Sayısal İşaret İşleme

DTFT, DFT, dairesel konvülasyon ve FFT

Orijinal not: Dsp-ders-13.pdf

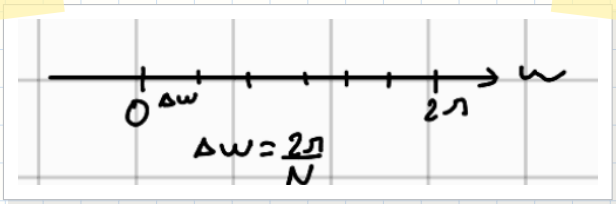
UYARI Bu belge, el yazısı ders notlarından Claude (Anthropic) yardımıyla dijitale aktarılmıştır. Yardımcı kaynak olarak hazırlanmıştır; hata veya eksik içerebilir. Lütfen orijinal notları ve ders kaynaklarını esas alın.

NOTICE *This document was transcribed from handwritten lecture notes with the help of Claude (Anthropic). It is meant as a supplementary resource and may contain errors or omissions. Always check against the original notes and official course material.*

orj. s. 1

DTFT

$$x[n] \longrightarrow X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$

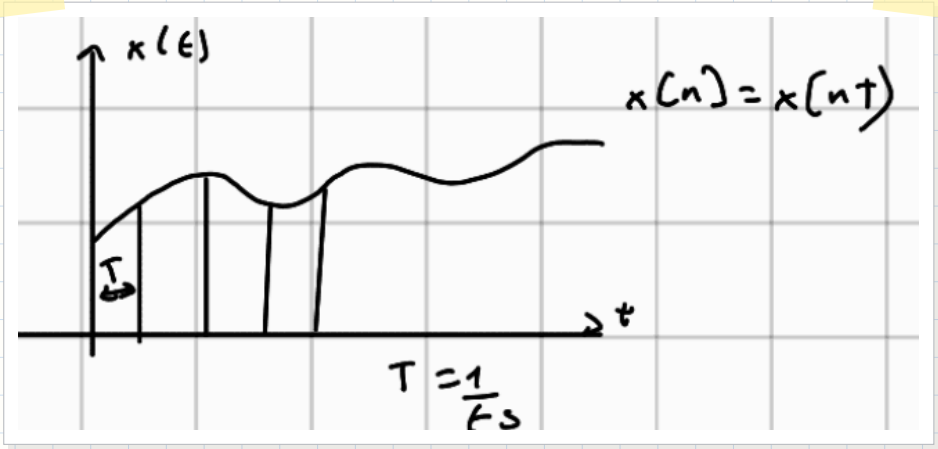


orijinal çizim, s. 1

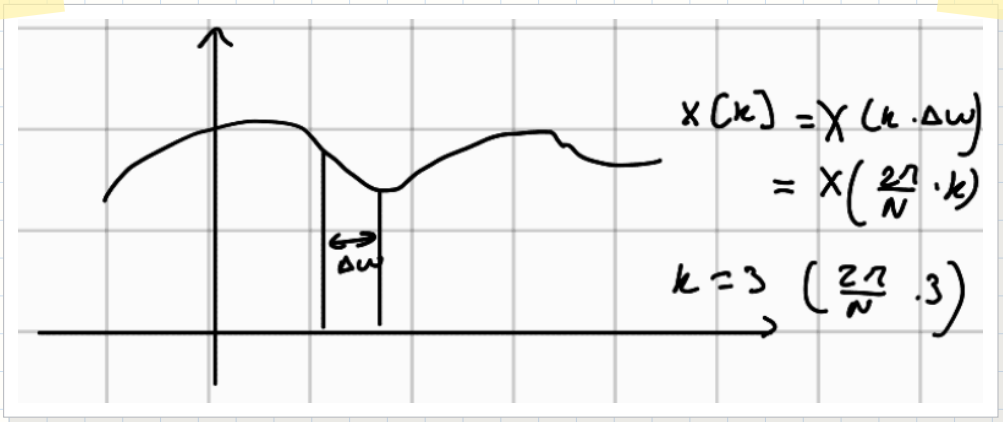
DFT

$$x[n] \longleftrightarrow X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j\left(\frac{2\pi}{N}k\right)n}$$

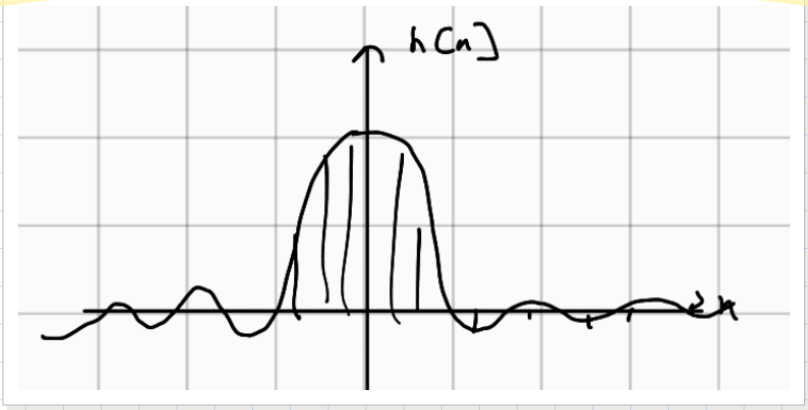
jω örneklendi



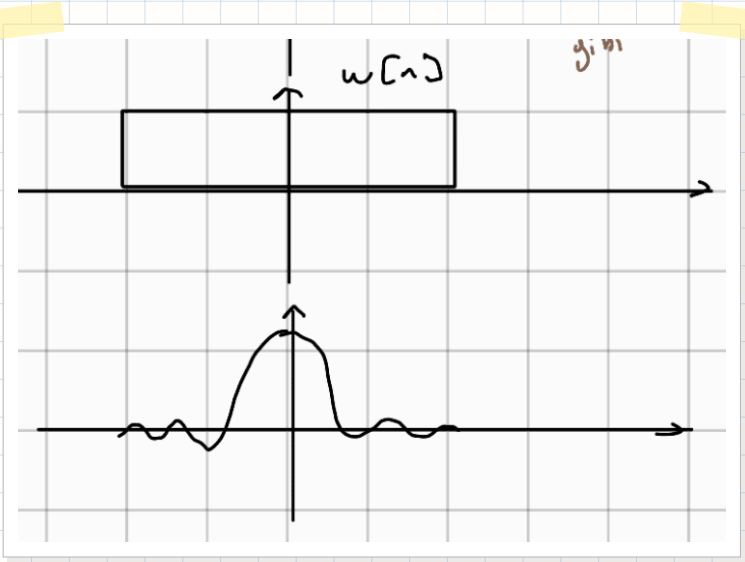
orijinal çizim, s. 1



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orijinal çizim, s. 1



orijinal çizim, s. 1

yeni bir konu sonra ilişkilendireceğiz

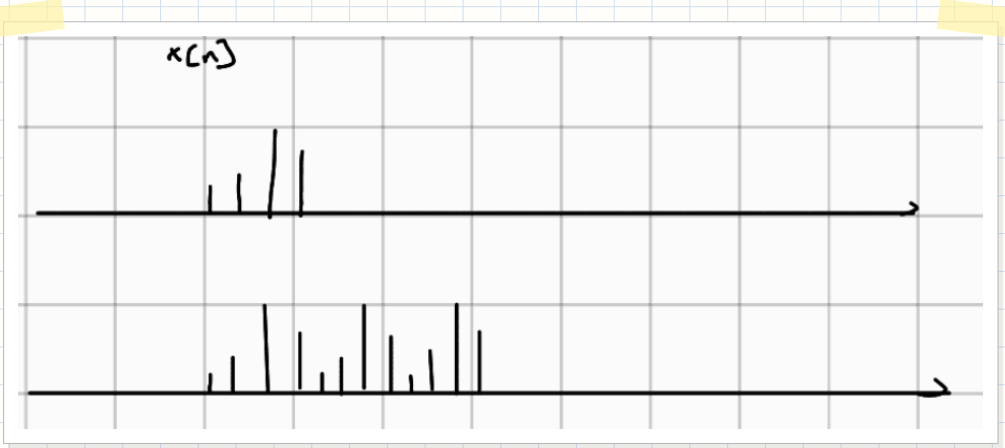
Periodic sequences

? periyodik, 1 dönem $\tilde{x}[n] = \tilde{x}[n+N]$ tüm n ler için sağlanıyorsa periyodiktir

Discrete fourier series

$$\tilde{X}[k] = \sum_{n=0}^{N-1} \tilde{x}[n]\, e^{-j\frac{2\pi}{N}kn}$$

$$\tilde{x}[n] = \frac{1}{N} \sum_{n=0}^{N-1} \tilde{X}[n]\, e^{j\frac{2\pi}{N}kn}$$



orijinal çizim, s. 1

[1] Claude notu: muhtemelen toplam indisi k ve $\tilde{X}[k]$ olmalı.

Periodic Convolution

$\tilde{x}_1[n]$, $\tilde{x}_2[n]$ with period N

DFS $\rightarrow \tilde{X}_1[k]$, $\tilde{X}_2[k]$

$$\tilde{X}_3[k] = \tilde{X}_1[k]\tilde{X}_2[k]$$

$$\tilde{x}_3[n] = \sum_{m=0}^{N-1} \tilde{x}_1[m]\tilde{x}_2[n-m]$$

periodic convolution with period N (Dairesel konvülasyon)

orj. s. 2

Linear Convolution

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$$

($y[n]$: x_3; $x[k]$: x_1; $h[n-k]$: x_2)

$$x_3[n] = \sum_{m=-\infty}^{\infty} x_1[m]\,x_2[n-m]$$

$$\tilde{X}_3[k] = \text{DFS}[\tilde{x}_1[n]] \cdot \text{DFS}[\tilde{x}_2[n]]$$

$$\tilde{X}_3[k] = \tilde{X}_1[k] \cdot \tilde{X}_2[k]$$

$$x_3[n] = \text{IFT}\Big(\text{DFT}(x_1[n])\ \text{DFT}(x_2[n])\Big)$$

($x_1[n]$: bilgisayarda; $x_2[n]$: bilgisayarda)



orijinal çizim, s. 2

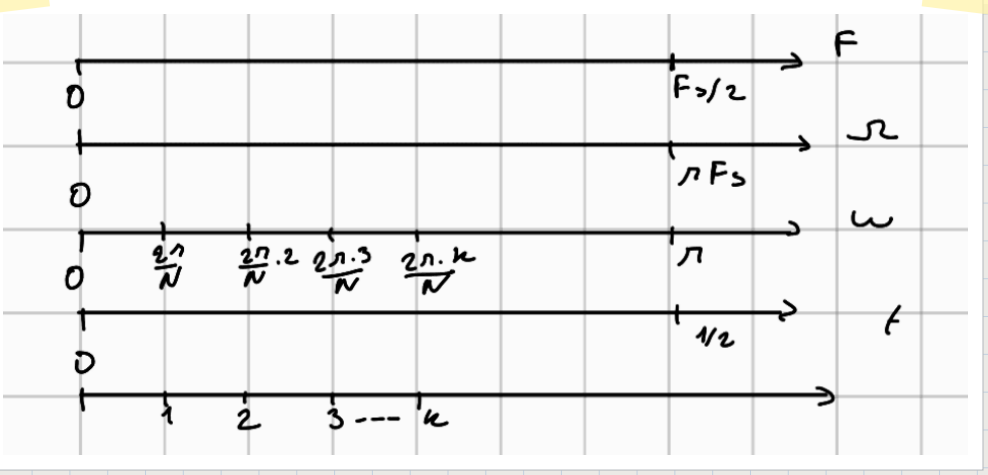
Bunun DFS'i ile bunun DFT'si arasında herhangi bir fark yok

DFT'nin formülü mecburen geldi o mecburiyet bizi DFS'e yöneltti. çıkan sonuç yine dairesel konvülasyon sonucudur.

$$\sin(2\pi 5t) = \sin(10\pi t)$$

$F_s = 100\,\text{Hz}$

$$x[n] = \sin\left(\frac{\pi}{10}n\right)$$

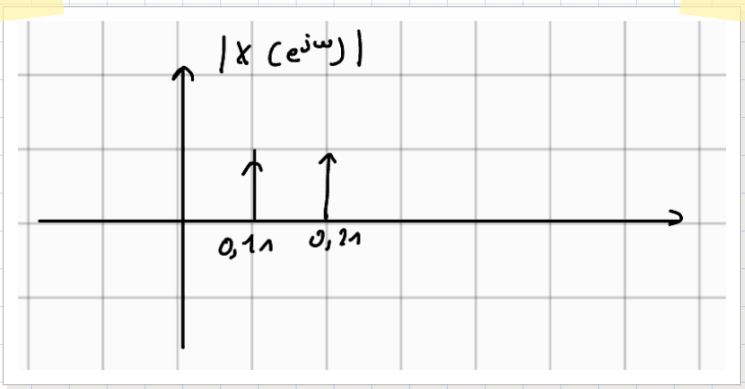


orijinal çizim, s. 2

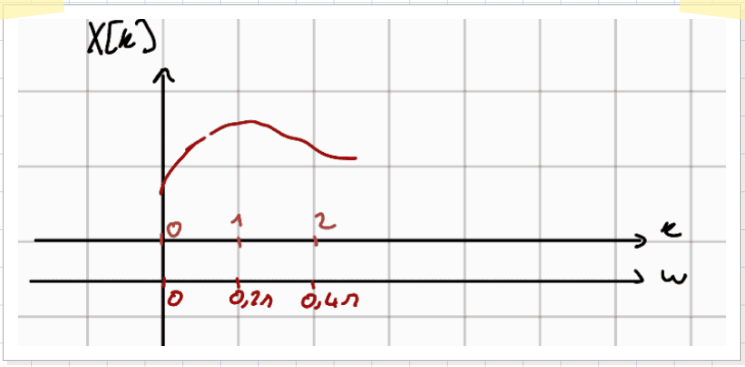
$$\omega_k = \frac{2\pi}{N}k \qquad x[n] = \sin\left(\frac{\pi}{10}n\right)$$

orj. s. 3

$$x[n] = \cos(0{,}1\pi n) + \cos(0{,}2\pi n)$$



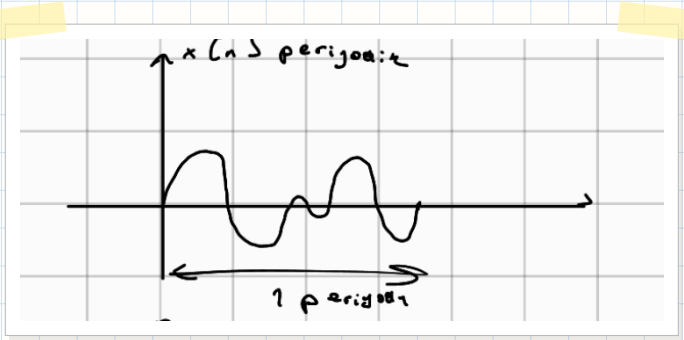
orijinal çizim, s. 3



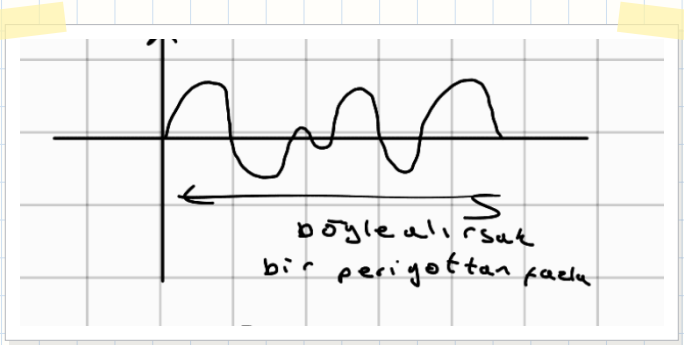
orijinal çizim, s. 3

$$N = 10 \Rightarrow \Delta\omega = \frac{2\pi}{10} = 0{,}2\pi$$

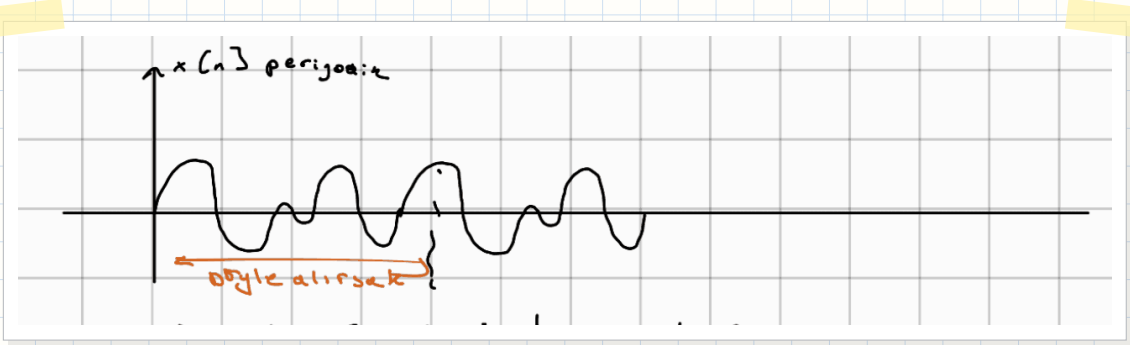
$$N = 20 \longrightarrow \Delta\omega = \frac{2\pi}{20} = 0{,}1\pi$$



orijinal çizim, s. 3

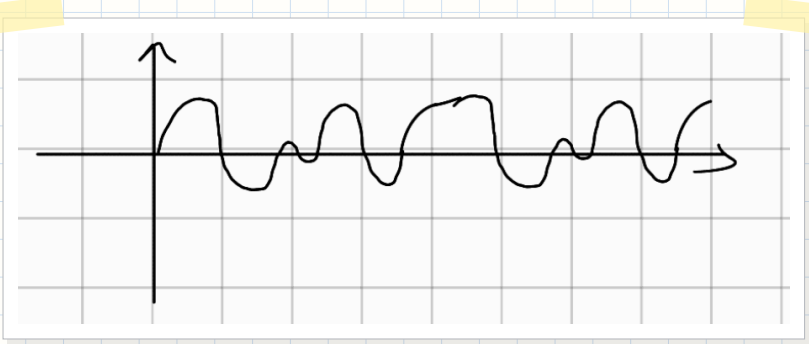


orijinal çizim, s. 3



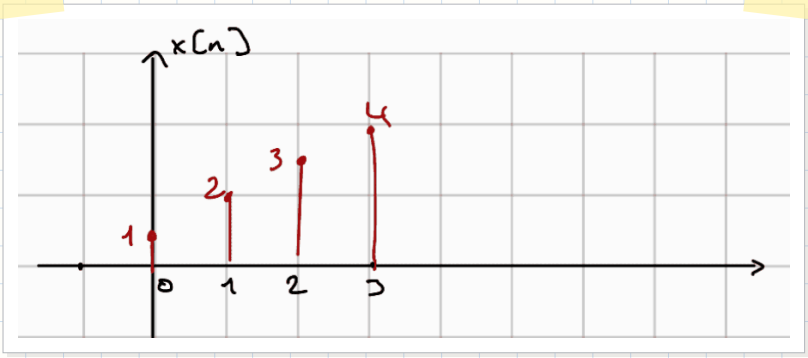
orijinal çizim, s. 3

Bu işaretin Fourieri alınmış olur



orijinal çizim, s. 3

orj. s. 4



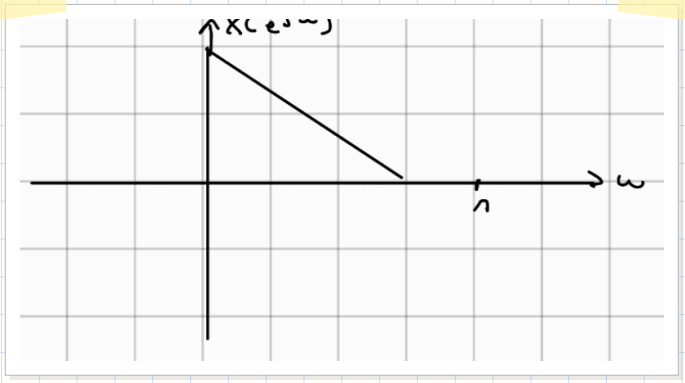
orijinal çizim, s. 4

$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi}{N}kn}$$

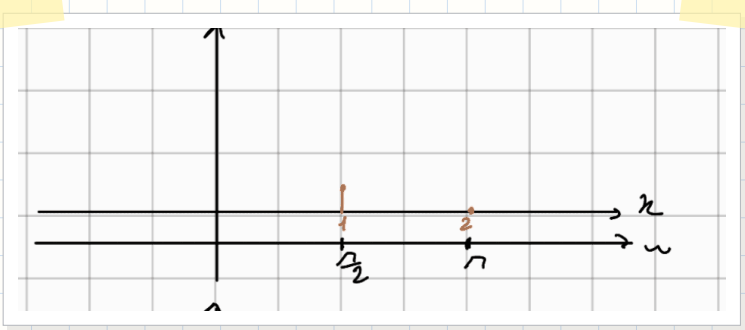
$$X[k] = \sum_{n=0}^{3} x[n]\, e^{-j\frac{2\pi}{4}kn} \qquad k = 0,1,2,3$$

$$X[k] = \sum_{n=0}^{4} x[n]\, e^{-j\frac{2\pi}{5}kn} \qquad k = 0,1,2,3,4$$

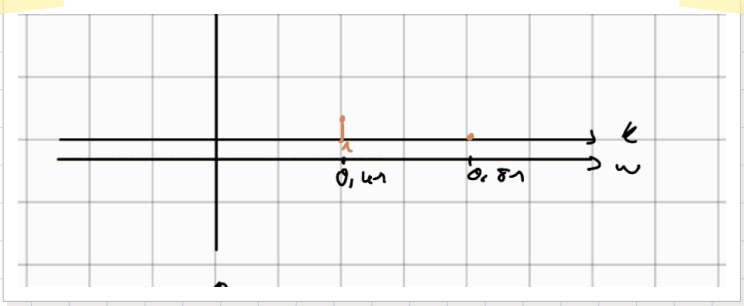
$$X[k] = \sum_{n=0}^{5} x[n]\, e^{-j\frac{2\pi}{6}kn} \qquad k = 0,1,2,3,4,5$$



orijinal çizim, s. 4



orijinal çizim, s. 4



orijinal çizim, s. 4



orijinal çizim, s. 4

orj. s. 5

Hesap yükünü azaltalım

$$X[k] = \sum_{n=0}^{N-1} x[n] \underbrace{\left(e^{-j\frac{2\pi}{N}}\right)}_{W_N}^{kn} \qquad k = 0, \ldots, N-1$$

$$= \sum_{n=0}^{N-1} x[n] W_N^{kn}$$

$$W_N = e^{-j\frac{2\pi}{N}}$$

$$y = x * h$$

$$y[n] = \sum_{\substack{k=0 \\ N \text{ tane}}} x[k]\, h[n-k]$$

$x[n]$ complex $\Rightarrow N^2$ complex mult.

$N(N-1)$ summation

for H N^2 complex multiplication

$Y = XH \rightarrow N^1$

$\hookrightarrow y$ $N^2 + N^2 + N + N^2 = 3N^2 + N$

belli koşullar altında Fourier dönüşümü ilişkisi bulmak, daha verimli olabilir

$$N = 2^v$$

	$x[0]$	$x[1]$	$x[2]$	$\cdots$	$x[N-1]$
even	$x[0]$	$x[2]$	$x[4]$	$\cdots$	$x[N-2]$
odd	$x[1]$	$x[3]$	$x[5]$	$\cdots$	$x[N-1]$

$$X[k] = \sum_{n=0}^{N-1} x[n]\, W_N^{kn} \qquad k = 0, \ldots, N-1$$

$$= \sum_{\substack{n=0 \\ n \text{ even} \\ \text{let } n=2r}}^{N-2} x[n]\, W_N^{kn} + \sum_{\substack{n=1 \\ n \text{ odd} \\ n=2r+1}}^{N-1} x[n]\, W_N^{kn}$$

$$n = 2r+1 \Rightarrow r = \frac{n-1}{2} \qquad \begin{aligned} n = 1 &\Rightarrow r = 0 \\ n = N-1 &\Rightarrow r = \frac{N}{2} - 1 \end{aligned}$$

$$= \sum_{r=0}^{N/2-1} x[2r]\, \underbrace{W_N^{2rk}}_{W_{N/2}^{rk}} + \sum_{r=0}^{N/2-1} x[2r+1]\, \underbrace{W_N^{(2r+1)k}}_{\substack{W_N^{2rk}\, W_N^k \\ W_{N/2}^{rk}}}$$

$$r = 0,1,2,\ldots$$

$$W_N = e^{-j\frac{2\pi}{N}}$$

$$(W_N)^{2rk} = \left(e^{-j\frac{2\pi}{N}}\right)^{2rk} = \underbrace{e^{-j\frac{2\pi}{N/2}rk}}_{W_{N/2}} = W_{N/2}^{rk}$$

$$= \underbrace{\sum_{r=0}^{N/2-1} x[2r]\, W_{N/2}^{rk}}_{\frac{N}{2} \text{ noktalı DFT}} + W_N^k \underbrace{\sum_{r=0}^{N/2-1} x[2r+1]\, W_{N/2}^{rk}}_{\frac{N}{2} \text{ point DFT}}$$

$$\underbrace{\frac{N}{2}\frac{N}{2} + \frac{N}{2}\frac{N}{2}}_{N^2/2}$$

orj. s. 6

FFT

$O(N \log_2 N)$

? N^2

N	N^2	$N \log_2 N$
4	16	8
16	256	64
1024	1E6	10240