Sayısal İşaret İşleme

Ders 8: Grup gecikmesi ve lineer faz

Orijinal not: Dsp-ders-8.pdf

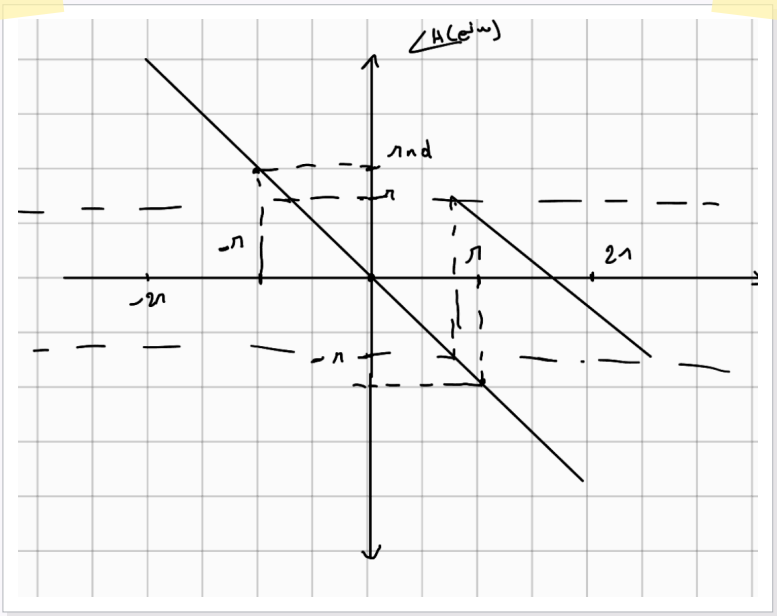
UYARI Bu belge, el yazısı ders notlarından Claude (Anthropic) yardımıyla dijitale aktarılmıştır. Yardımcı kaynak olarak hazırlanmıştır; hata veya eksik içerebilir. Lütfen orijinal notları ve ders kaynaklarını esas alın.

NOTICE *This document was transcribed from handwritten lecture notes with the help of Claude (Anthropic). It is meant as a supplementary resource and may contain errors or omissions. Always check against the original notes and official course material.*

orj. s. 1

Örnek

$$h_{id}[n] = \delta[n - n_d]$$
$$H_{id}(e^{j\omega}) = e^{-j\omega n_d}$$
$$|H(e^{j\omega})| = 1$$
$$\angle H(e^{j\omega}) = -\omega n_d$$



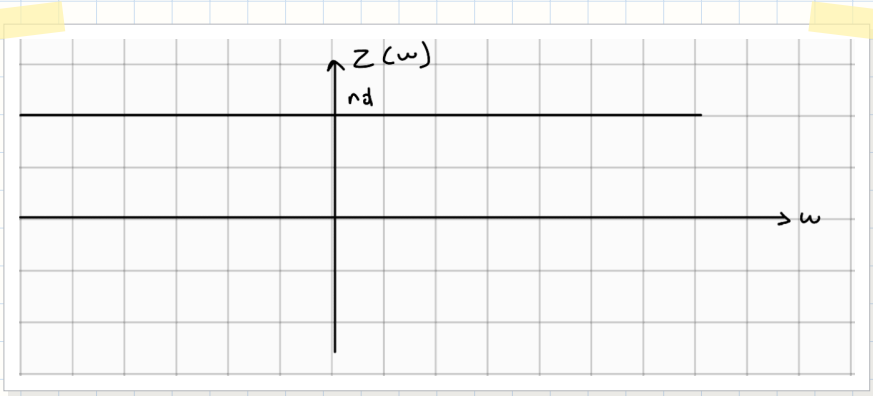
orijinal çizim, s. 1

Groupdelay $\tau(\omega)$

$$\tau(\omega) \triangleq -\frac{d}{d\omega}\angle H(e^{j\omega})$$

For ideal delay system

$$\tau(\omega) = -\frac{d}{d\omega}(-\omega n_d) = n_d \text{ [sample]}$$



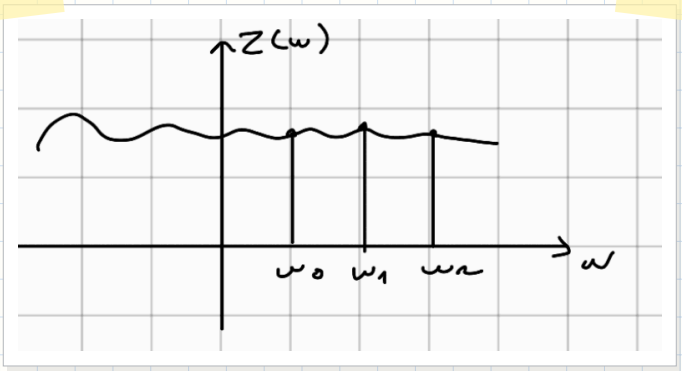
orijinal çizim, s. 1

$$\begin{cases} h_{id}[n] = \delta[n - n_d] \\ x[n] = \cos(\omega_0 n) \\ \rightarrow\ H(e^{j\omega}) = e^{-j\omega n_d} \end{cases}$$

orj. s. 2

$$\rightarrow\ g[n] = \cos(\omega_0 n - \omega_0 n_d) = \cos(\omega_0 (n - n_d))$$

$$x[n] = \cos(\omega_0 n) + \cos(\omega_1 n) + \cos(\omega_2 n)$$
$$g[n] = \cos(\omega_0 (n - n_d)) + \cos(\omega_1 (n - n_d)) + \cos(\omega_2 (n - n_d))$$



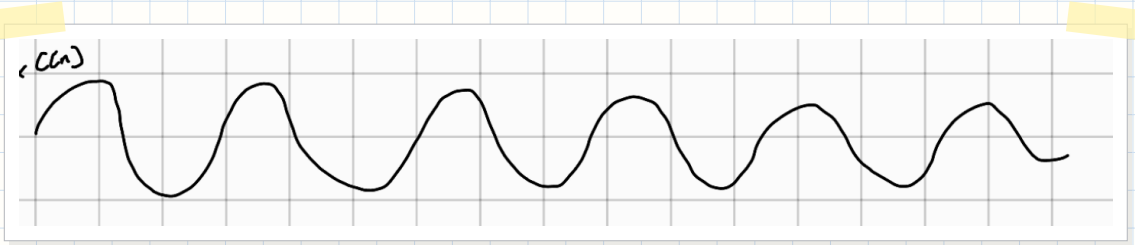
orijinal çizim, s. 2

IIR hiç bir zaman lineer faza sahip değildir.

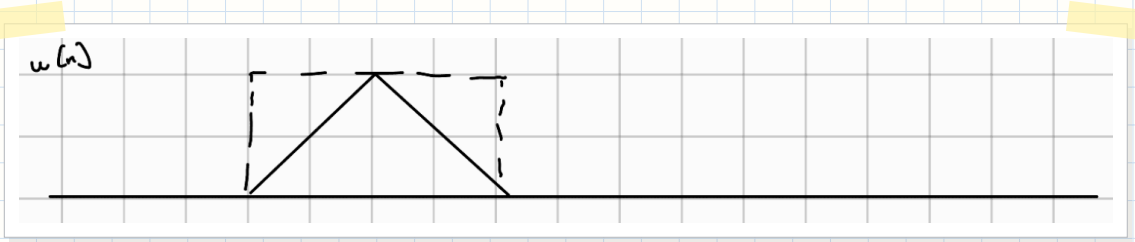
FIR sahip olabilmektedir.

$$\angle H(e^{j\omega}) = -\alpha\omega$$
$$\angle H(e^{j\omega}) = -\alpha\omega + \beta$$

$$x_1[n] = w[n]\cos(0{,}2\pi n)$$
$$x_2[n] = w[n]\cos(0{,}4\pi n - \pi/2)$$
$$x_3[n] = w[n]\cos(0{,}8\pi n + \pi/5)$$



orijinal çizim, s. 2



orijinal çizim, s. 2



orijinal çizim, s. 2

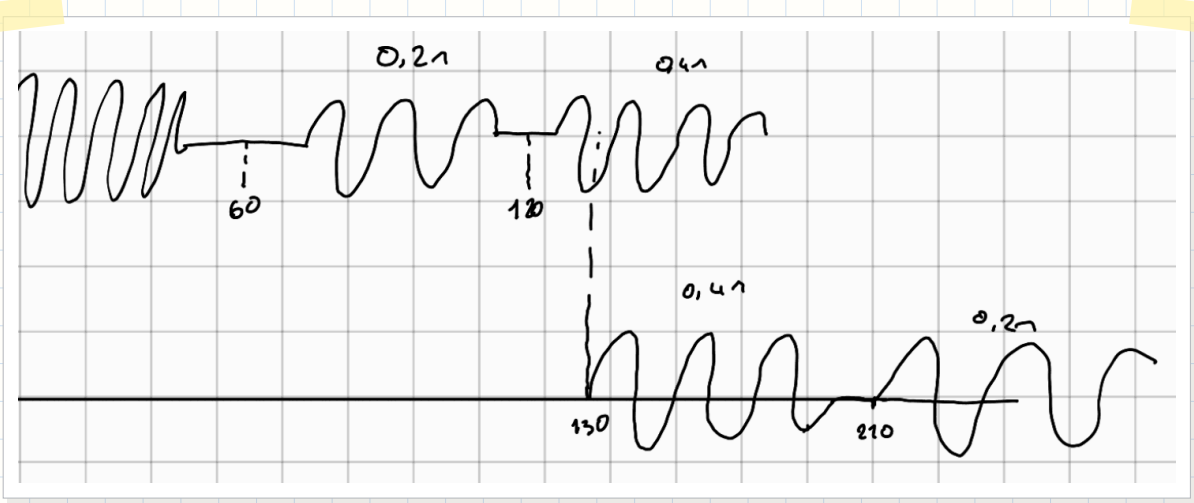
orj. s. 3

$$x[n] = x_3[n] + x_1[n - M - 1] + x_2[n - 2M - 2]$$

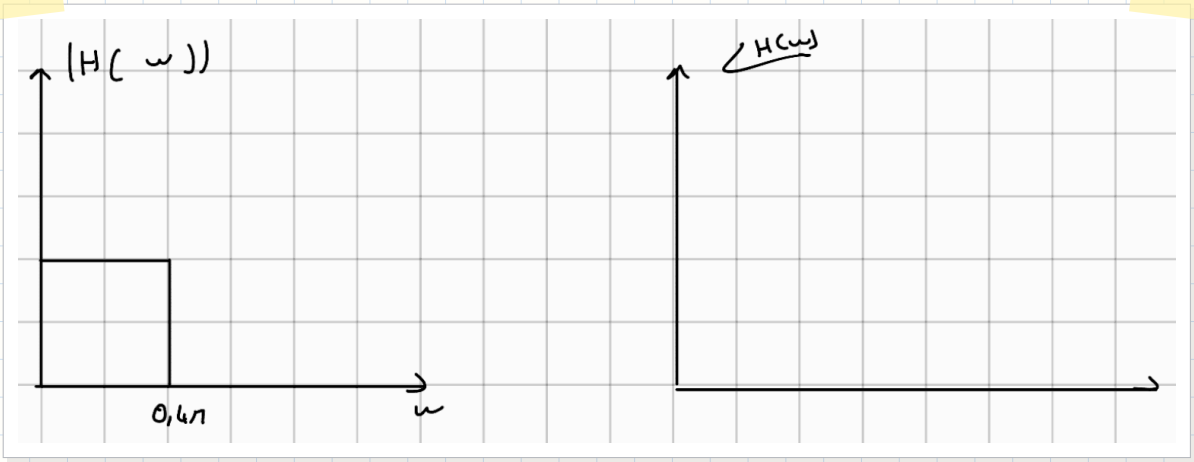
$M = 60$

? kesişim kalmasın diye

$$x[n] = x_3[n] + x_1[n - 60] + x_2[n - 120]$$



orijinal çizim, s. 3



orijinal çizim, s. 3

$0{,}4\pi$'de $n_d = 10$, $\angle H(e^{j0{,}4\pi}) = -0{,}4\pi \cdot 10 = -4\pi$

Frequency Response

$$\sum_{k=0}^{N} a_k\, y[n-k] = \sum_{k=0}^{M} b_k\, x[n-k]$$

$$H(e^{j\omega}) = \frac{\sum_{k=0}^{M} b_k e^{-j\omega k}}{\sum_{k=0}^{N} a_k e^{-j\omega k}} = \left(\frac{b_0}{a_0}\right) \frac{\prod_{k=1}^{M}(1 - c_k e^{-j\omega})}{\prod_{k=1}^{N}(1 - d_k e^{-j\omega})}$$

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{k=0}^{N} a_k z^{-k}}$$

orj. s. 4

$$20 \log |H(e^{j\omega})| = 10 \log |H(e^{j\omega})|^2$$

$$|H(e^{j\omega})|^2 = H(e^{j\omega})\, H^*(e^{j\omega})$$

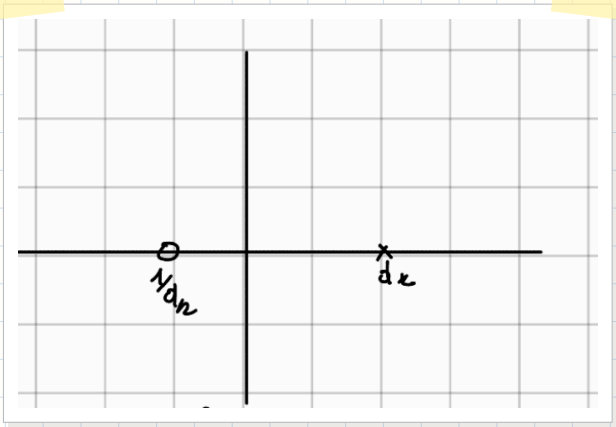
$|H(e^{j\omega})| = 1 \Rightarrow 0$ dB

$|H(e^{j\omega})| = \frac{1}{\sqrt{2}} \Rightarrow -3$ dB

$$20 \log |H(e^{j\omega})| = 20 \log \left(\frac{b_0}{a_0}\right) + \sum_{k=1}^{M} 20 \log |1 - c_k e^{-j\omega}| - \sum_{k=1}^{N} 20 \log |1 - d_k e^{-j\omega}|$$

$$d_k = r e^{j\theta}$$

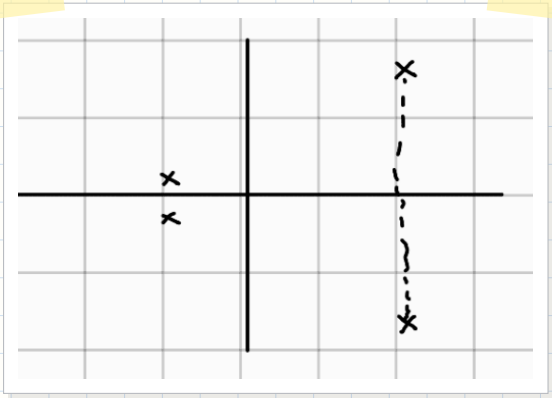
$$|1 - r e^{j\theta} e^{-j\omega}|^2 = (1 - r e^{j\theta} e^{-j\omega})(1 - r e^{-j\theta} e^{j\omega}) = 1 + r^2 - 2r \cos(\omega - \theta)$$



orijinal çizim, s. 4

$$H(z) = \prod_{k=1}^{p} \frac{z^{-1} - d_k^*}{1 - d_k z^{-1}}$$

$$H(z) = \frac{(z^{-1} - e_k^*)(z^{-1} - e_k)}{(1 - e_k z^{-1})(1 - e_k^* z^{-1})}$$

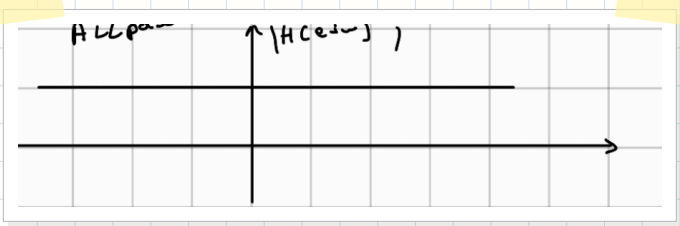


orijinal çizim, s. 4

$$|H(e^{j\omega})| = 1$$

$$H(e^{j\omega}) = \frac{e^{-j\omega} - d_k}{1 - d_k e^{-j\omega}}$$

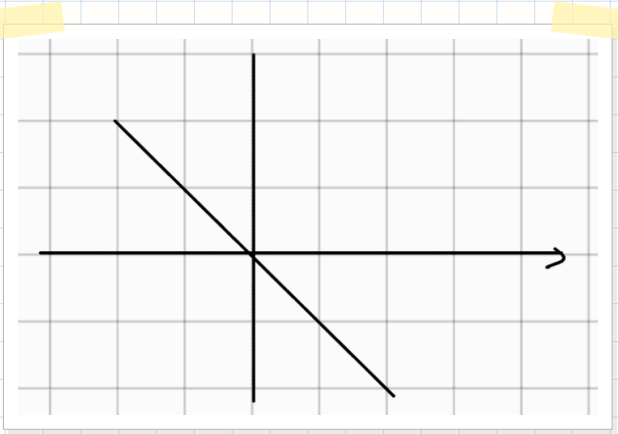
$$|H(e^{j\omega})|^2 = \frac{e^{-j\omega} - d_k}{1 - d_k e^{-j\omega}} \frac{e^{j\omega} - d_k}{1 - d_k e^{j\omega}}$$



orijinal çizim, s. 4

orj. s. 5

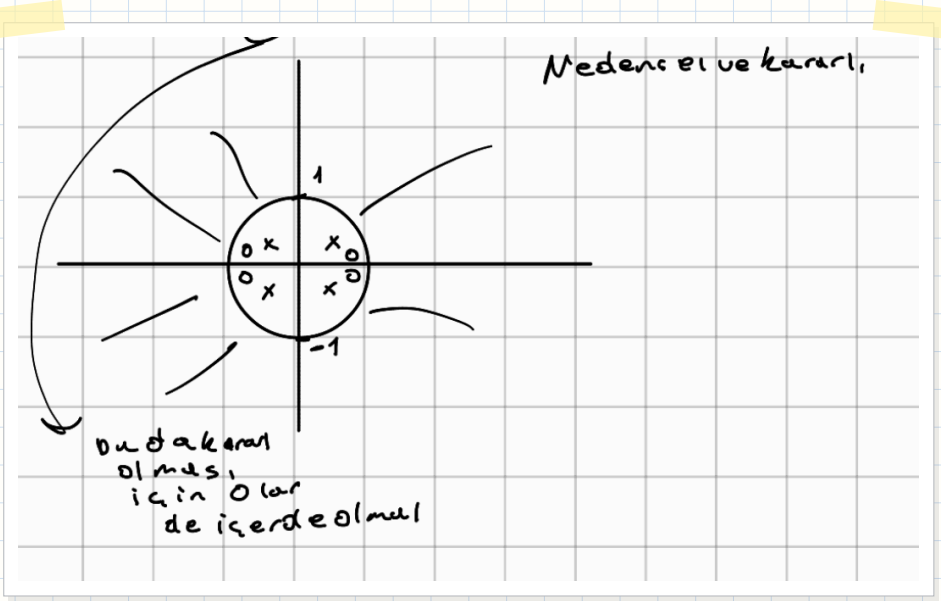
Ama fazı farklı olabilir.



orijinal çizim, s. 5

Minimum phase systems

$$H(z) \longrightarrow \frac{1}{H(z)}$$



orijinal çizim, s. 5

Hem kutupların hem O'ların birim çemberin içinde olması şart

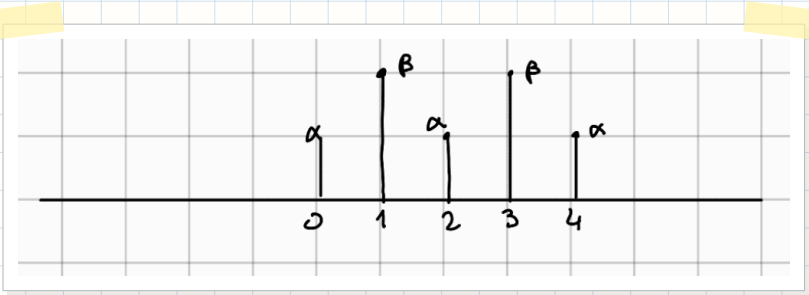
Linear Phase Filters

Type-I

$$h[n] = h[M-n] \qquad 0 \le n \le M \qquad M \text{ even integer}$$

$$M = 4 \Rightarrow 0 \le n \le 4$$

$$h[0] = h[4]$$
$$h[1] = h[3]$$



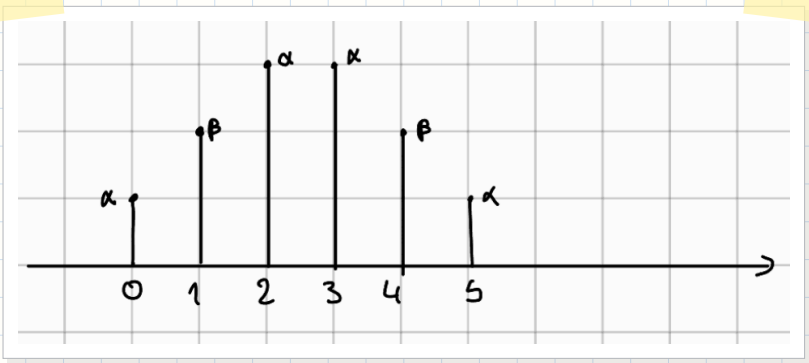
orijinal çizim, s. 5

orj. s. 6

Type-II

$$h[n] = h[M-n] \qquad 0 \le n \le M \rightarrow \text{odd integer}$$

$$M = 5$$
$$h[0] = h[5]$$
$$h[1] = h[4]$$
$$h[2] = h[3]$$



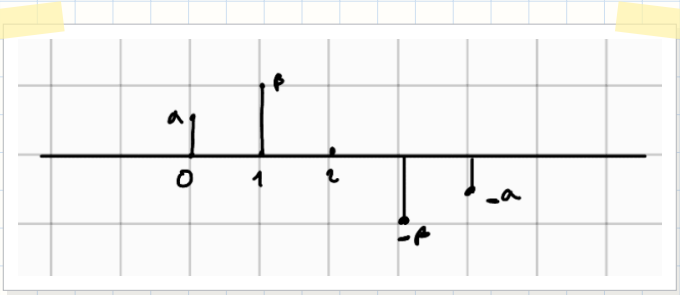
orijinal çizim, s. 6

Type III

$$h[n] = -h[M-n]$$

M even odd

$$M = 4$$
$$h[0] = -h[4]$$
$$h[1] = -h[3]$$
$$h[2] = 0$$



orijinal çizim, s. 6

Type IV

$$h[n] = -h[M-n]$$

M odd integer

orj. s. 7 Show $h[n] = h[M-n]$, M odd (tip 2)

$\Rightarrow$ get linear phase

$$M = 5 \qquad h[0] = h[5]$$
$$h[1] = h[4]$$
$$h[2] = h[3]$$

$$H(e^{j\omega}) = \sum_{n=0}^{M} h[n]e^{-j\omega n}$$

$$= \sum_{n=0}^{\frac{M-1}{2}} h[n]e^{-j\omega n} + \sum_{n=\frac{M+1}{2}}^{M} h[n]e^{-j\omega n}$$

$$= \sum_{n=0}^{\frac{M-1}{2}} h[n]e^{-j\omega n} + \sum_{k=0}^{\frac{M-1}{2}} h[M-k]e^{-j\omega(M-k)}$$

$$= \sum_{n=0}^{\frac{M-1}{2}} h[n]e^{-j\omega n} + \left(\sum_{n=0}^{\frac{M-1}{2}} h[n]e^{+j\omega n}\right) e^{-j\omega M}$$

$$= \sum_{n=0}^{\frac{M-1}{2}} h[n]\left[e^{-j\omega n} + e^{j\omega n}e^{-j\omega M}\right]$$

$$= \sum_{n=0}^{\frac{M-1}{2}} h[n]\Big[e^{-j\omega n}e^{-j\omega\frac{M}{2}}e^{j\omega\frac{M}{2}} + e^{j\omega n}e^{-j\omega\frac{M}{2}}e^{-j\omega\frac{M}{2}}\Big]$$

$$= e^{-j\omega\frac{M}{2}} \sum_{n=0}^{\frac{M-1}{2}} h[n]\left[e^{-j\omega(n-\frac{M}{2})} + e^{j\omega(n-\frac{M}{2})}\right]$$

$$H(e^{j\omega}) = e^{-j\omega\frac{M}{2}}\, 2\sum_{n=0}^{\frac{M-1}{2}} h[n]\cos\left(\omega n - \omega\frac{M}{2}\right)$$

$$|H(e^{j\omega})| = \underbrace{\left| 2 \sum_{n=0}^{\frac{M-1}{2}} h[n] \cos\left(\omega n - \omega \frac{M}{2}\right) \right|}_{\downarrow}$$

$$\angle H(e^{j\omega}) = \begin{cases} -\omega \frac{M}{2} & > 0 \\ -\omega \frac{M}{2} + \pi & < 0 \end{cases}$$