Sayısal İşaret İşleme

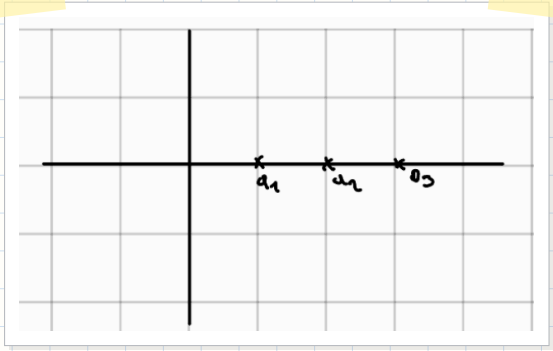
Z dönüşümü, fark denklemi ve örnekleme

Orijinal not: Dsp-ders-6.pdf

UYARI Bu belge, el yazısı ders notlarından Claude (Anthropic) yardımıyla dijitale aktarılmıştır. Yardımcı kaynak olarak hazırlanmıştır; hata veya eksik içerebilir. Lütfen orijinal notları ve ders kaynaklarını esas alın.

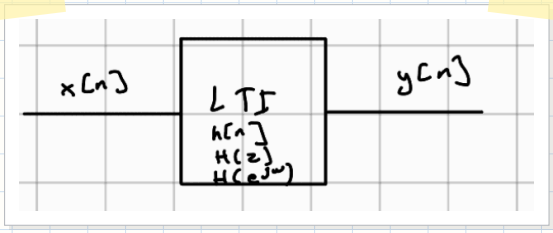
NOTICE *This document was transcribed from handwritten lecture notes with the help of Claude (Anthropic). It is meant as a supplementary resource and may contain errors or omissions. Always check against the original notes and official course material.*

orj. s. 1



orijinal çizim, s. 1

$$|z| > a_3$$
$$a_2 < |z| < a_3$$
$$a_1 < |z| < a_2$$
$$|z| < a_1$$



orijinal çizim, s. 1

$$y[n] = x[n] * h[n]$$
$$Y(z) = X(z)\,H(z)$$
$$H(z) = \frac{Y(z)}{X(z)}$$

$$y[n] = -\sum_{k=1}^{N} a_k\, y[n-k] + \sum_{k=0}^{M} b_k\, x[n-k]$$

$$Y(z) = -\sum_{k=1}^{N} a_k z^{-k}\, Y(z) + \sum_{k=0}^{M} b_k z^{-k}\, X(z)$$

$$Y(z)\left[1 + \sum_{k=1}^{N} a_k z^{-k}\right] = X(z)\left[\sum_{k=0}^{M} b_k z^{-k}\right]$$

$$\Rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{k=0}^{N} a_k z^{-k}} \qquad a_0 = 1 \text{ olmalıdır}$$

$$y[n] = \tfrac{1}{2}\, y[n-1] + 2x[n] - x[n-2]$$

$$Y(z) = \tfrac{1}{2} z^{-1} Y(z) + 2X(z) - z^{-2} X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{2 - z^{-2}}{1 - \frac{1}{2} z^{-1}} = 2z^{-1} + 4 + \frac{-2}{1 - \frac{1}{2} z^{-1}}$$

$$\Rightarrow h[n] = 2\delta[n-1] + 4\delta[n] - 2\left(\tfrac{1}{2}\right)^n u[n]$$

fark denklemi aksi belirtilmedikçe nedensel olarak kabul edilir

orj. s. 2

Örnek

$$y[n] = 0{,}5\, y[n-1] + x[n]$$

$$x[n] = 10\cos\left(\tfrac{\pi}{4} n\right) \underbrace{u[n]}_{\text{transient}} \qquad \rightarrow \text{sağ yanlı}$$

$$y[n] = ?$$

$$H(z) = \frac{1}{1 - 0{,}5 z^{-1}} \qquad |z| > \tfrac{1}{2}$$

$$h[n] = \left(\tfrac{1}{2}\right)^n u[n]$$

$$r^n x[n] \longleftrightarrow ?$$

$$\cos(\omega n)\, u[n] \longleftrightarrow$$

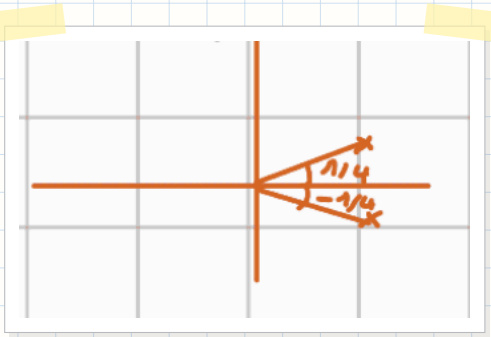
$$\frac{e^{j\omega n} + e^{-j\omega n}}{2}\, u[n]$$

$$\tfrac{1}{2} e^{j\omega n} u[n] + \tfrac{1}{2} e^{-j\omega n} u[n]$$

$$\tfrac{1}{2} a_1^n u[n] + \tfrac{1}{2} a_2^n u[n]$$

$$X(z) = \frac{10\left(1 - \frac{1}{\sqrt{2}} z^{-1}\right)}{1 - \sqrt{2}\, z^{-1} + z^{-2}}$$

$$\left(1 - e^{j\pi/4} z^{-1}\right)\left(1 - e^{-j\pi/4} z^{-1}\right) \rightarrow |z| > 1$$



orijinal çizim, s. 2

$$ax^2 + bx + c \qquad \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$Y(z) = H(z)\,X(z)$$

$$Y(z) = \frac{10\left(1 - \frac{1}{\sqrt{2}}z^{-1}\right)}{(1 - 0{,}5z^{-1})\left(1 - \sqrt{2}\,z^{-1} + z^{-2}\right)}$$

$$= \frac{10\left(1 - \frac{1}{\sqrt{2}}z^{-1}\right)}{(1 - 0{,}5z^{-1})\left(1 - e^{j\pi/4}z^{-1}\right)\left(1 - e^{-j\pi/4}z^{-1}\right)}$$

$$= \frac{A}{1 - 0{,}5z^{-1}} + \frac{B}{1 - e^{j\pi/4}z^{-1}} + \frac{C}{1 - e^{-j\pi/4}z^{-1}}$$

? kısmi kesirler yöntemi

$$A = Y(z)\left(1 - 0{,}5z^{-1}\right)\Big|_{z=1/2}$$

$$y[n] = 6{,}3\,(0{,}5)^n u[n] + 6{,}78\,e^{-j28{,}7^\circ}\left(e^{j\pi/4}\right)^n u[n]$$
$$+ 6{,}78\,e^{j28{,}7^\circ}\left(e^{-j\pi/4}\right)^n u[n]$$

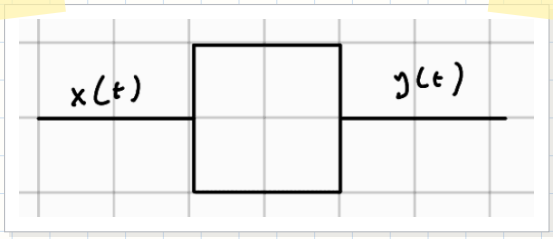
$$= \underbrace{6{,}3\,(0{,}5)^n u[n]}_{\text{transient cevabı}} + \underbrace{13{,}56\cos\left(\tfrac{\pi}{4}n - 28{,}7^\circ\right)u[n]}_{\text{girişin değişmiş hali, steady state}}$$

$$\sum |h[n]| = \sum_{n=-\infty}^{\infty}\left|\left(\tfrac{1}{2}\right)^n u[n]\right| = \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^n = \frac{1}{1 - \frac{1}{2}} = 2$$

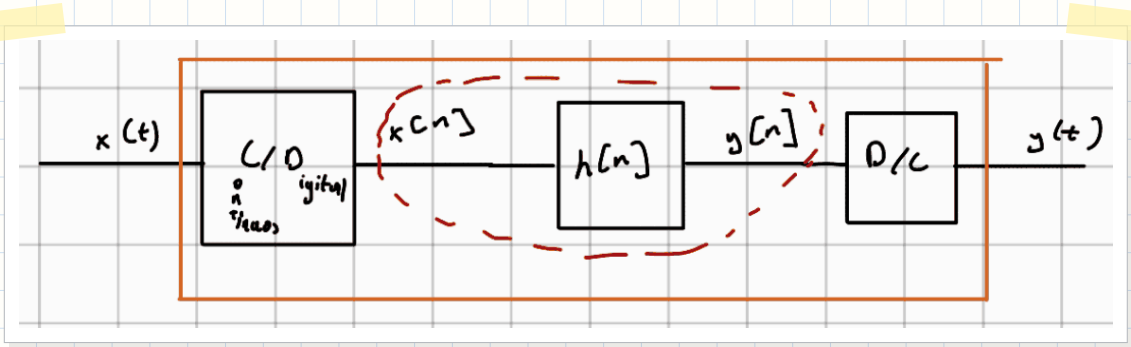
$$= 1 + \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{8}$$

orj. s. 3

Sampling

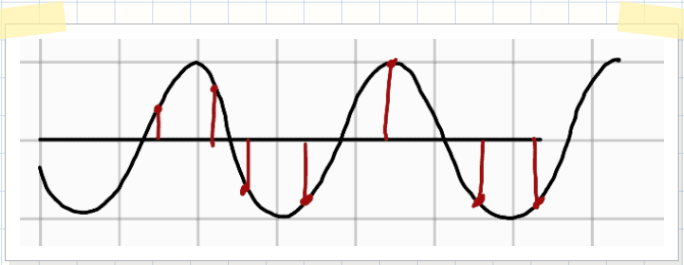


orijinal çizim, s. 3

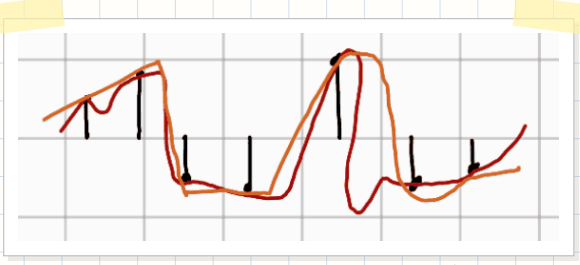


orijinal çizim, s. 3

$$x(t)\Big|_{t=nT} = x(nT) = x[n]$$



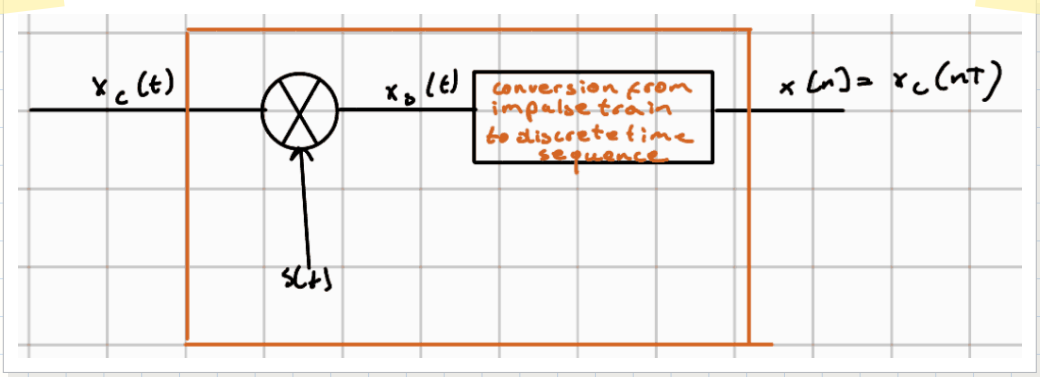
orijinal çizim, s. 3



orijinal çizim, s. 3

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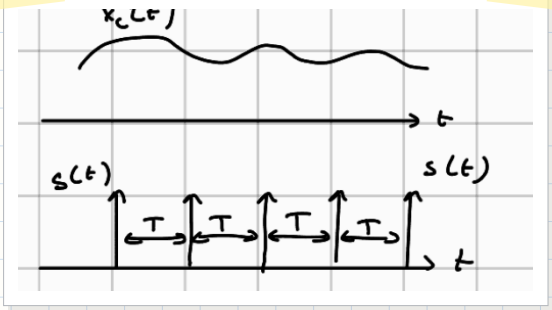
$$x(t) = \cos(12\pi t)$$
$$x[n] = \cos\left(\frac{\pi}{2}n\right) \quad \longleftarrow F_s = 24 \text{ Hz}$$
$$x[n] = \cos(2\pi n) = 1 \quad \longleftarrow F_s = 6 \text{ Hz}$$



orijinal çizim, s. 3

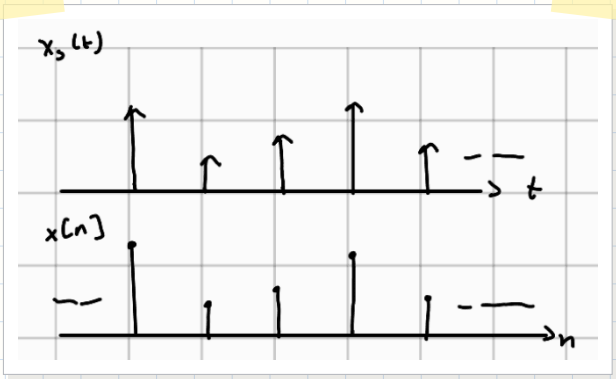
$$s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

$$x_s(t) = x_c(t) \cdot s(t)$$

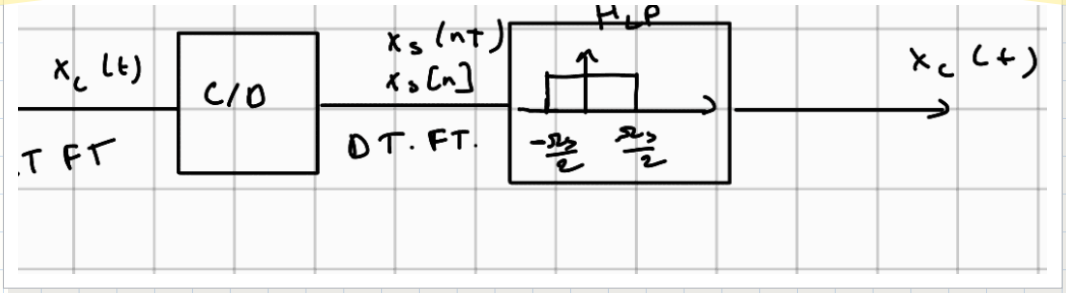


orijinal çizim, s. 3

orj. s. 4



orijinal çizim, s. 4



orijinal çizim, s. 4

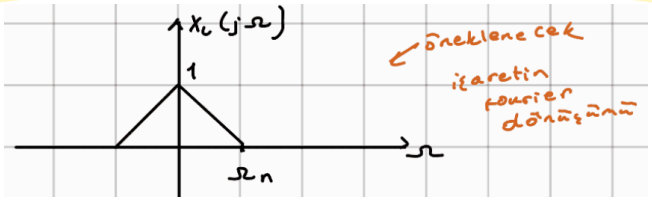
$$X_c(j\Omega) = \int_{-\infty}^{\infty} x_c(t)\, e^{-j\Omega t}\, dt$$

$$X_s(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x_s[n]\, e^{-j\omega n}$$

$$X_s(e^{j\omega}) = \frac{1}{T}\sum_{k} X_c(j(\Omega - k\Omega_s)) \qquad \rightarrow\ \Omega_s = 2\pi f_s,\ T = \frac{1}{f_s}$$

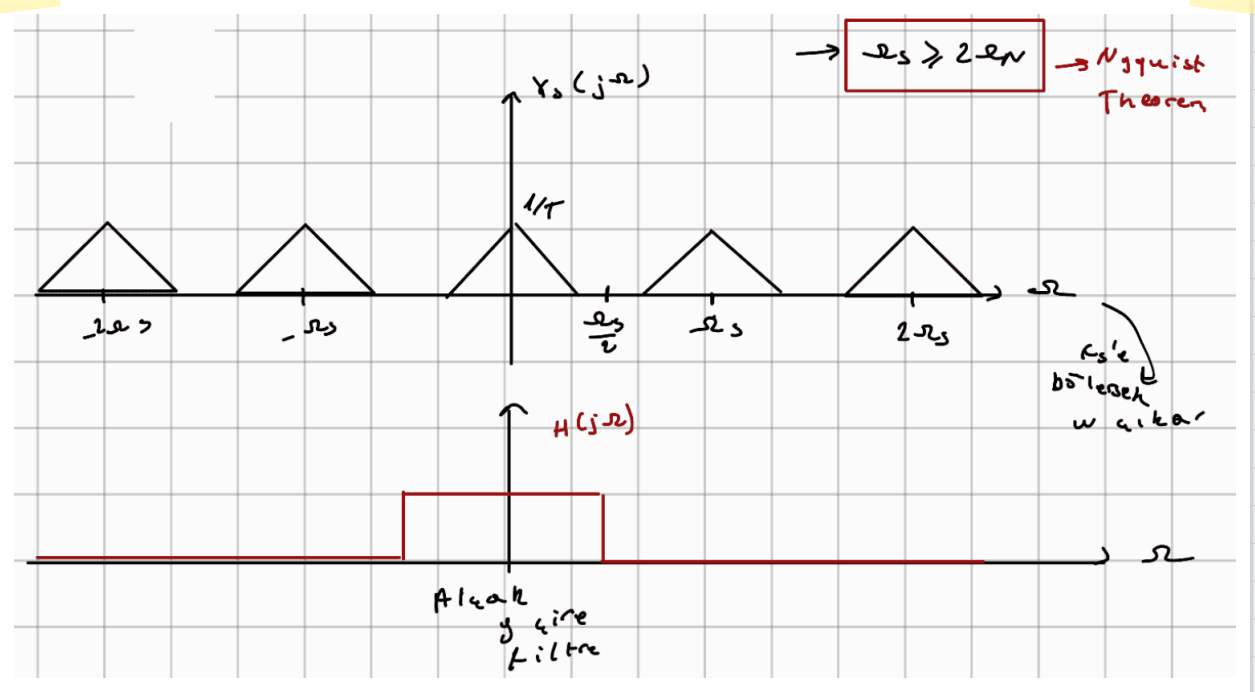
$$= \frac{1}{T}\sum_{k} X_c\left(\frac{\omega - 2\pi k}{T}\right) \qquad \text{olduğunu kabul edelim}$$

$$X_s(e^{j\omega}) = \frac{1}{T}\Big[\cdots + X_c(j(\Omega + 2\Omega_s)) + X_c(j(\Omega + \Omega_s)) + X_c(j\Omega) + X_c(j(\Omega - \Omega_s)) + X_c(j(\Omega - 2\Omega_s)) + \cdots\Big]$$



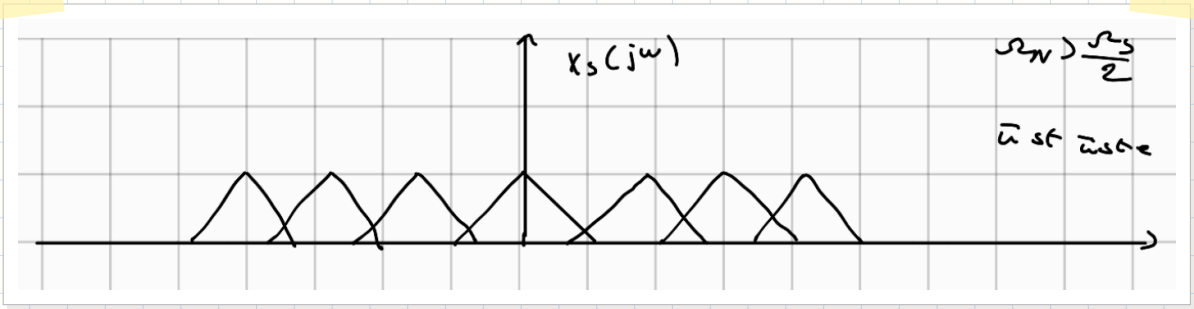
orijinal çizim, s. 4

$$\Omega_n < \frac{\Omega_s}{2}$$

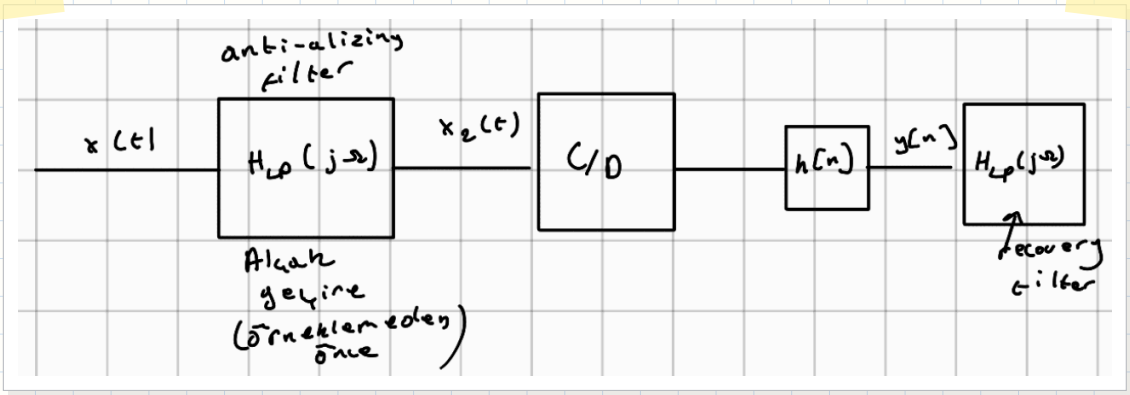


orijinal çizim, s. 4

orj. s. 5

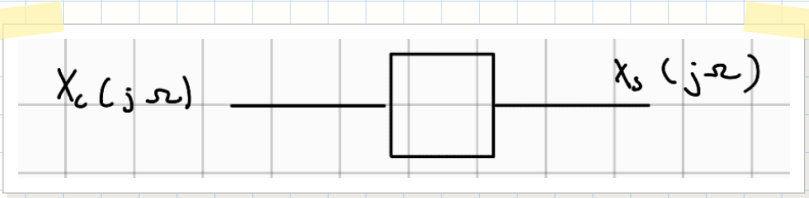


orijinal çizim, s. 5



orijinal çizim, s. 5

örtüşme aslında iki farklı işaretin örneklerinin aynı olması.



orijinal çizim, s. 5

$$x_s(t) = x_c(t)\,s(t) \qquad s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT) \qquad \Omega_s = \frac{2\pi}{T} = 2\pi f_s$$

$$X_s(j\Omega) = \frac{1}{2\pi} X_c(j\Omega) * S(j\Omega) \quad \rightarrow \; S(j\Omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\Omega - k\Omega_s)$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} X_c(j\theta)\, S(j(\Omega - \theta))\, d\theta \qquad x(t) * h(t) = \int x(\tau)\, h(t - \tau)\, d\tau$$

$$= \frac{1}{2\pi} \int X_c(j\theta) \cdot \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\Omega - \theta - k\Omega_s)\, d\theta$$

$$= \frac{1}{T} \sum_{k=-\infty}^{\infty} X_c(j(\Omega - k\Omega_s))$$

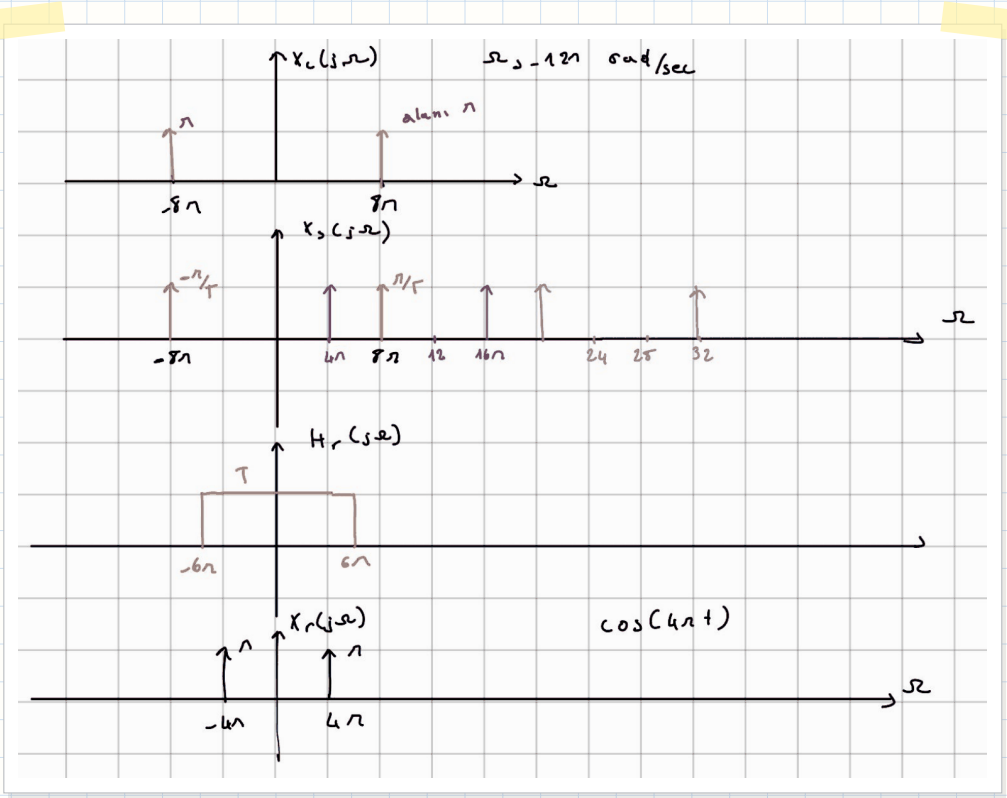
$$\omega = \frac{\Omega}{F_s} \implies \Omega = \omega \cdot F_s \implies \Omega = \frac{\omega}{T}$$

$$X_s(j\omega) = \frac{1}{T} \sum_k X_c\left(j\,\frac{\omega - 2\pi k}{T}\right)$$

? 2π

orj. s. 6

$x_c(t) = \cos(8\pi t) \qquad F_s = 6\ \text{Hz} \qquad \Omega_s = 12\pi\ \text{rad/sec}$



orijinal çizim, s. 6

örtüşme var cos(4πt) ve cos(8πt) örnekleri aynı oldu



orijinal çizim, s. 6

$$\cos(8\pi t)\Big|_{t=n\frac{1}{6}} = \cos\left(\frac{4\pi}{3}n\right)$$

$$\cos(4\pi t)\Big|_{t=n\frac{1}{6}} = \cos\left(\frac{2\pi}{3}n\right)$$