Sayısal İşaret İşleme

Quiz çözümü, FIR/IIR filtre tasarımı

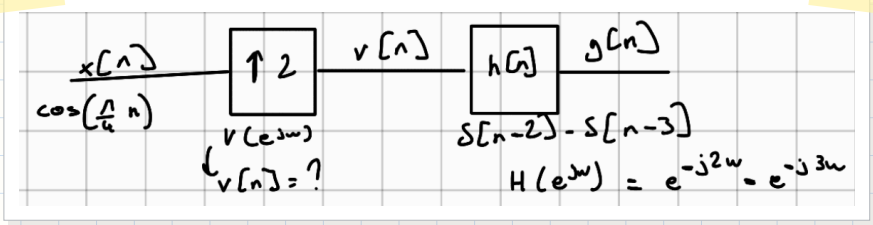
Orijinal not: Dsp-ders-11.pdf

UYARI Bu belge, el yazısı ders notlarından Claude (Anthropic) yardımıyla dijitale aktarılmıştır. Yardımcı kaynak olarak hazırlanmıştır; hata veya eksik içerebilir. Lütfen orijinal notları ve ders kaynaklarını esas alın.

NOTICE *This document was transcribed from handwritten lecture notes with the help of Claude (Anthropic). It is meant as a supplementary resource and may contain errors or omissions. Always check against the original notes and official course material.*

orj. s. 1

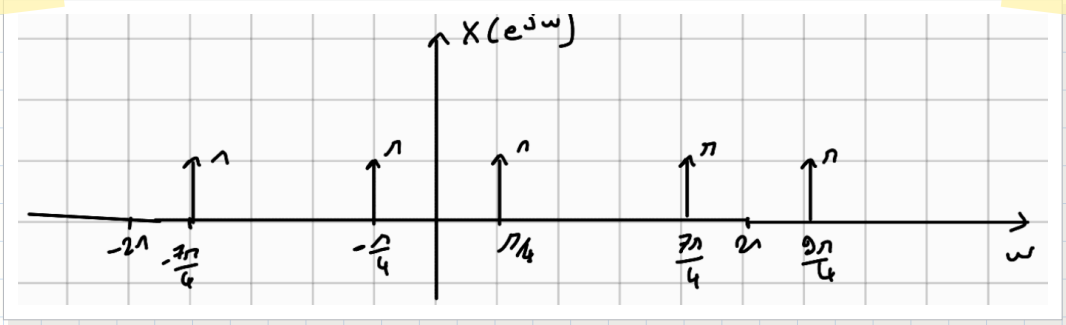
Quiz çözümü



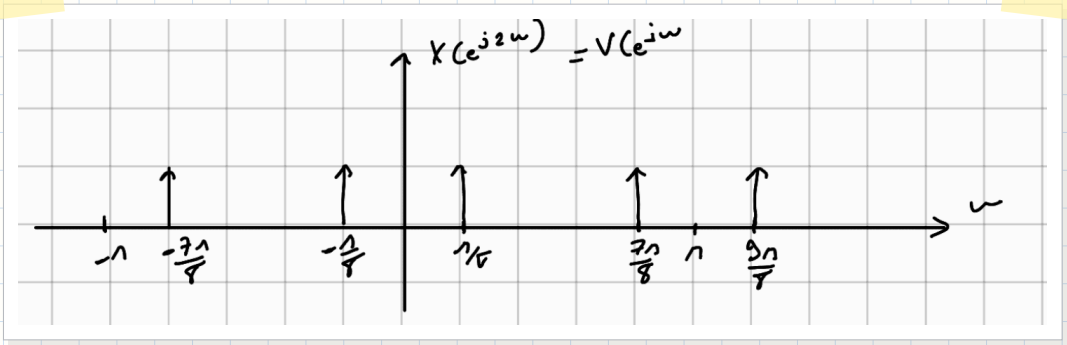
orijinal çizim, s. 1

$$V(e^{j\omega}) = X(e^{j2\omega})$$

$$V(z) = X(z^2)$$



orijinal çizim, s. 1



orijinal çizim, s. 1

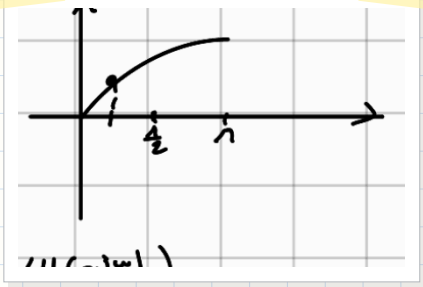
$$v[n] = \cos\left(\frac{\pi}{8}n\right) + \cos\left(\frac{7\pi}{8}n\right)$$

$$H(e^{j\omega}) = e^{-j2\omega} - e^{-j3\omega} = 2j\,e^{-j\frac{5}{2}\omega}\left(\frac{e^{j\frac{1}{2}\omega} - e^{-j\frac{1}{2}\omega}}{2j}\right)$$

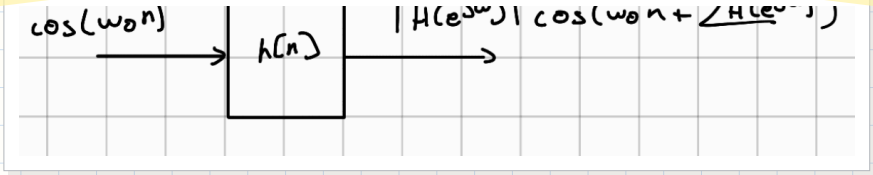
$$= \underbrace{e^{j\frac{\pi}{2}}\,e^{-j\frac{5}{2}\omega}}_{e^{-j\left(\frac{5}{2}\omega - \frac{\pi}{2}\right)}}\,2\sin\left(\frac{\omega}{2}\right)$$

$$|H(e^{j\omega})| = 2\left|\sin\frac{\omega}{2}\right|$$

$$\angle H(e^{j\omega}) = -\frac{5}{2}\omega + \frac{\pi}{2} \qquad 0 < \omega < \pi$$



orijinal çizim, s. 1



orijinal çizim, s. 1

orj. s. 2

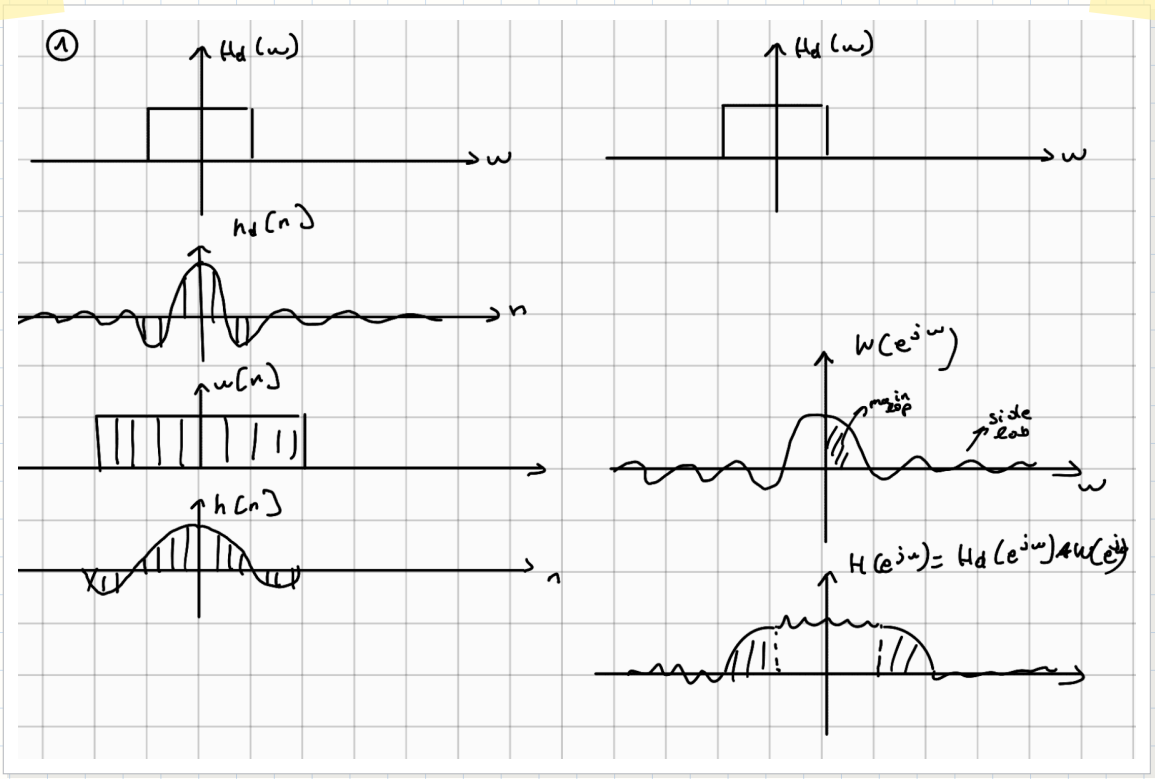
FIR Filter Design Using windows

① start with desired freq response $H_d(\omega)$

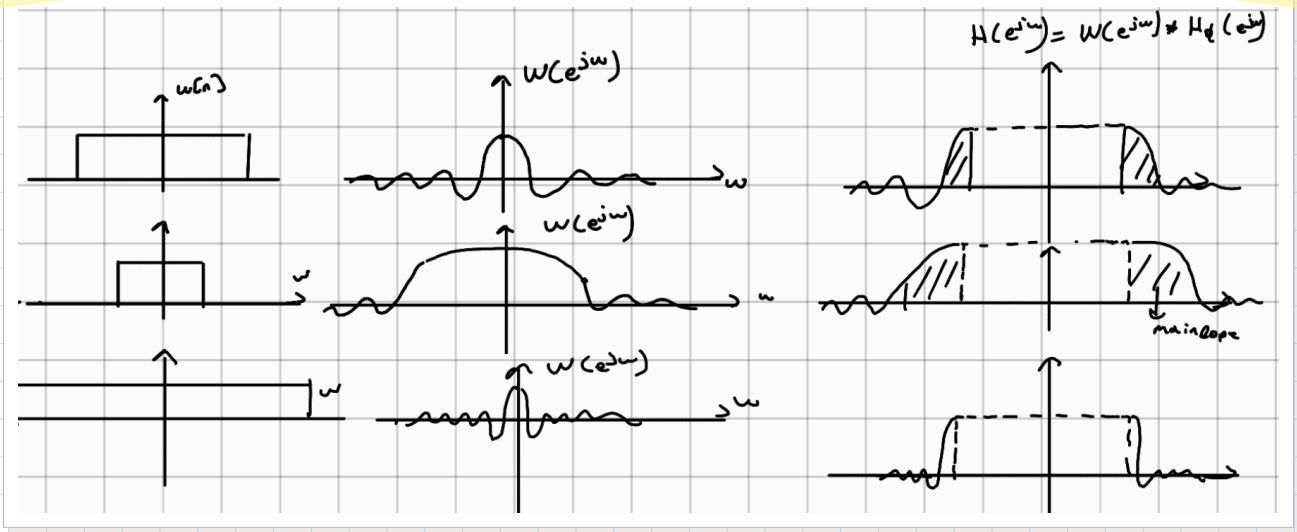
② compute IDTFT $\{H_d(\omega)\} = h_d[n]$

③ $h[n] = h_d[n]\, w[n]$

$H(e^{j\omega})$, $H(\omega)$ } Ayrık $H(\Omega)$, $H(j\Omega)$ } sürekli

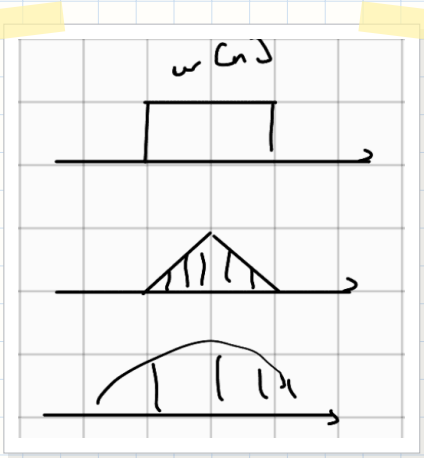


orijinal çizim, s. 2

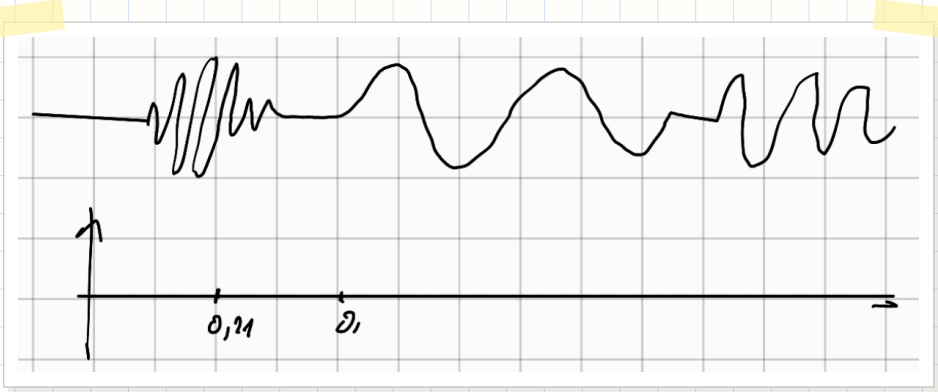


orijinal çizim, s. 2

orj. s. 3



orijinal çizim, s. 3



orijinal çizim, s. 3

Fig 4.14

Üçgen kullanıldığında side loblar daha az, main lobe ne kadar ? durursa dahiyiği

1- shape ↗ main lobe
→ side lobe ①

2- size → main lobe ②

DTFT DFT → kullanılıyorsa periyodik var sayılıyor



orijinal çizim, s. 3

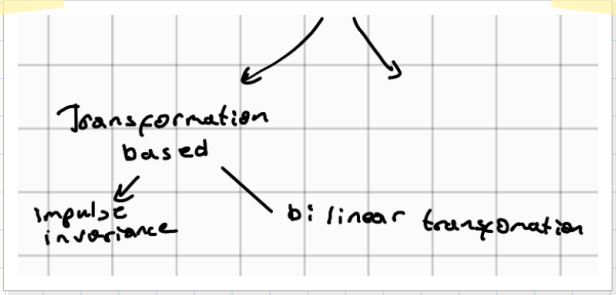


orijinal çizim, s. 3

resolüsyon bileşenlerin birbirine karışması

orj. s. 4

IIR Filter Design



orijinal çizim, s. 4

① Given set of digital filter specs
② Transform specs to analog filter specs
③ Design analog filter $H_a(s)$
④ Transform $H(s) \rightarrow H(z)$

Impulse Invariance

$h_a(t)$
$h_d[n]$
$h_d[n] = h_a(t)|_{t=nT}$
$H(z) = \mathcal{Z}[h_d[n]]$

Ex:

$$H_a(s) = \frac{A}{s+a}$$
$$h_a(t) = Ae^{-at}u(t)$$
$$h_d[n] = Ae^{-aTn}$$

Ex:

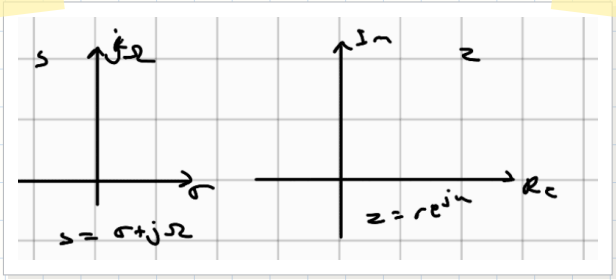
$$H_a(s) = \frac{A}{s+a}$$
$$h_a(t) = Ae^{-at}u(t)$$
$$h_d[n] = Ae^{-aTn}u(nT)$$
$$= A(e^{-aT})^n u[n] \implies H(z) = \frac{A}{1-e^{-aT}z^{-1}}$$

$$H_a(s) = \frac{Y(s)}{X(s)} = \frac{\sum_{k=0}^{M} b_k s^k}{\sum_{k=0}^{N} a_k s^k}$$

$$H_{LP}(s) = \frac{1}{s^3 + 2s^2 + 2s + 1} = \frac{Y(s)}{X(s)}$$

orj. s. 5

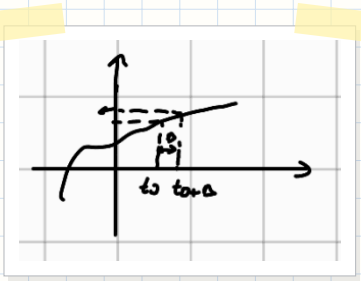
$$y'''(t) + 2y''(t) + 2y'(t) + y(t) = x(t)$$



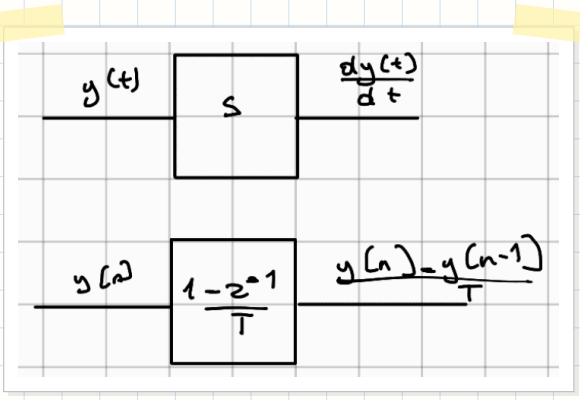
orijinal çizim, s. 5

The Filter Design by Approximation of Derivation

$$\left.\frac{dy(t)}{dt}\right|_{t=nT} = \frac{y(nT) - y((n-1)T)}{T} = \frac{y[n] - y[n-1]}{T}$$

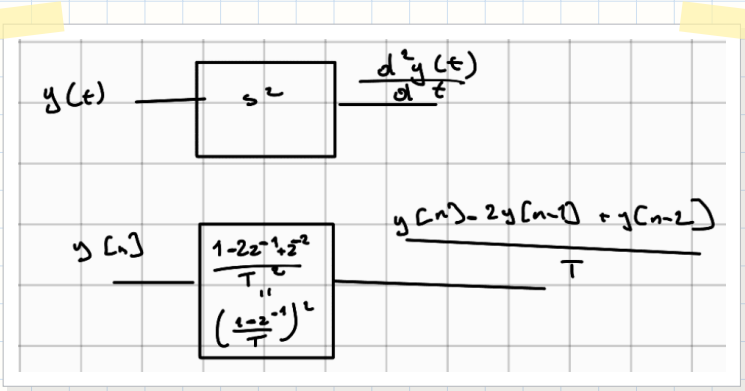


orijinal çizim, s. 5

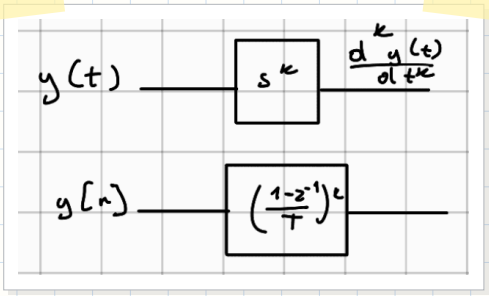


orijinal çizim, s. 5

$$\left.\frac{d^2y(t)}{dt^2}\right|_{t=nT} = \left.\frac{d}{dt}\left(\frac{dy(t)}{dt}\right)\right|_{t=nT} = \frac{y(nT) - 2y((n-1)T) + y((n-2)T)}{T^2}$$
$$= \frac{y[n] - 2y[n-1] + y[n-2]}{T^2}$$



orijinal çizim, s. 5



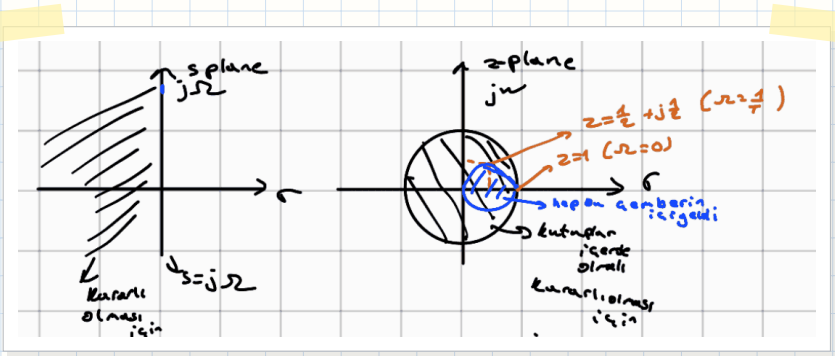
orijinal çizim, s. 5

$$s = \frac{1 - z^{-1}}{T}$$

orj. s. 6

$$H(z) = H_a(s)\Big|_{s \leftarrow \frac{1-z^{-1}}{T}}$$

$$s = \frac{1 - z^{-1}}{T} \implies z = \frac{1}{1 - sT}$$



orijinal çizim, s. 6

$$z = \underset{1}{r} e^{j\omega}$$

$$s = \sigma + j\Omega$$

$s = j\Omega$ sınır

$$z = \frac{1}{1 - sT}\Big|_{s=j\Omega} = \frac{1}{1 - j\Omega T} \cdot \frac{1 + j\Omega T}{1 + j\Omega T}$$

$$= \frac{1}{1 + \Omega^2 T^2} + j\frac{\Omega T}{1 + \Omega^2 T^2}$$

Burada sınırda bir kutup orda (z'de) nerede ↗

$$\Omega = 0 \Longrightarrow z = 1$$
$$\Omega = \frac{1}{T} \Longrightarrow z = \frac{1}{2} + \frac{1}{2}j$$
$$\Omega = \infty \Longrightarrow \frac{1}{T}^{1}$$

Çok iyi bir yöntem değilmiş.

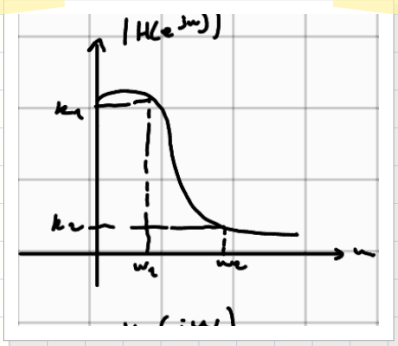
IIR Filter Design by the Bilinear Transformation

$$s = \frac{2}{T}\frac{1 - z^{-1}}{1 + z^{-1}} \Longrightarrow z = \frac{1 - s\frac{T}{2}}{1 + s\frac{T}{2}}^{2}$$

$$H_a(s)\Big|_{s \leftarrow \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}} = H(z)$$

$$\Omega = \frac{2}{T}\tan\left(\frac{\omega}{2}\right)$$

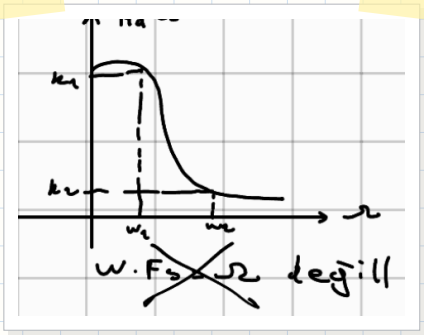
$$\omega = 2\tan^{-1}\left(\frac{\Omega T}{2}\right)$$



orijinal çizim, s. 6

[1] Claude notu: $\Omega \to \infty$ için $z \to 0$ beklenir; muhtemelen $z = 0$ olmalı.

[2] Claude notu: muhtemelen $z = \frac{1 + sT/2}{1 - sT/2}$ olmalı.



orijinal çizim, s. 6

$$\Omega_1 = \frac{2}{T}\tan\left(\frac{\omega_1}{2}\right)k_1$$

$$\Omega_2 = \frac{2}{T}\tan\left(\frac{\omega_2}{2}\right)k_2$$

$$H_a(s)\Big|_{s\leftarrow \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}}$$