Sayısal İşaret İşleme

Quiz çözümü, IIR filtre tasarımı (B.T.), Butterworth

Orijinal not: Dsp-ders-12.pdf

UYARI Bu belge, el yazısı ders notlarından Claude (Anthropic) yardımıyla dijitale aktarılmıştır. Yardımcı kaynak olarak hazırlanmıştır; hata veya eksik içerebilir. Lütfen orijinal notları ve ders kaynaklarını esas alın.

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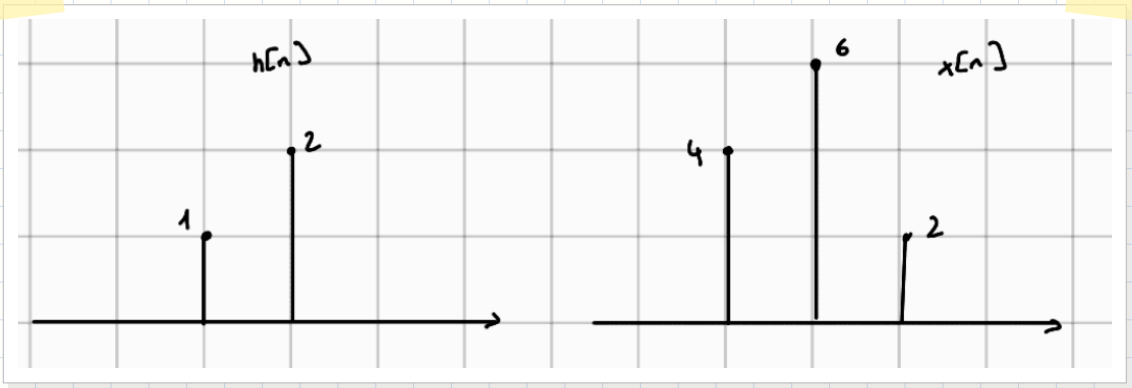
orj. s. 1

Quiz çözümü

$$x(t) = \cos(0{,}1\pi t + 0{,}2)$$

$f_s = 5\,\text{Hz}$ $\quad x[2] = ?$

$$x[2] = \cos\left(0{,}1\pi \cdot \frac{2}{5} + 0{,}2\right)$$

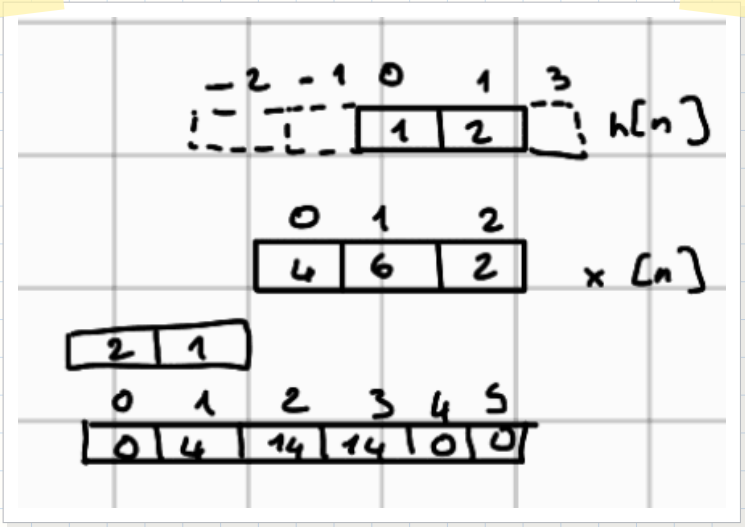


orijinal çizim, s. 1

$$H(z) = 1 + 2z^{-1} \qquad X(z) = 4 + 6z^{-1} + 2z^{-2}$$

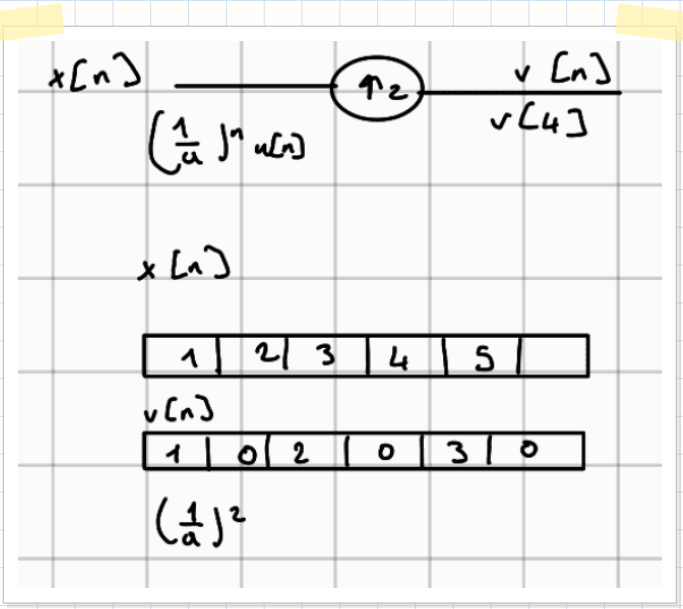
$$X(z)\,H(z) \Rightarrow$$

Başka bir yol

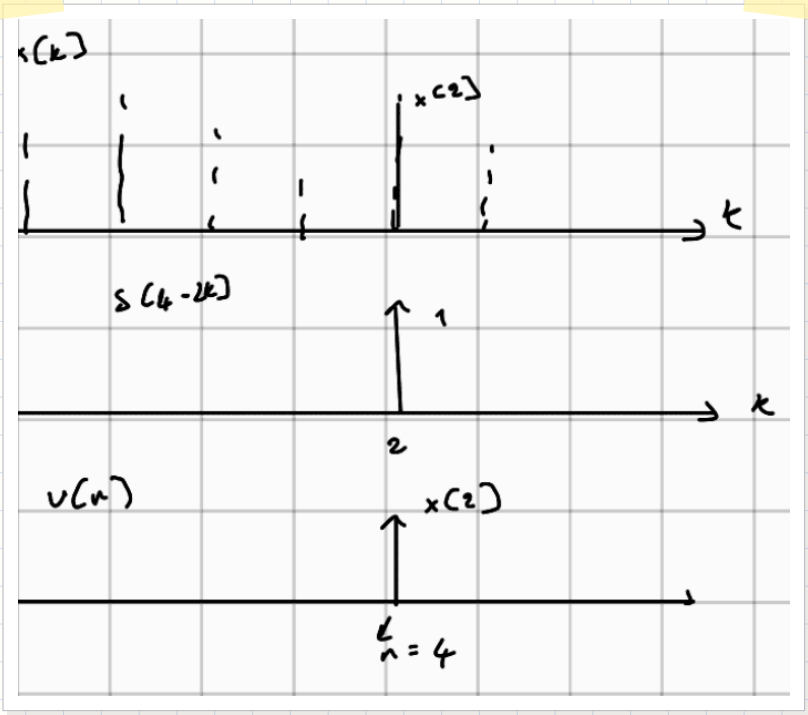


orijinal çizim, s. 1

3. soru



orijinal çizim, s. 1



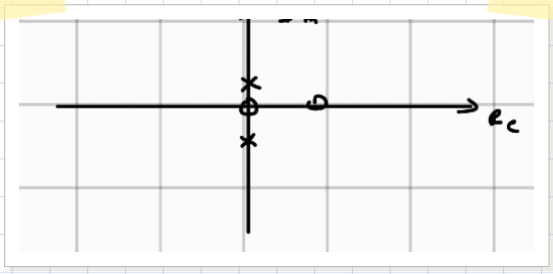
orijinal çizim, s. 1

$$v[n] = \sum_{k=-\infty}^{\infty} \delta[n-2k]\,x[k]$$

$$v[4] = \delta[4-2\cdot 2]\,x[2] = \sum_{k=-\infty}^{\infty} \delta[4-2k]\,x[k]$$

$$X(z) = \frac{1-0{,}81z^{-1}}{1+0{,}25z^{-2}}, \quad |z| > 0{,}5$$

$$= \frac{z^2-0{,}81z}{z^2+0{,}25} = \frac{z(z-0{,}81)}{(z+j0{,}5)(z-j0{,}5)}$$

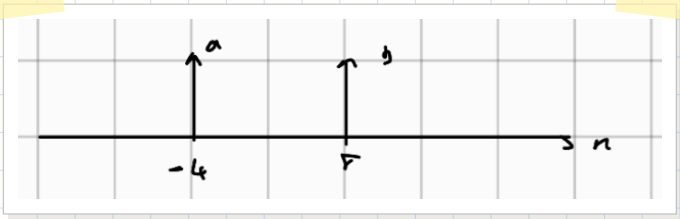


orijinal çizim, s. 1

orj. s. 2

$$h[n] = \cos\left(\frac{\pi}{2}n\right)\left[\delta[n+4]-\delta[n-8]\right]$$

$$h[n] = \underbrace{\cos\left(\frac{\pi}{2}(-4)\right)}_{a}\,\delta[n+4] \underbrace{-\cos\left(\frac{\pi}{2}8\right)}_{b}\,\delta[n-8]$$



orijinal çizim, s. 2

$$H(z) = az^4 + bz^{-8} = az^4 + \frac{b}{z^8}$$

$$X(z) = \frac{0{,}4}{1 - 3z^{-1}}, \qquad |z| < 3$$

$x[n] * \delta[n+2] = ?$

$$(0{,}4)(-3^n u[-n-1])$$

$$X(z) = \frac{1}{1 - az^{-1}}, \qquad |z| < a$$

$$x[n] = -a^n u[-n-1]$$

IIR Filter Design by the B.T.

$$H(s) = \frac{b}{s+a} = \frac{Y(s)}{X(s)}$$

$$H(s) = \frac{1}{s^3 + 2s^2 + 3s + 4} = \frac{k_1}{s-a} + \frac{k_2}{s-b} + \cdots$$

$$\frac{dy(t)}{dt} + a\,y(t) = b\,x(t)$$? okunamadı

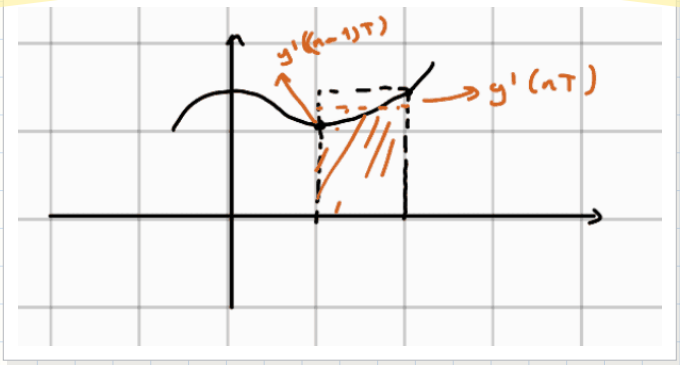
$$y'(t) + a\,y(t) = b\,x(t)$$

$$y'(t) = b\,x(t) - a\,y(t)$$

$t = nT \qquad y'(nT) = b\,x(nT) - a\,y(nT)$ ①

$$y(t) = \int_{t_0}^{t} y'(t)\,dt + y(t_0)$$

$t = nT,\ t_0 = (n-1)T \qquad y(t) = \int_{t_0}^{t} y'(t)\,dt + y(t_0)$



orijinal çizim, s. 2

$$y'(nT) = \frac{T}{2}\left[y'(nT) + y'((n-1)T)\right] + y((n-1)T) \qquad ②^{1}$$

Substitute ① into ②

$$\left(1 + a\frac{T}{2}\right) y[n] + \left(-1 + a\frac{T}{2}\right) y[n-1] = b\frac{T}{2}\left[x[n] + x[n-1]\right]$$

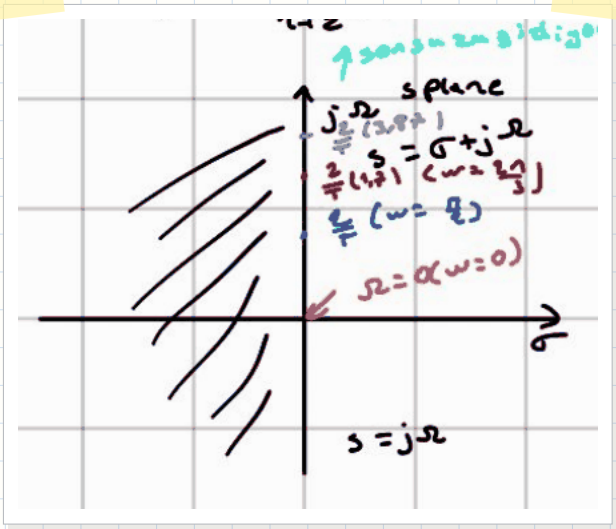
orj. s. 3

$$H(z) = \frac{Y(z)}{X(z)} = b\,\frac{T}{2}\,\frac{1 + z^{-1}}{\left(1 + a\frac{T}{2}\right) + \left(-1 + a\frac{T}{2}\right) z^{-1}} = \frac{b}{\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}} + a}$$

$$H(s) = \frac{b}{s + a}$$

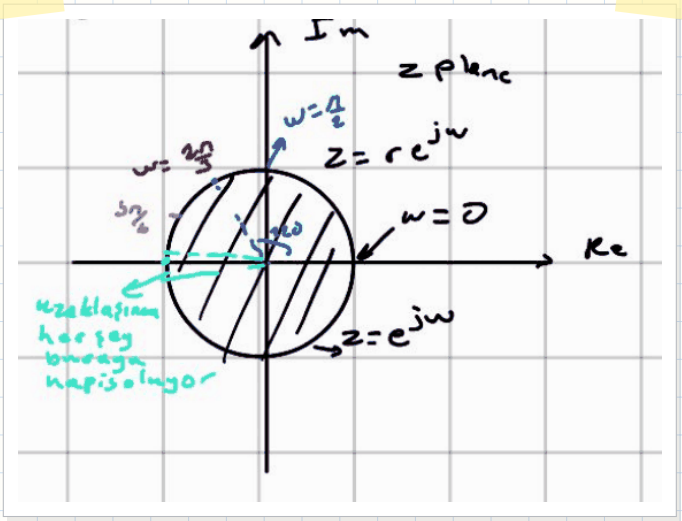
$$H(z) = H(s)\Big|_{s=\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}}$$

$$s = \frac{2}{T}\frac{1 - z^{-1}}{1 + z^{-1}} \qquad z = \frac{1 + sT/2}{1 - sT/2}$$



orijinal çizim, s. 3

[1] Claude notu: muhtemelen soldaki terim $y(nT)$ olmalı.



orijinal çizim, s. 3

s yerine $j\Omega$ z yerine $e^{j\omega}$

$$j\Omega = \frac{2}{T}\frac{e^{j\omega}-1}{e^{j\omega}+1}$$

$$= \frac{2}{T}\frac{e^{j\omega/2}\left(e^{j\omega/2}-e^{-j\omega/2}\right)}{e^{j\omega/2}\left(e^{j\omega/2}+e^{-j\omega/2}\right)}$$

$$= \frac{2}{T}\frac{2j\sin(\omega/2)}{2\cos(\omega/2)} = j\frac{2}{T}\tan\left(\frac{\omega}{2}\right)$$

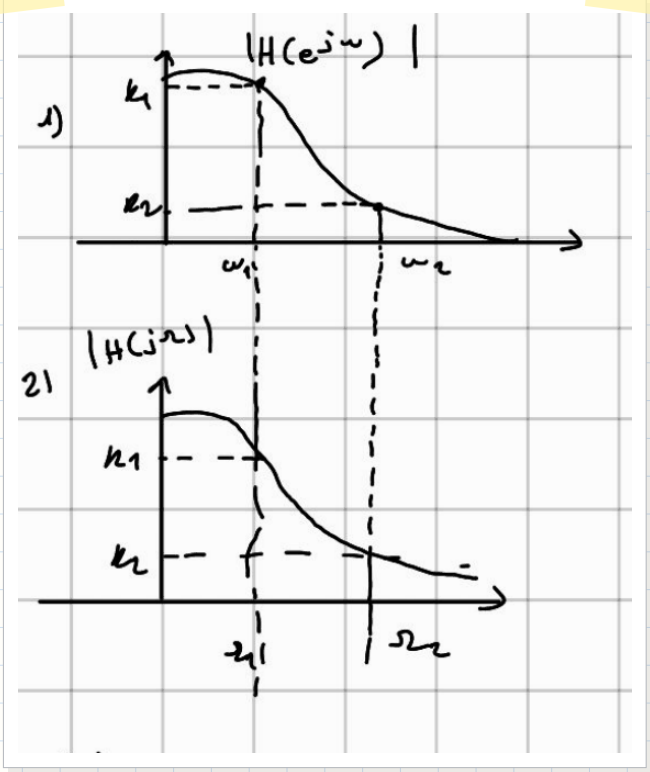
$$j\Omega = j\frac{2}{T}\tan\left(\frac{\omega}{2}\right)$$

$$\Omega = \frac{2}{T}\tan\left(\frac{\omega}{2}\right)$$

$$\omega = 2\arctan\left(\frac{\Omega\cdot T}{2}\right)$$

Örnek

Specleri belirleyelim



orijinal çizim, s. 3

k_1 @ ω_1
k_2 @ ω_2
k_1 @ $\Omega_1 = \frac{2}{T}\tan\left(\frac{\omega_1}{2}\right)$
k_2 @ $\Omega_2 = \frac{2}{T}\tan\left(\frac{\omega_2}{2}\right)$
3) design $H(s)$
4) $H(z) = H(s)\Big|_{s=\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}}$

orj. s. 4

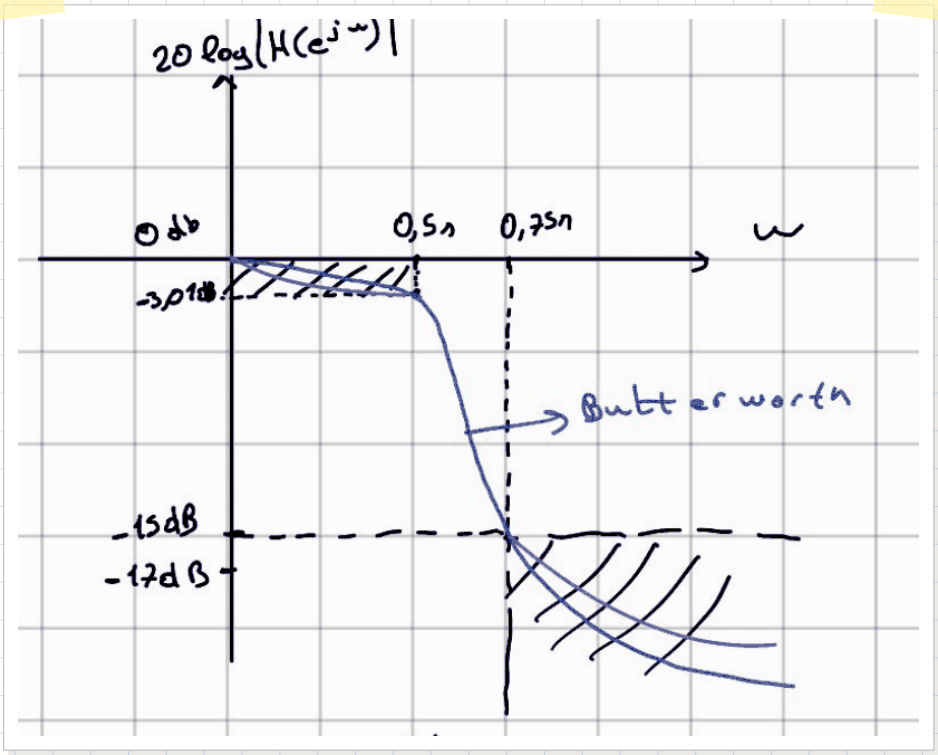
EX: Design LPF, B.T

sadece Butterworth

a) Monotonic stop band, pass band
b) −3,01 dB cutoff at 0,5π rad
c) 0,75π rad ⇒ at least 15 db olsun



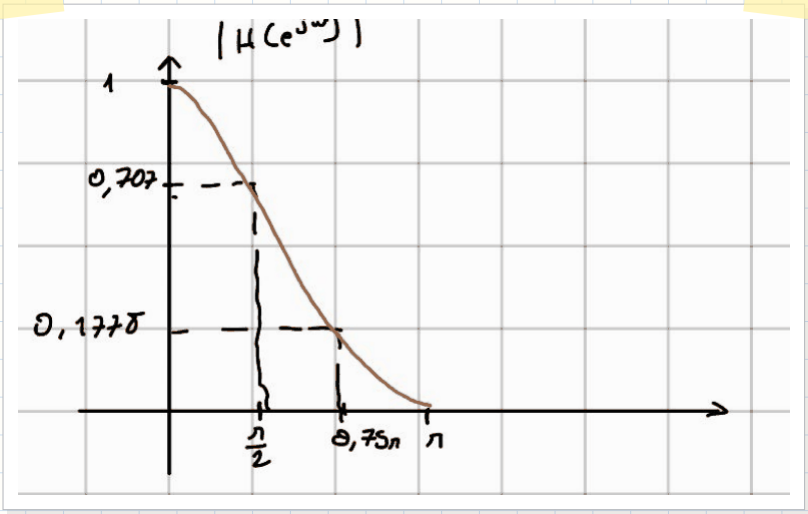
orijinal çizim, s. 4



orijinal çizim, s. 4

$F_s = 1000\,Hz \qquad T = 10^{-3}\ sec$

$$\frac{1}{T} = F_s$$



orijinal çizim, s. 4

$$20\log\left|H(e^{j\omega})\right| = -3{,}01 \text{ dB}$$

$$20\log\left|H(e^{j\omega})\right| < -15 \text{ dB}$$

$F_s = 1000$ Hz $\quad T = 10^{-3}$ sec

$$\Omega_1 = \frac{2}{T}\tan\left(\frac{\omega_1}{2}\right)\Big|_{\omega_1=0{,}5\pi} \qquad k_1@\omega_1$$

$$\Omega_2 = \frac{2}{T}\tan\left(\frac{\omega_2}{2}\right)\Big|_{\omega_2=0{,}75\pi}$$

$$k_1 \text{ @ } \Omega_1 = \frac{2}{T}\tan\left(\frac{\omega_1}{2}\right)$$
$$k_2 \text{ @ } \Omega_2 = \frac{2}{T}\tan\left(\frac{\omega_2}{2}\right)$$

$$\Omega_1 = 2000 \text{ rad/sec}$$

$$\Omega_2 = 4828{,}42 \text{ rad/sec}$$

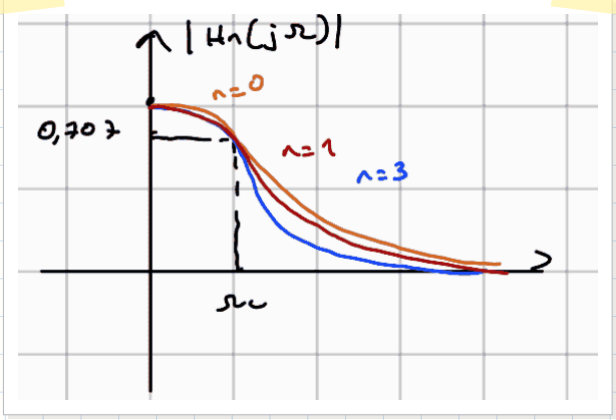
$$k_1 = 0{,}707 \qquad k_2 = 0{,}1778$$

orj. s. 5

Butterworth

$$\left|H_n(j\Omega)\right|^2 = \frac{1}{1+\left(\frac{\Omega}{\Omega_c}\right)^{2n}}$$

n ne olursa olsun 1'den başlar, n'den bağımsız olarak Ω_c'de genlik $\frac{1}{\sqrt{2}}$



orijinal çizim, s. 5

$$|H_n(j\Omega)|^2 = \frac{1}{1+\left(\frac{\Omega}{\Omega_c}\right)^{2n}}$$

$$|H_n(j\Omega_1)|^2 = \frac{1}{1+\left(\frac{\Omega_1}{\Omega_c}\right)^{2n}}$$

$$① \quad |H_n(j\Omega_1)|^2 = \frac{1}{1+\left(\frac{2000}{\Omega_c}\right)^{2n}} = (0{,}707)^2 \quad \Rightarrow \quad \Omega_c = 2000 \text{ rad/sec}$$

$$② \quad |H_n(j\Omega_2)|^2 = \frac{1}{1+\left(\frac{\Omega_2}{\Omega_c}\right)^{2n}}$$

$$(0{,}1778)^2 \geq \frac{1}{1+\left(\frac{4828{,}42}{2000}\right)^{2n}}$$

$$\Rightarrow \quad n \geq 1{,}942$$

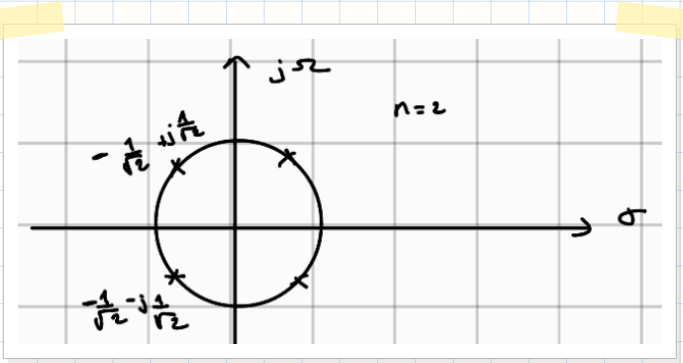
n: 2, 3, 4 ⟶ seçebiliriz

$\boxed{n = 2}$ seçelim

$$H(s) = \frac{k}{(s-s_0)(s-s_1)\cdots(s-s_{n-1})}$$

$$= \frac{k}{\left(s-\left(-\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}\right)\right)\left(s-\left(-\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}}\right)\right)}$$

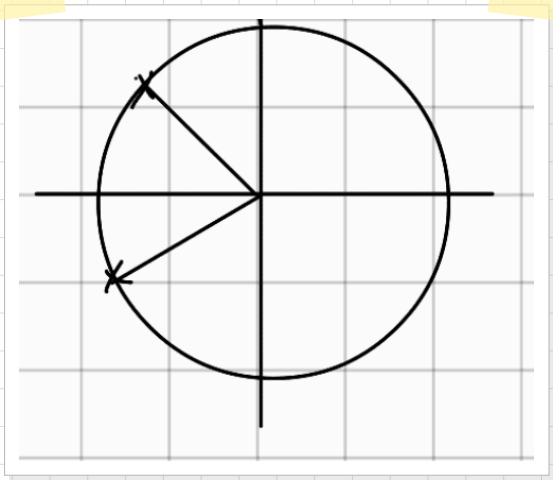
$$= \frac{\Omega_c^2}{s^2+\sqrt{2}\,\Omega_c s+\Omega_c^2}$$



orijinal çizim, s. 5

orj. s. 6

n=3 olsun



orijinal çizim, s. 6

Tablo

n	
1	$s+1$
2	$1+\sqrt{2}s+s^2$
3	$(s^2+s+1)(s+1)$

$n = 2$

$$H(s) = \frac{1}{1+\sqrt{2}s+s^2}$$

$$H(s) = \frac{k}{s^2 + \underset{\uparrow}{\Omega_c}\sqrt{2}s + \Omega_c^2}$$

2000

$$H(z) = H(s)\Big|_{s=2000\frac{1-z^{-1}}{1+z^{-1}}}$$

$$H(z) = \frac{1+2z^{-1}+z^{-2}}{3{,}4142 + 0{,}58\,z^{-2}}$$