Sayısal İşaret İşleme

Altörnekleme, üstörnekleme ve LTI frekans yanıtı

Orijinal not: Dsp-ders-7.pdf

UYARI Bu belge, el yazısı ders notlarından Claude (Anthropic) yardımıyla dijitale aktarılmıştır. Yardımcı kaynak olarak hazırlanmıştır; hata veya eksik içerebilir. Lütfen orijinal notları ve ders kaynaklarını esas alın.

NOTICE *This document was transcribed from handwritten lecture notes with the help of Claude (Anthropic). It is meant as a supplementary resource and may contain errors or omissions. Always check against the original notes and official course material.*

orj. s. 1



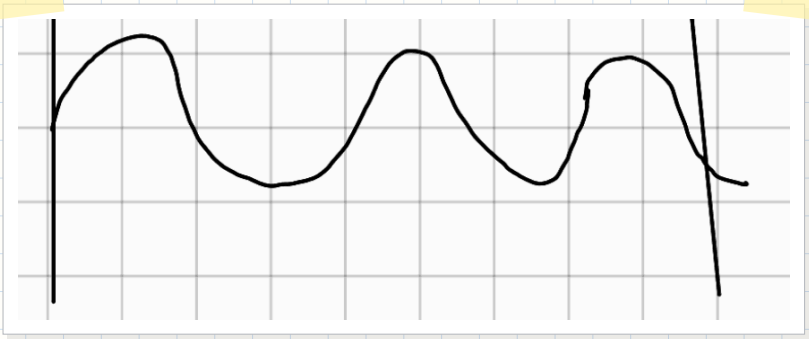
orijinal çizim, s. 1

orijinal

$F_s = 200$ Hz

$T_{yeni} = \overset{\text{tamsayı}}{M}\, T = T'$

$F_{s,yeni} = 100$ Hz $\quad M = 2$



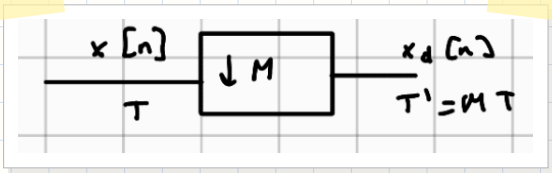
orijinal çizim, s. 1

$$x[n] = x(nT)$$

$$x_d[n] = x[nM]$$

down samples

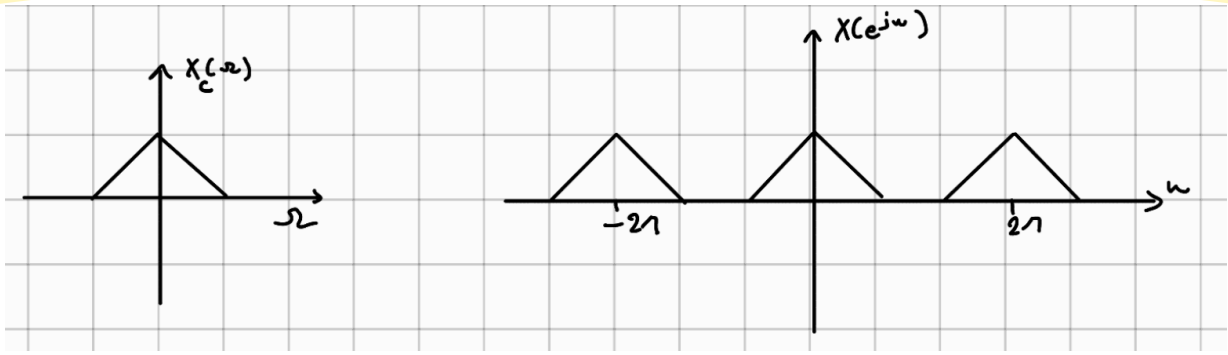
	$M = 2$	$M = 3$
$x[0] = A$	$x_d[0] = A$	$x_d[0] = A$
$x[1] = B$	$x_d[1] = C$	$x_d[1] = D$
$x[2] = C$	$x_d[2] = E$	$x_d[2] = G$
$x[3] = D$		
$x[4] = E$		
$x[5] = F$		
$x[6] = G$		



orijinal çizim, s. 1

$$x[n] \longleftrightarrow X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$

$$X(e^{j\omega}) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X_c\left(j\left(\underbrace{\frac{\omega}{T}}_{\Omega} - \underbrace{\frac{2\pi k}{T}}_{\Omega_s}\right)\right)$$



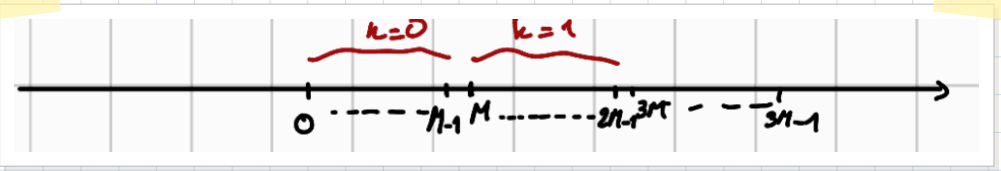
orijinal çizim, s. 1

$$X_d(e^{j\omega}) = \frac{1}{T'} \sum_{k=-\infty}^{\infty} X_c\left(j\left(\frac{\omega}{T'} - \frac{2\pi k}{T'}\right)\right)$$

$$= \frac{1}{MT} \sum_{r=-\infty}^{\infty} X_c\left(j\left(\frac{\omega}{MT} - \frac{2\pi r}{MT}\right)\right) \qquad r \text{ yazalım}$$

orj. s. 2

$$r = i + kM$$
$$0 \le i \le M - 1$$
$$-\infty < k \le +\infty$$



orijinal çizim, s. 2

Görüldüğü üzere $r = -\infty$ ile $+\infty$ arasında değer alıyor

$$X_d(e^{j\omega}) = \frac{1}{MT}\sum_{r=-\infty}^{\infty} X_c\left(j\left(\frac{\omega}{MT} - \frac{2\pi r}{MT}\right)\right)$$

$$= \frac{1}{M}\sum_{i=0}^{M-1}\frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\left(j\left(\frac{\omega}{MT} - \frac{2\pi(i+kM)}{MT}\right)\right)$$

$$\frac{2\pi i + 2\pi kM}{MT} = \frac{2\pi i}{MT} + \frac{2\pi k}{T}$$

$$= \frac{1}{M}\sum_{i=0}^{M-1}\frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\left(j\left(\frac{\omega - 2\pi i}{MT} - \frac{2\pi k}{T}\right)\right)$$

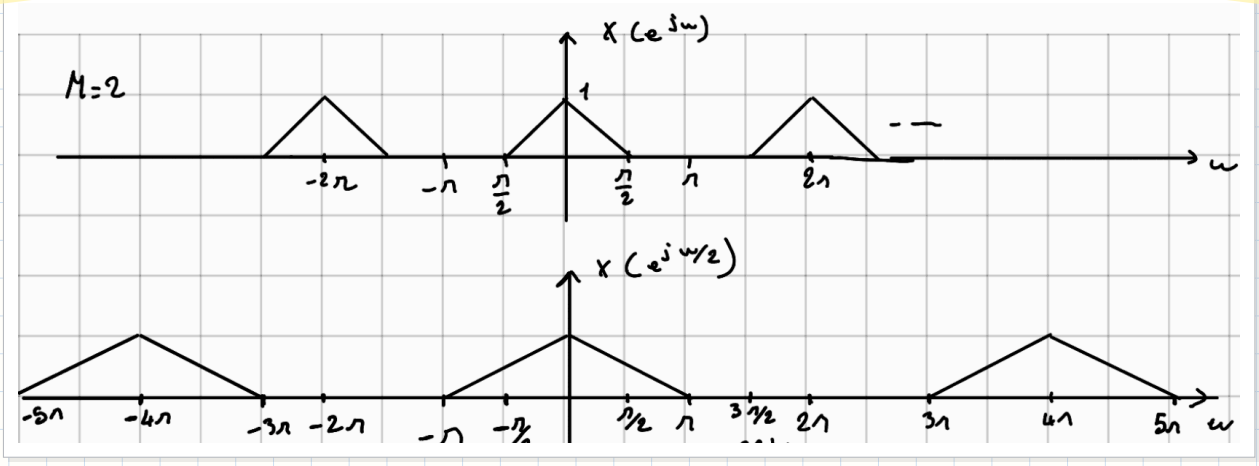
$$X_d(e^{j\omega}) = \frac{1}{M}\sum_{i=0}^{M-1} X(e^{j\omega})\Big|_{\omega=\frac{\omega-2\pi i}{M}}$$

$$X_d(e^{j\omega}) = \frac{1}{M}\sum_{i=0}^{M-1} X\left(e^{j\frac{\omega-2\pi i}{M}}\right)$$

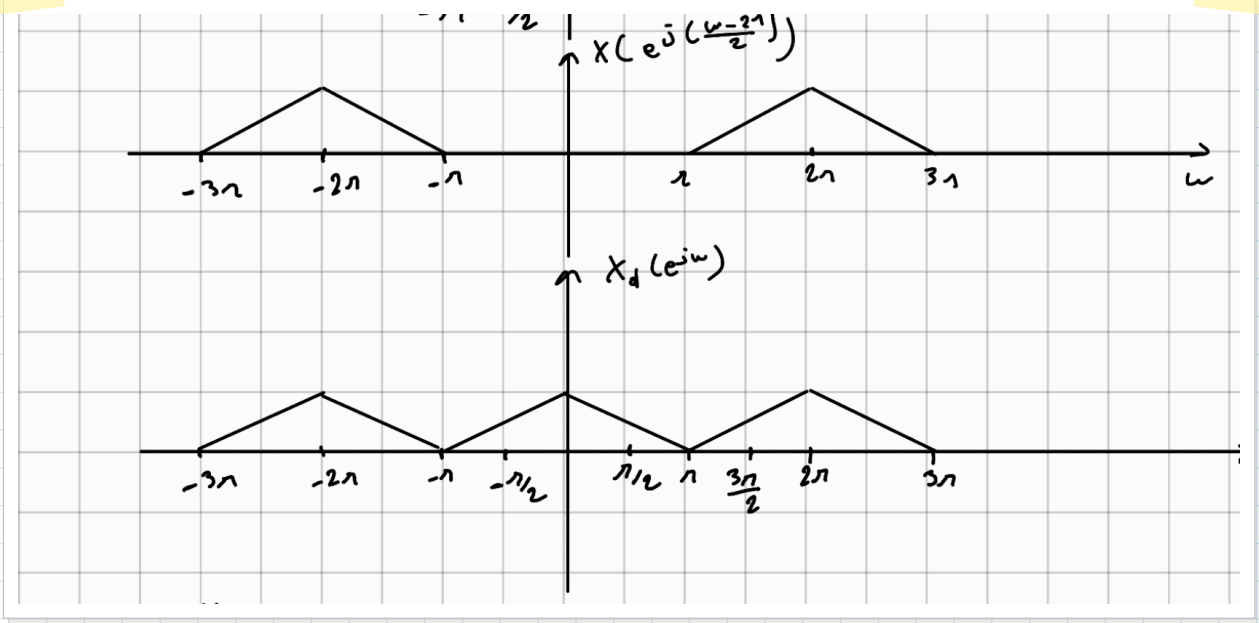
$$M = 2 \rightarrow X_d(e^{j\omega}) = \frac{1}{2}X\left(e^{j\frac{\omega}{2}}\right) + \frac{1}{2}X\left(e^{j\frac{\omega-2\pi}{2}}\right)$$

$$X_d(e^{j\omega}) = \frac{1}{3}X\left(e^{j\frac{\omega}{3}}\right) + \frac{1}{3}X\left(e^{j\frac{\omega-2\pi}{3}}\right) + \frac{1}{3}X\left(e^{j\frac{\omega-4\pi}{3}}\right)$$

orj. s. 3



orijinal çizim, s. 3

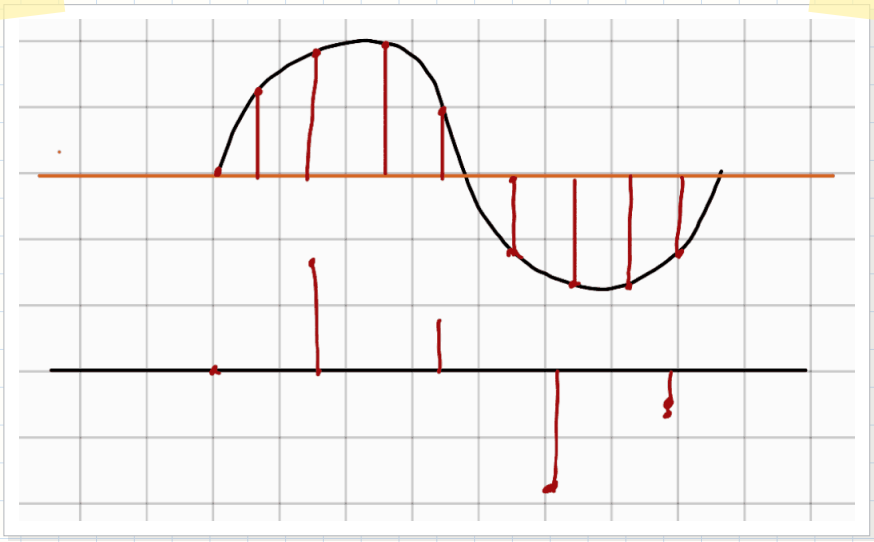


orijinal çizim, s. 3

$M = 2$ için $BG = \frac{\pi}{2}$

$M = 3$ " $BG = \frac{\pi}{3}$

M " $BG = \frac{\pi}{M}$ olmalı



orijinal çizim, s. 3

frekans artıyor.

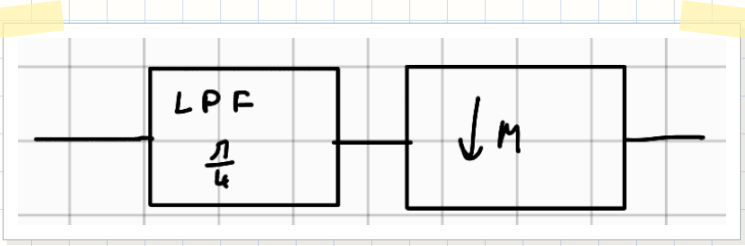
⚠ Dikkat!

* İşaretin bant genişliği en fazla $\frac{\pi}{M}$ kadar olmalı,

Seyrek örneklediğimizde geçişler keskinleşiyor, frekans artıyor.

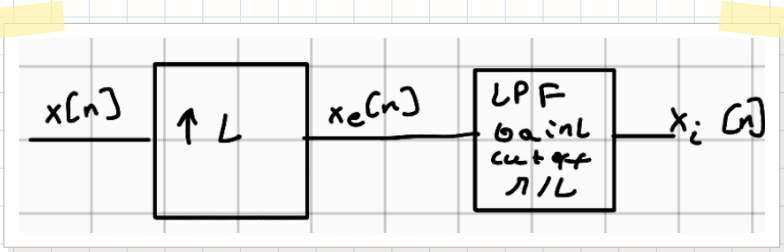
orj. s. 4

Down sampling



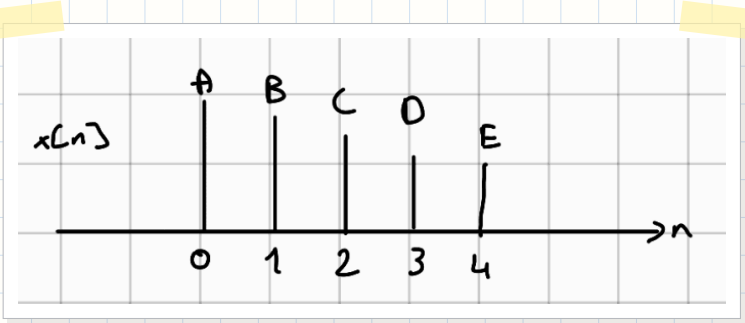
orijinal çizim, s. 4

Up sampling



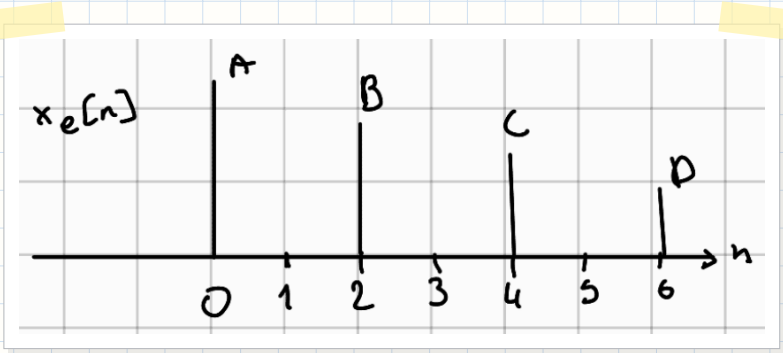
orijinal çizim, s. 4

$$x_e[n] \triangleq \begin{cases} x\left[\frac{n}{L}\right], & n = 0, \pm L, \pm 2L, \pm 3L \\ 0, & \text{o.w} \end{cases}$$



orijinal çizim, s. 4

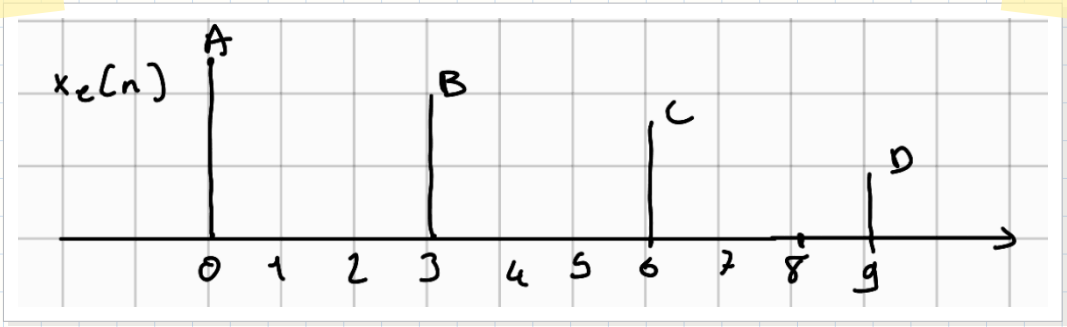
$L = 2$



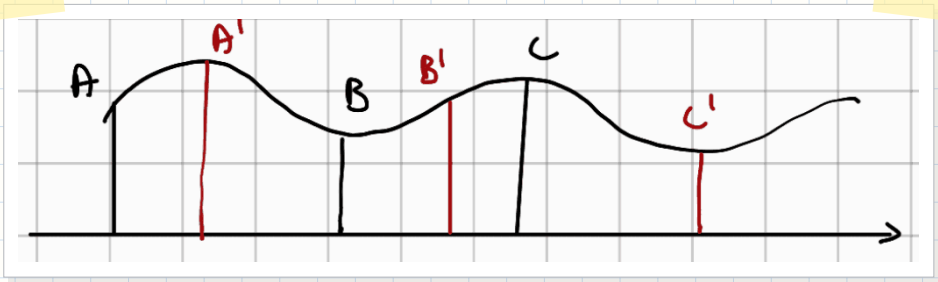
orijinal çizim, s. 4

araya $L-1$ ekliyor

$L = 3$



orijinal çizim, s. 4



orijinal çizim, s. 4

$$x_e[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-kL]$$

$$X_e(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x_e[n]\,e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \left(\sum_{k=-\infty}^{\infty} x[k]\,\delta[n-kL] \right) e^{-j\omega n}$$

orj. s. 5

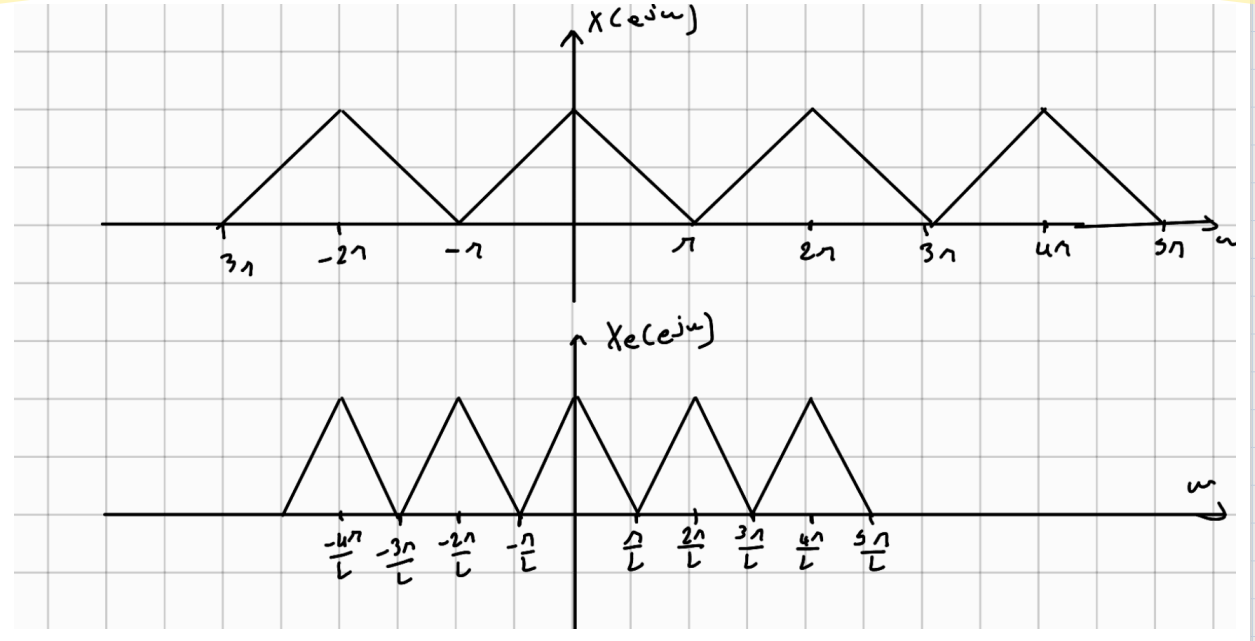
$$= \sum_{k=-\infty}^{\infty} x[k] \sum_{n=-\infty}^{\infty} \delta[n-kL]\,e^{-j\omega n}$$

$$X_e(e^{j\omega}) = \sum_{k=-\infty}^{\infty} x[k]\,e^{-j\omega Lk}$$

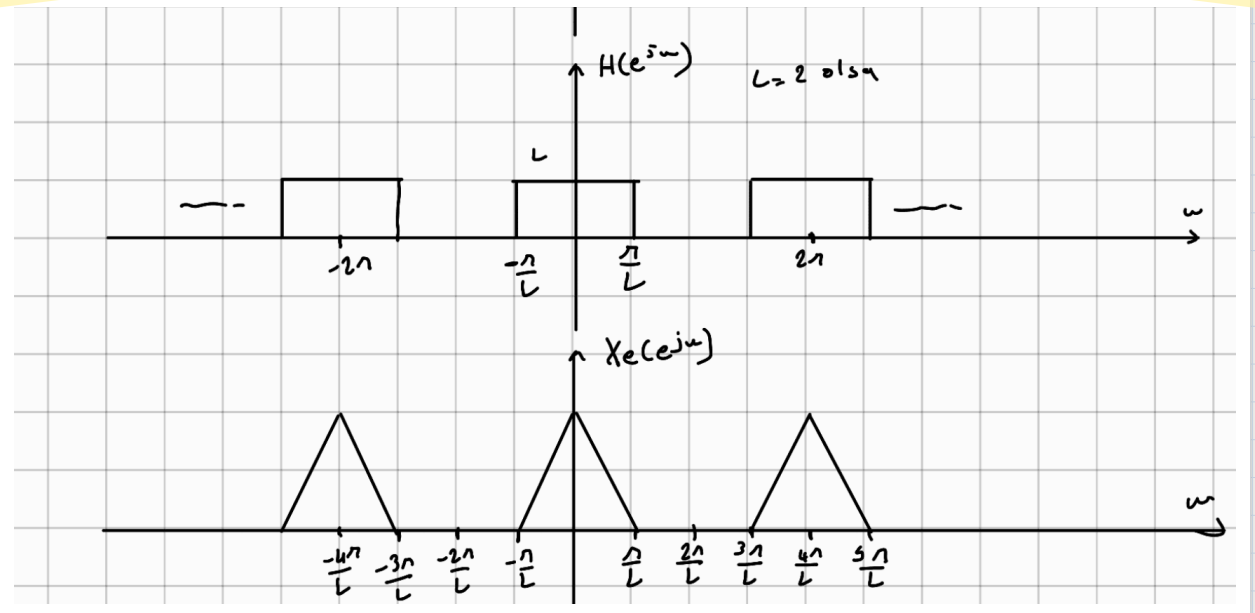
$$X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} x[k]\, e^{-j\omega k}$$

$$X_e(e^{j\omega}) = X(e^{j\omega})\Big|_{\omega \leftarrow \omega L}$$

$$X_e(e^{j\omega}) = X(e^{j\omega L})$$



orijinal çizim, s. 5



orijinal çizim, s. 5

orj. s. 6

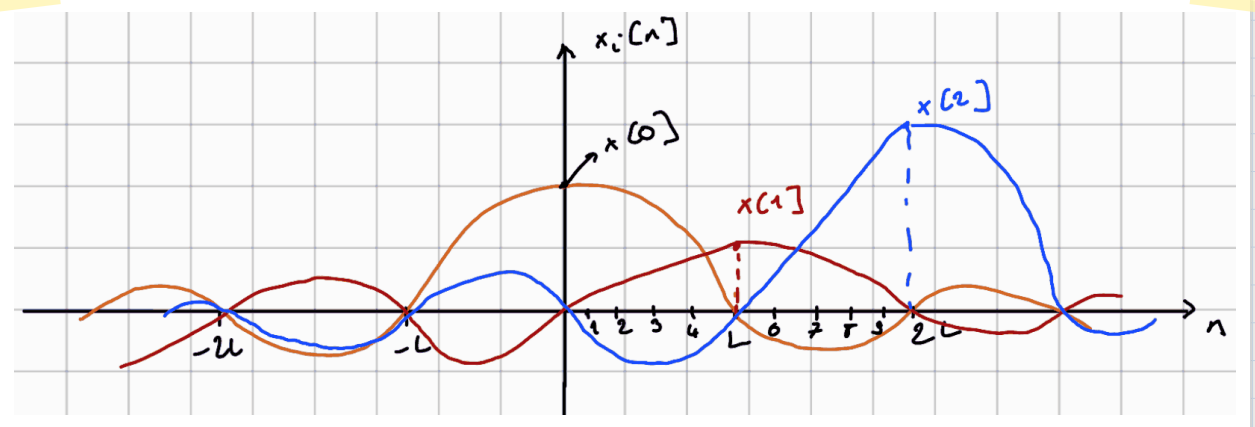
$$x_i[n] = x_e[n] * h[n]$$

$$h[n] = \frac{\sin(\pi n/L)}{\pi n/L}$$

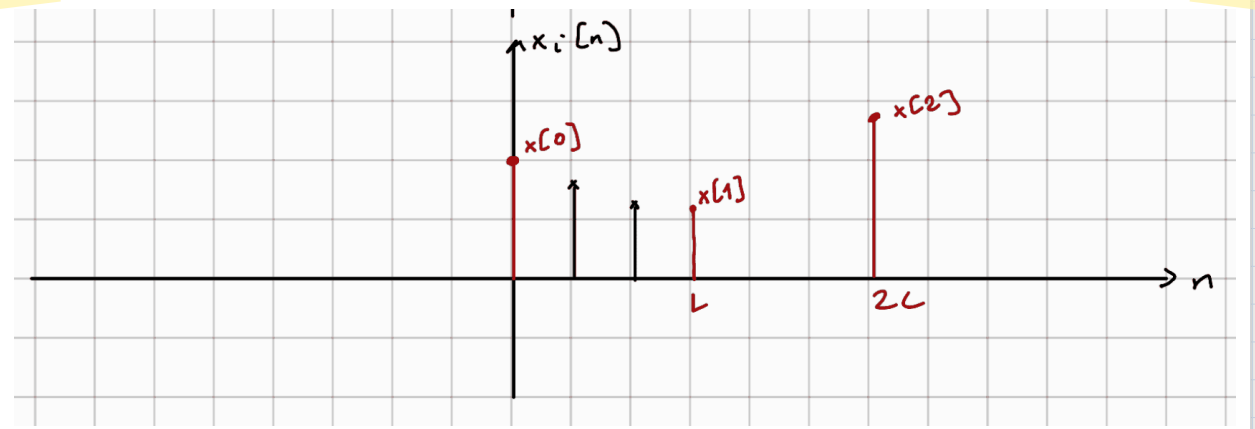
$$x_i[n] = \sum_{k=-\infty}^{\infty} x_e[k]\,h[n-k] \qquad = \sum_{k=-\infty}^{\infty} x[k]\,h[n-kL]$$

$$= \cdots + \overbrace{x_e[0]\,h[n]}^{x[0]} + \overbrace{x_e[1]\,h[n-1]}^{0} + \overbrace{x_e[2]\,h[n-2]}^{0} + x_e[3]\,h[n-3]$$
$$+ \overbrace{x_e[L]\,h[n-L]}^{x[1]} + \cdots$$
$$+ \overbrace{x_e[2L]\,h[n-2L]}^{x[2]} + \cdots$$
$$\vdots$$

$$x_i[n] = \sum_{k=-\infty}^{\infty} x[k]\,\frac{\sin \pi(n-kL)/L}{\pi(n-kL)/L}$$



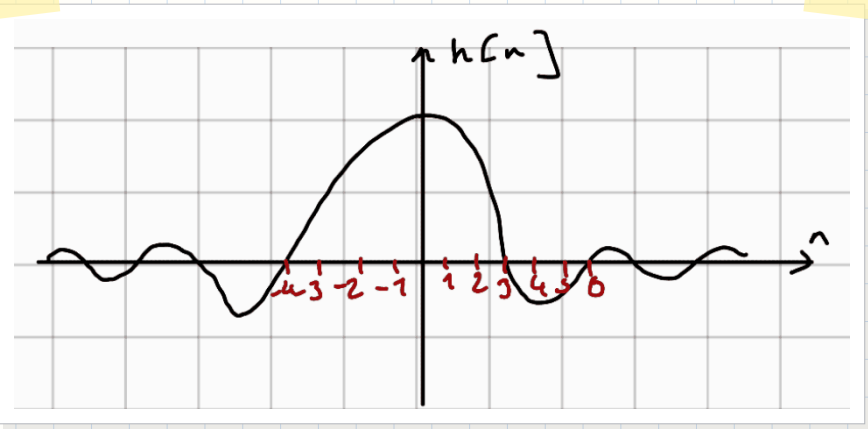
orijinal çizim, s. 6



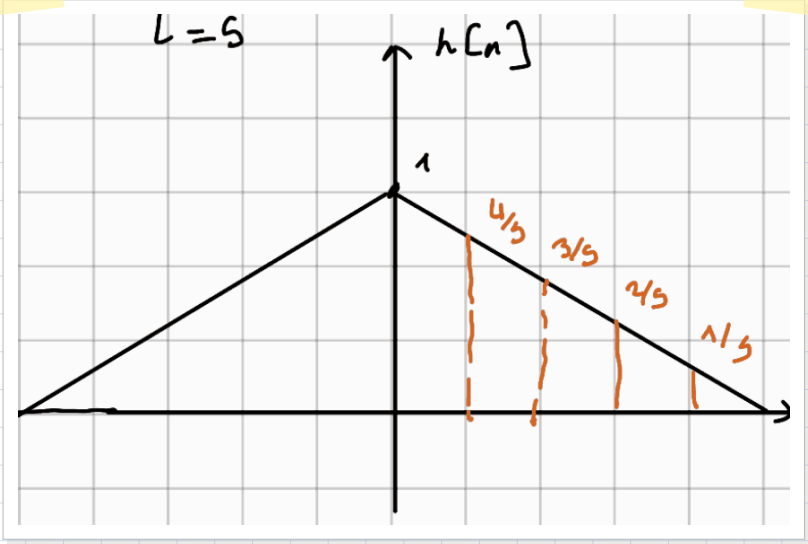
orijinal çizim, s. 6

orj. s. 7

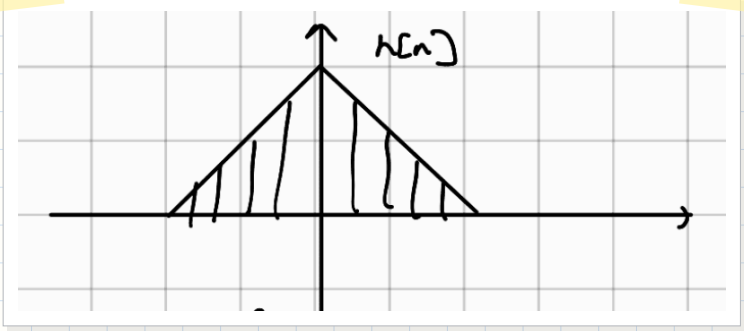
$$x_i[n] = x_e[n] * h[n]$$



orijinal çizim, s. 7

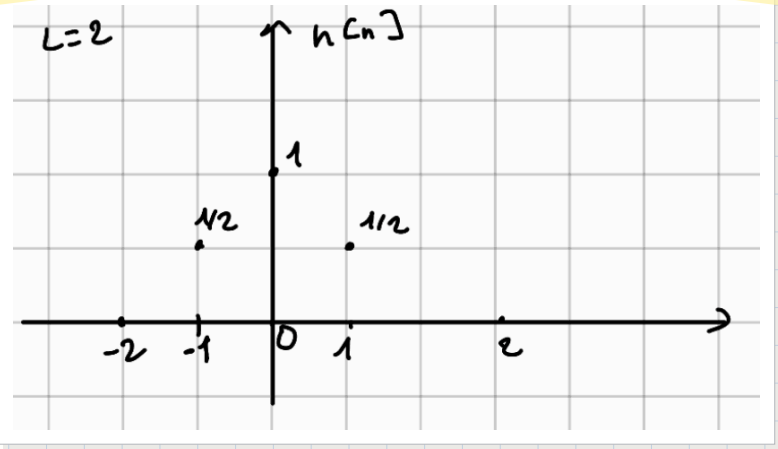


orijinal çizim, s. 7

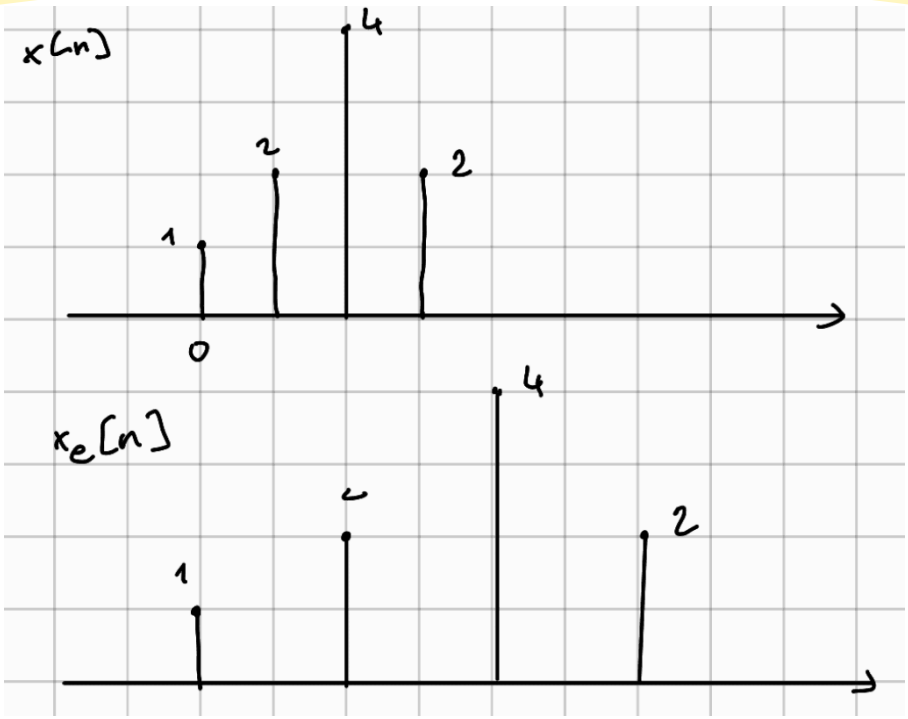


orijinal çizim, s. 7

$$h[n] = \begin{cases} 1 - \dfrac{|n|}{L}, & |n| < L \\ 0, & \text{o.w} \end{cases}$$

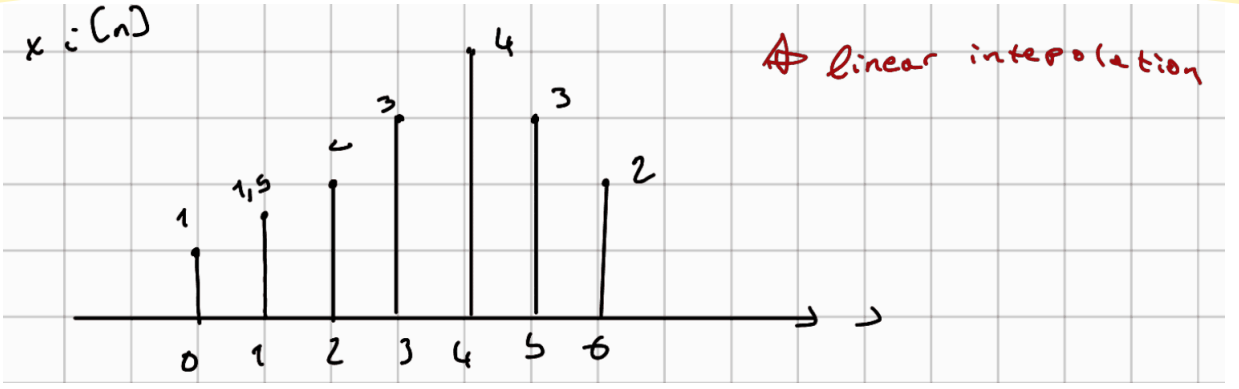


orijinal çizim, s. 7

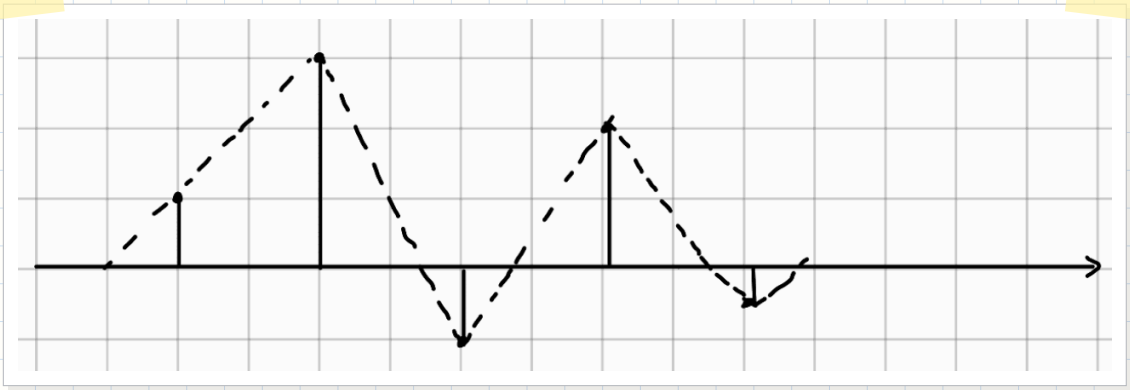


orijinal çizim, s. 7

orj. s. 8



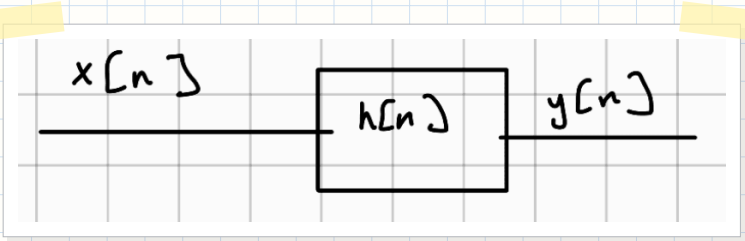
orijinal çizim, s. 8



orijinal çizim, s. 8

Her zaman up yapılır başta

Frequency Response of LTI systems



orijinal çizim, s. 8

$$y[n] = x[n] * h[n]$$
$$Y(z) = X(z)\,H(z)$$

$$Y(e^{j\omega}) = X(e^{j\omega})\,H(e^{j\omega})$$
$$\left|Y(e^{j\omega})\right| = \left|X(e^{j\omega})\right|\left|H(e^{j\omega})\right|$$
$$\angle Y(e^{j\omega}) = \angle X(e^{j\omega}) + \angle H(e^{j\omega})$$

$$x[n] = A\cos(\omega_0 n + \phi)$$
$$y[n] = A\left|H(e^{j\omega_0})\right|\cos\left(\omega_0 n + \phi + \angle H(e^{j\omega_0})\right)$$
$$h_{id}[n] = \delta[n - n_d]$$

orj. s. 9

$$H_{id}(e^{j\omega}) = e^{-j\omega n_d}$$

$$\left|H_{id}(e^{j\omega})\right| = 1$$
$$\angle H_{id}(e^{j\omega_0}) = -\omega_0 n_d$$

$$\begin{aligned} y[n] &= A\cos(\omega_0 n + \phi - \omega_0 n_d) \\ &= A\cos\left(\omega_0(n - n_d) + \phi\right) \end{aligned}$$