Sayısal İşaret İşleme

FIR filtre tipleri ve tasarımı, STFT

Orijinal not: Dsp-ders-10.pdf

UYARI Bu belge, el yazısı ders notlarından Claude (Anthropic) yardımıyla dijitale aktarılmıştır. Yardımcı kaynak olarak hazırlanmıştır; hata veya eksik içerebilir. Lütfen orijinal notları ve ders kaynaklarını esas alın.

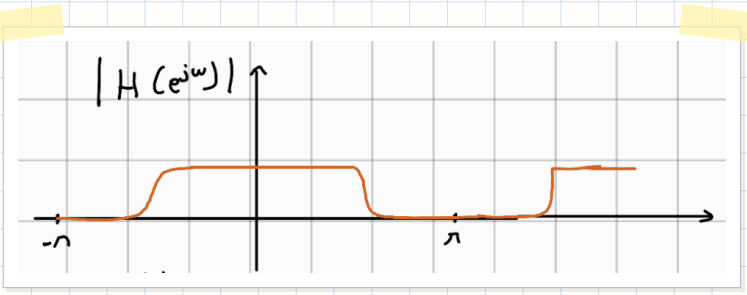
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orj. s. 1

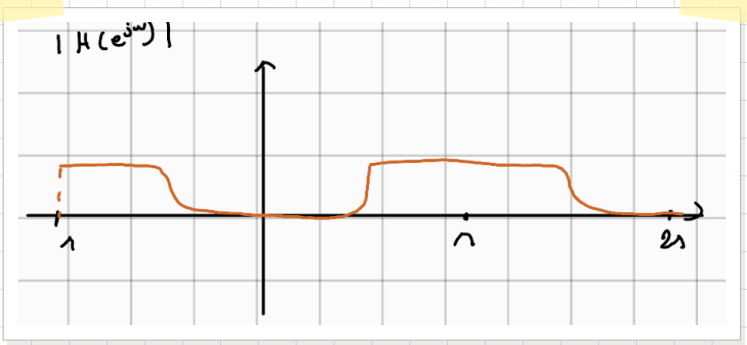
Tip 2

$$H(e^{j\omega}) = e^{-j\omega\frac{M}{2}} \sum_{n=0}^{\frac{M-1}{2}} \underbrace{h[n]\left[2\cos\left(\omega n - \omega\frac{M}{2}\right)\right]}_{A \gtrless 0}$$

$$\angle H(e^{j\omega}) = \begin{cases} -\omega\dfrac{M}{2}, & A > 0 \\ -\omega\dfrac{M}{2} + \pi, & A < 0 \end{cases}$$



orijinal çizim, s. 1



orijinal çizim, s. 1

$$|H(e^{j\omega})|\Big|_{\omega=0} = \left|\sum_{n=0}^{\frac{M-1}{2}} 2h[n]\right|$$

$$|H(e^{j\omega})|\Big|_{\omega=\pi} = \left|\sum_{n=0}^{\frac{M-1}{2}} h[n]\underbrace{\cos\left(\pi n - \frac{\pi}{2}M\right)}_{\cos\left(\frac{\pi}{2}(2n-M)\right)}\right| = 0$$

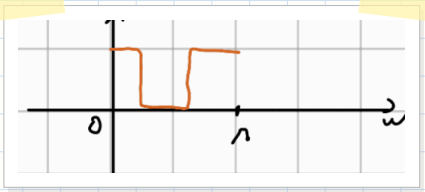
⋆ M tip 2'de tektir.

çift − tek = tek

1
2
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5

Bant söndüren

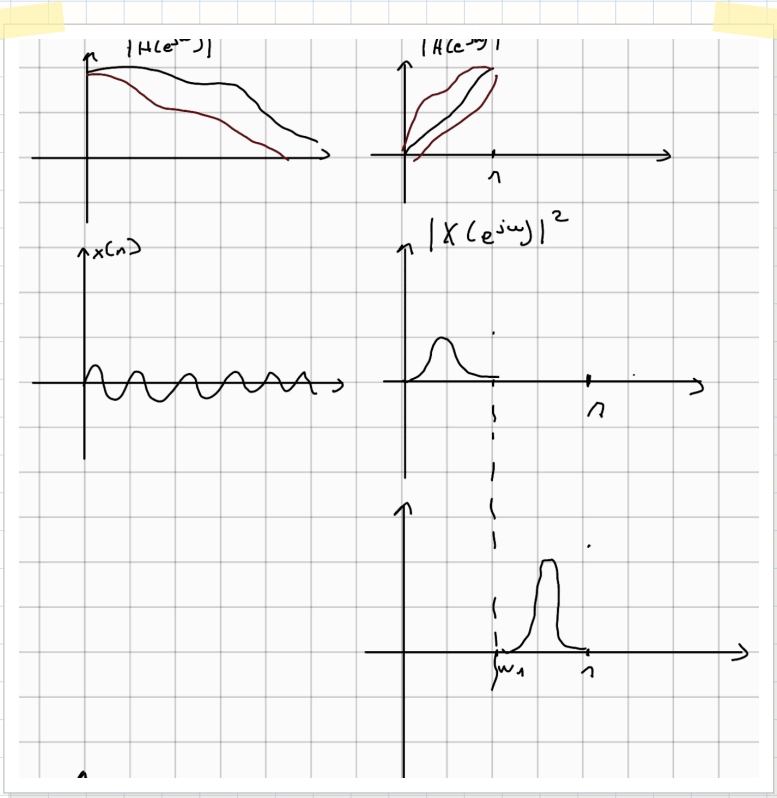
$\omega = 0$ da ve π de $|H(e^{j\omega})| > 0$



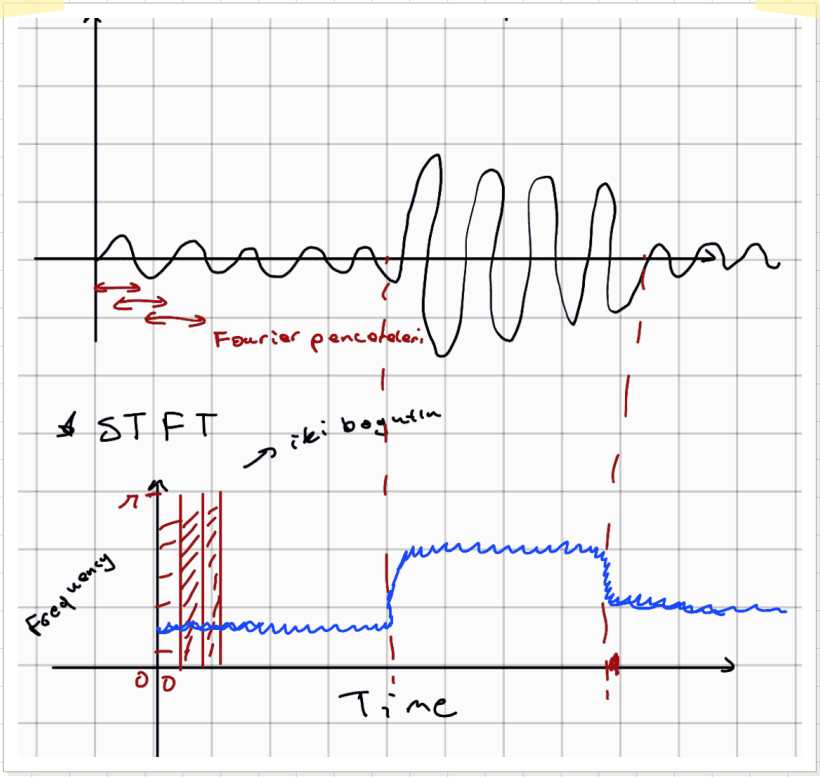
orijinal çizim, s. 1

	LP	HP	BP	BS (Band stop)
Type I	✓	✓	✓	✓
Type II	✓	×	✓	×
Type III	×	×	✓	×
Type IV	×	✓	✓	×

orj. s. 2 artık $H(e^{j\omega})$, $h[n]$?ta sağlayacağız



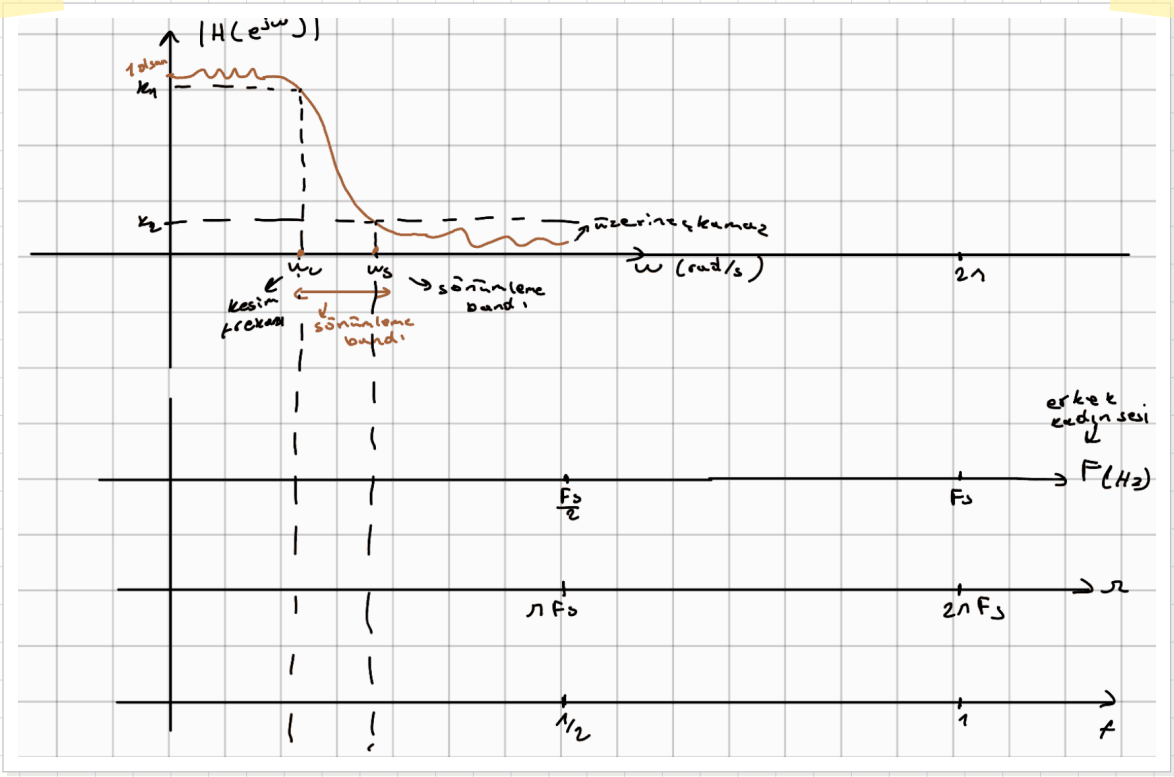
orijinal çizim, s. 2



orijinal çizim, s. 2

orj. s. 3

Filter Specifications



orijinal çizim, s. 3

$$\Omega = F_s\,\omega \qquad \Omega = 2\pi F$$

$$F = F_s \cdot f \qquad \omega = 2\pi f$$

$$\omega = \frac{F}{F_s}\,2\pi$$

Tip 1

$$h[n] = h[M-n] \qquad M \text{ even (çift)}$$

$$H(e^{j\omega}) = e^{-j\omega\frac{M}{2}}\left(\sum_{k=0}^{M/2} a_k \cos \omega k\right)$$

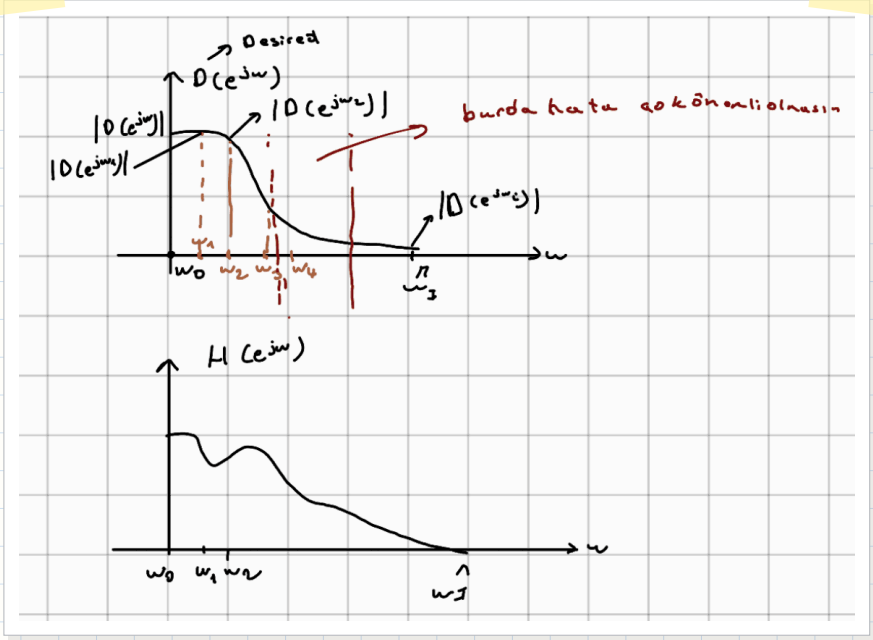
$$a_0 = h[M/2]$$

$$a_k = 2h[M/2-k] \qquad k = 1,\ldots,M/2$$

Tip 1 için

$$H(e^{j\omega}) = e^{-j\omega\frac{M}{2}}\left(\sum_{m=0}^{M/2} a_m \cos \omega m\right)$$

orj. s. 4

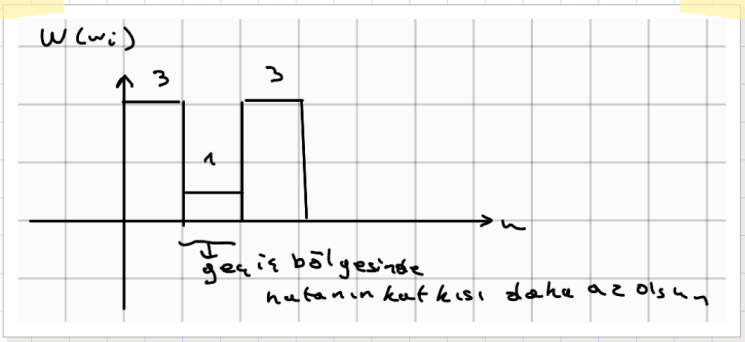


orijinal çizim, s. 4

$$\underset{\text{k noktasındaki}}{\text{Hata}} = \left|D(e^{j\omega_k}) - H(e^{j\omega_k})\right|$$

$$\text{minimize etme gerek} \quad \sum_{k=0}^{I}\left(D(e^{j\omega_k}) - H(e^{j\omega_k})\right)^2$$

kare işlemi yüksek hataları güçlendirir, onlara önem verir



orijinal çizim, s. 4

$$\Longrightarrow \min \sum_{k=0}^{I} W(\omega_k)\left(D(e^{j\omega_k}) - H(e^{j\omega_k})\right)^2$$

$$e(\omega_i) = W(\omega_i)\left[H(\omega_i) - D(\omega_i)\right]$$

$$\omega_i = \quad i = 0, \ldots, I$$

$$\begin{aligned} E &= \sum_{i=0}^{I} |e(\omega_i)|^2 \\ &= \sum_{i=0}^{I} \Big[W(\omega_i)\big[H(\omega_i) - D(\omega_i)\big] \Big]^2 \\ &= \sum_{i=0}^{I} \Big[\underbrace{\sum_{m=0}^{M/2} W(\omega_i)\, e^{-j\omega_i \frac{M}{2}} \cos(\omega_i m)\, a_m - W(\omega_i) D(\omega_i)}_{e_i} \Big]^2 \end{aligned}$$

orj. s. 5

Minimize E

$$= e_0^2 + e_1^2 + \cdots + e_I^2 = \underbrace{\begin{bmatrix} e_0 & e_1 & \cdots & e_I \end{bmatrix}}_{e^T} \underbrace{\begin{bmatrix} e_0 \\ e_1 \\ \vdots \\ e_I \end{bmatrix}}_{e}$$

$$= e^T \cdot e$$

$$e_i = \Big[W(\omega_i)\, e^{-j\omega_i \frac{M}{2}}, \;\; W(\omega_i)\, e^{-j\omega_i \frac{M}{2}} \cos(\omega_i 1) \Big] \underbrace{\begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_I \end{bmatrix}}_{a \text{ vektörü}} - \underbrace{W(\omega_i) D(\omega_i)}_{d_i \text{ diyelim}}$$

$$h_i = \begin{bmatrix} 1 \\ \cos(\omega_i) \\ \cos(2\omega_i) \\ \cos(3\omega_i) \\ \vdots \\ \cos\left(\frac{M}{2}\omega_i\right) \end{bmatrix}$$

$$e_i = W(\omega_i)\, e^{-j\omega_i \frac{M}{2}}\, h_i^T \cdot a - d_i$$

$$e = \begin{bmatrix} e_0 \\ e_1 \\ \vdots \\ e_I \end{bmatrix} \quad \underbrace{\begin{bmatrix} W(\omega_0)\, e^{-j\omega_0 \frac{M}{2}}\, h_0^T \\ W(\omega_1)\, e^{-j\omega_1 \frac{M}{2}}\, h_1^T \\ \vdots \\ W(\omega_I)\, e^{-j\omega_I \frac{M}{2}}\, h_I^T \end{bmatrix}}_{H \text{ vektörü}} a - \overbrace{\begin{bmatrix} d_0 \\ d_1 \\ d_2 \\ \vdots \\ d_I \end{bmatrix}}^{d}$$

$$e = Ha - d$$

$$\begin{aligned}\underline{\text{minimize}}\ E = e^T \cdot e &= (H \cdot a - d)^T (H \cdot a - d) \\ &= a^T H^T H a - \underbrace{a^T}_{a^T} H^T \underbrace{d}_{b} - d^T H a + d^T d \\ &= a^T H^T H a - 2a^T H^T d + d^T d\end{aligned}$$

$$a^T b = b^T a \quad \checkmark$$

$$\begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} y_1 & y_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\frac{\partial E}{\partial a} = 0 \qquad \frac{\partial E}{\partial a} = 2H^T H a - 2H^T d$$

$$H^T H a = H^T d \implies \boxed{a = (H^T H)^{-1} H^T d}$$

orj. s. 6 Eğer M'yi çok kısıtlı tutarsanız filtrenizin çözünürlüğü düşük

[1] Claude notu: iki köşeli parantez arasında muhtemelen "=" olmalı.