^{j2\pi fT}\, df\right|^2}{\frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2\, df}$$

Cauchy-Schwarz eşitsizliği

$$\left|\int_{-\infty}^{\infty} f_1(x)f_2(x)\, dx\right|^2 \le \int_{-\infty}^{\infty} |f_1(x)|^2 dx \int_{-\infty}^{\infty} |f_2(x)|^2 dx$$

$$\hookrightarrow f_1(x) = k \cdot f_2^*(x) \qquad \text{? şartı}$$

$$f_1(x) = H(f)$$

$$f_2(x) = S(f)e^{j2\pi fT}$$

$$\Rightarrow SNR \le \frac{\int_{-\infty}^{\infty} |H(f)|^2 df \int_{-\infty}^{\infty} |S(f)|^2 df}{\frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2 df}$$

$$SNR \le \frac{2}{N_0}\underbrace{\int_{-\infty}^{\infty}|S(f)|^2\,df}_{E_s} \Rightarrow SNR \le \frac{2E_s}{N_0}$$

SNR'ın maksimum değeri ? budur

orj. s. 5

Biz max bulmak istiyoruz → $SNR = \frac{2E_s}{N_0}$ olması için

$$f_1(x) = k f_2^*(x)$$
$$H(f) = k S^*(f) e^{-j2\pi fT}$$

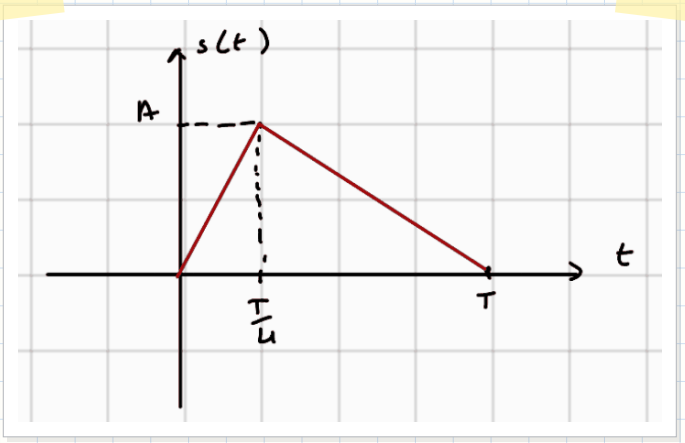
↓

$$h(t) = k\int_{-\infty}^{\infty}\underbrace{S^*(f)e^{-j2\pi fT}}e^{j2\pi ft}\,df$$

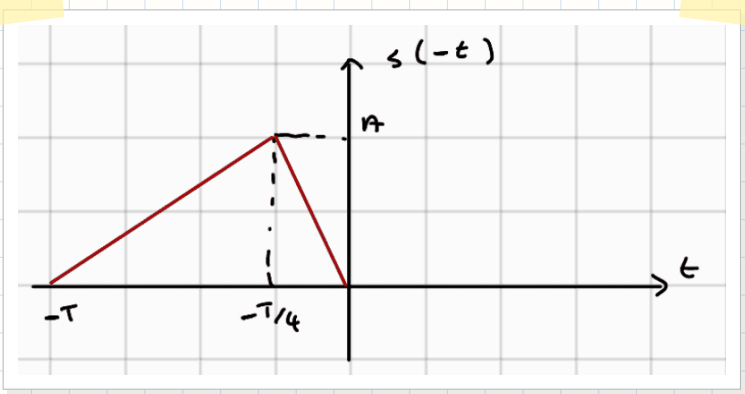
$$= k\left[\underbrace{\int_{-\infty}^{\infty}S(f)\,e^{j2\pi f\overbrace{(T-t)}^{t'}}\,df}_{s(t')}\right]^*$$

$$h(t) = k\cdot s^*(t') = k\cdot s^*(T-t) \rightarrow \boxed{h(t) = s^*(T-t)}$$

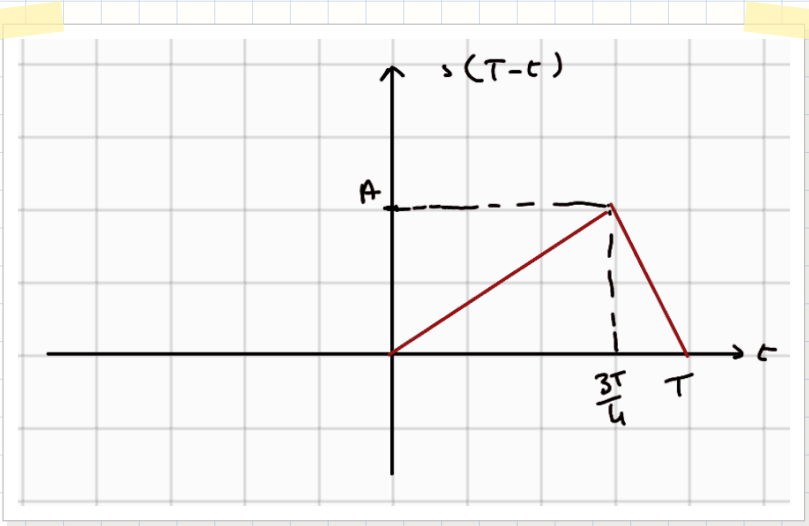
$$h(t) = s(T-t) \rightarrow SNR = \frac{2E_s}{N_0}$$




orijinal çizim, s. 5



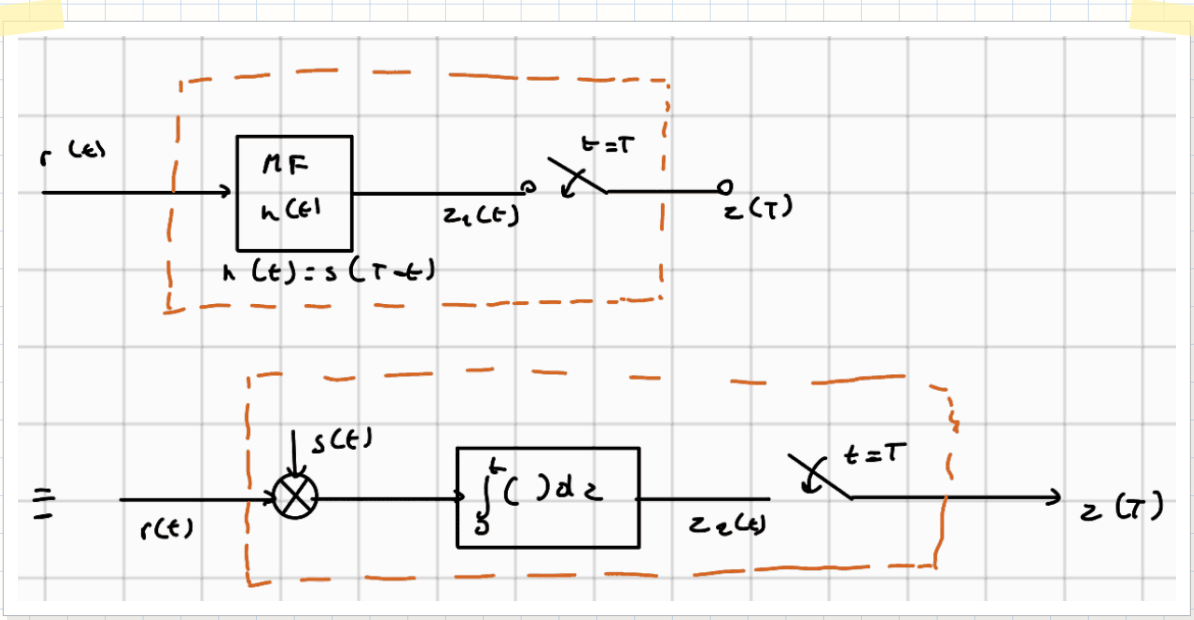

orijinal çizim, s. 5




orijinal çizim, s. 5

orj. s. 6

## İlişkilendirici (Correlator)




orijinal çizim, s. 6

$$z_1(t) = r(t) \circledast h(t) = \int_0^t r(z)\, h(t-z)\, dz$$

$$z_1(t) = \int_0^t r(z)\, s(T-(t-z))\, dz$$

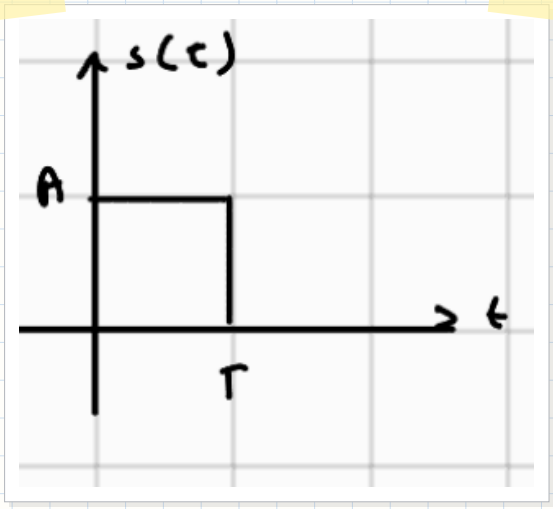
$$z_1(T) = \int_0^T r(z)\, s(z)\, dz = z(T)$$

$$z_2(t) = \int_0^t r(z)\, s(z)\, dz$$

$$z_2(T) = \int_0^T r(z)\,s(z)\,dz = z(T)$$

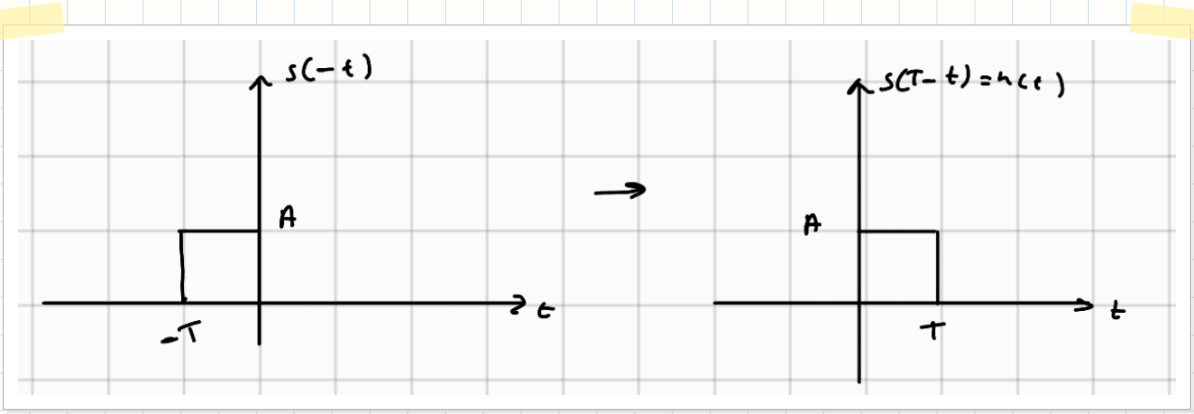
[Kırmızı yaylı okla $z_1(T)$ ve $z_2(T)$ satırları birbirine bağlanmış.]

**Örnek:**




orijinal çizim, s. 6

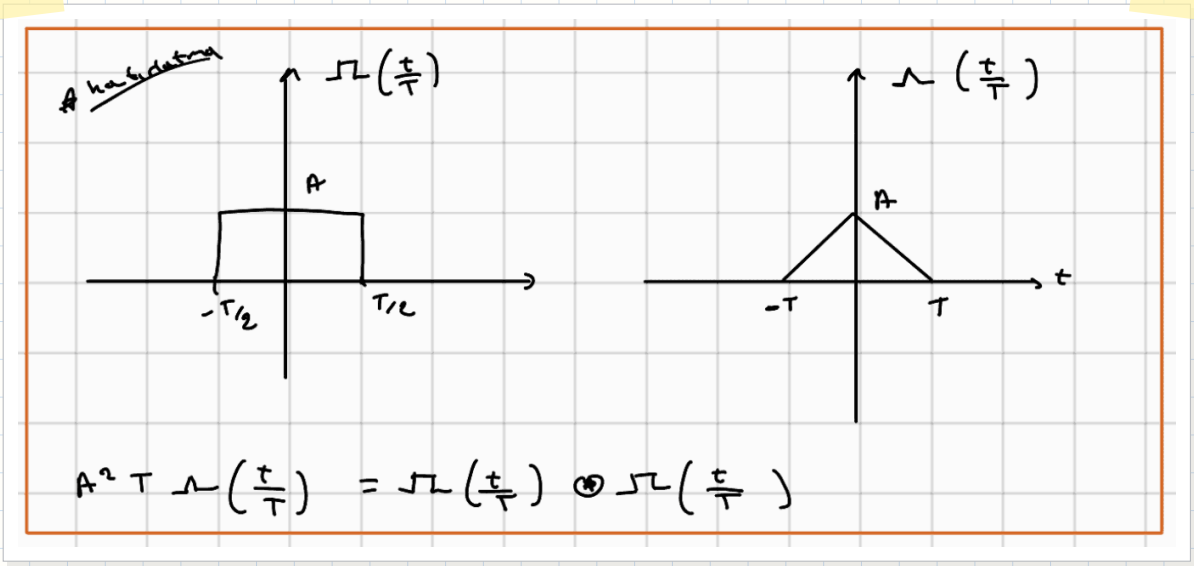
$r(t) = s(t)$ (no noise)




orijinal çizim, s. 6

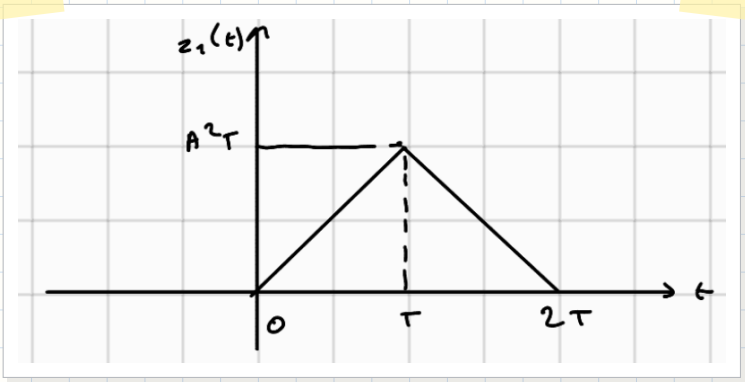
orj. s. 7

MF: $z_1(t) = r(t) \circledast h(t) = s(t) \circledast s(t)$




orijinal çizim, s. 7

$\Rightarrow z_1(t)$



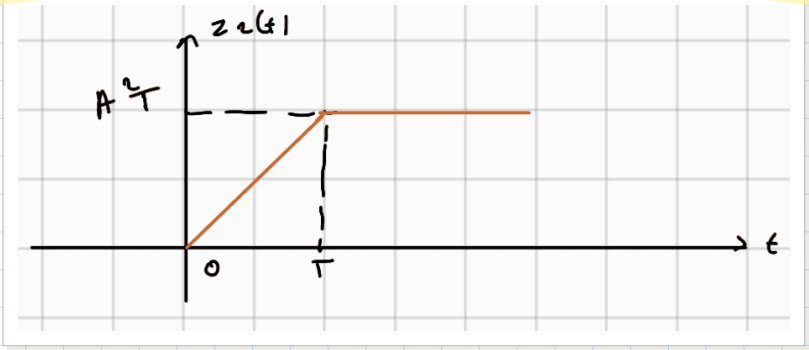

orijinal çizim, s. 7

## İlişkilendirici

$$z_2(t) = \int_0^t \underbrace{r(z)}_{s(z)}\, s(z)\, dz = \int_0^t s^2(z)\, dz \quad \begin{cases} = 0, & t < 0 \\ & 0 < t < T \end{cases}$$

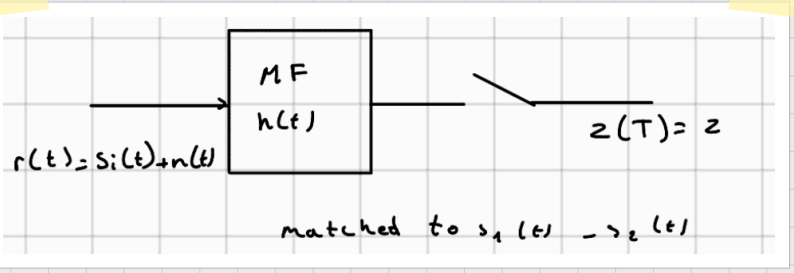
$$= \int_0^t A^2\, dz = A^2 t, \quad 0 < t < T$$

$$= \int_0^T A^2\, dz = A^2 T, \quad t > T$$




orijinal çizim, s. 7

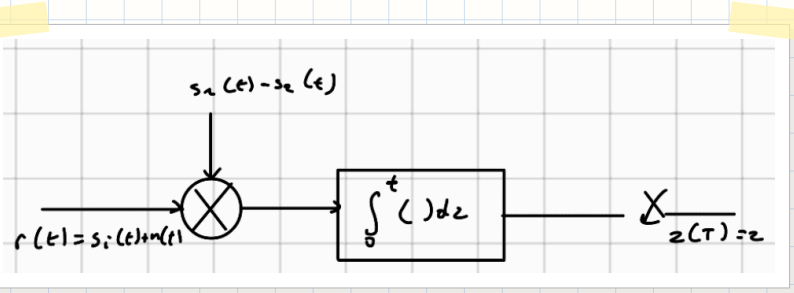
$$z_2(T) = z(T) = A^2T$$




orijinal çizim, s. 7

orj. s. 8

$$h(t) = s_1(T-t) - s_2(T-t)$$




orijinal çizim, s. 8

$$P_b = Q\left(\frac{a_1 - a_2}{2\sigma_0}\right) \qquad \left(\frac{a_1 - a_2}{\sigma_0}\right)^2 = \frac{(a_1 - a_2)^2}{\sigma_0^2}$$

$$\left(\frac{S}{N}\right)_T = \frac{a^2}{\sigma_0^2} = \frac{2\overbrace{E_s}^{s(t)\text{'nin enerjisi}}}{N_0}$$

← $h(t)$ tek $s(t)$'yi karşıladığı için

$$\left(\frac{S}{N}\right)_T = \frac{(a_1 - a_2)^2}{\sigma_0^2} = \frac{2\overbrace{E_d}^{\text{Fark işaretin enerjisi (Difference signal)}}}{N_0}$$

← $h(t)$ $s_1(t) - s_2(t)$'yi karşılıyor

$$E_d = \int_0^T [s_1(t) - s_2(t)]^2 dt = \underbrace{\int_0^T s_1^2(t)\,dt}_{E_b\text{: bit başına energy per bit}} + \underbrace{\int_0^T s_2^2(t)\,dt}_{E_b} - 2\underbrace{\int_0^T s_1(t)s_2(t)\,dt}_{\rho \cdot E_b}$$

$$\rho = \frac{1}{E_b}\int_0^T s_1(t)s_2(t)\,dt$$

↓ karşı-ilişki katsayısı
(Normalized cross correlation coefficient)
$-1 \le \rho \le 1$

*[Ok, "(Normalized cross correlation coefficient)" ifadesinden $\rho \cdot E_b$ ifadesine gidiyor.]*

$$E_d = 2E_b - 2\rho E_b = 2E_b(1-\rho)$$

$$P_b = Q\left(\frac{a_1 - a_2}{2\sigma_0}\right) = Q\left(\sqrt{\frac{2E_d}{4N_0}}\right)$$
$$= Q\left(\sqrt{\frac{E_d}{2N_0}}\right) = Q\left(\sqrt{\frac{E_b}{N_0}(1-\rho)}\right)$$