Sayısal İşaret İşleme

Ders 9: Filtre yapıları (structures)

Orijinal not: Dsp-ders-9.pdf

UYARI Bu belge, el yazısı ders notlarından Claude (Anthropic) yardımıyla dijitale aktarılmıştır. Yardımcı kaynak olarak hazırlanmıştır; hata veya eksik içerebilir. Lütfen orijinal notları ve ders kaynaklarını esas alın.

NOTICE *This document was transcribed from handwritten lecture notes with the help of Claude (Anthropic). It is meant as a supplementary resource and may contain errors or omissions. Always check against the original notes and official course material.*

orj. s. 1

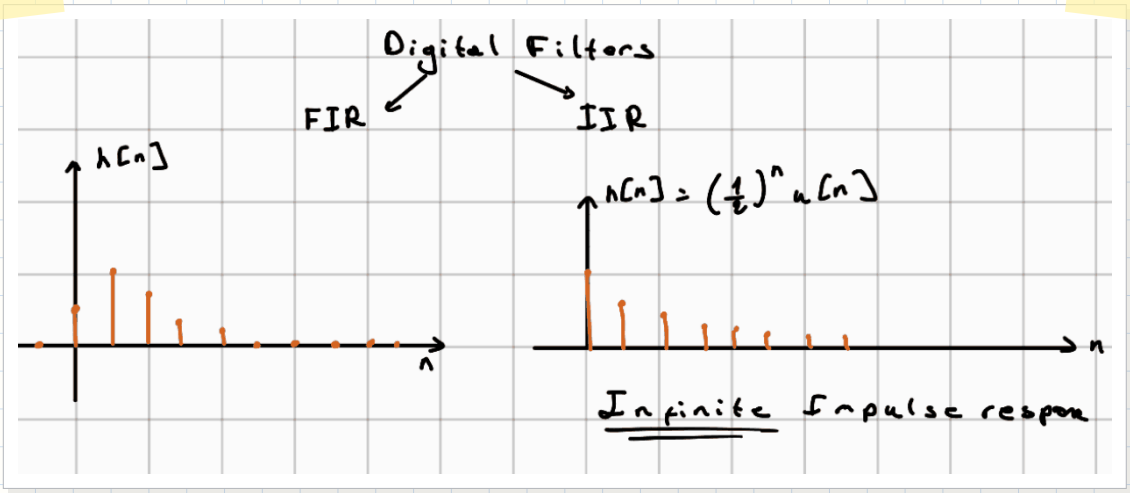
Dsp ders 9

kitapta 6. bölümdeyiz

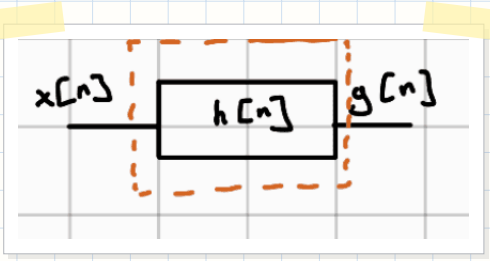
Structures for discrete-time system

1- Direct form I
2- Direct form II
3- Cascade
4- Parallel structures
5- Transposed form (flow graph reversal)

Önceden filtreleri ikiye ayırmıştık



orijinal çizim, s. 1



orijinal çizim, s. 1

$$y[n] = x[n] * h[n]$$

bilgisayarla yapılabilir ↑ FIR

$$\Rightarrow y[n] = x[n] * h[n] \leftarrow \text{IIR}$$

bilgisayarda yapabilmek için belli bir noktadan sonra kesmek gerekiyor o zaman aslında FIR olur böylece yaklaşıklık yapılır. Ama biz tam olarak bulmak istiyoruz, o zaman başka bir yöntem kullanacağız.

IIR Filter Design

1- specs. (Teknik özellikler)
2- $h[n]$
3- realize (gerçeklemek) (this week)
4- implementation (gerçekleştirmek)

orj. s. 2

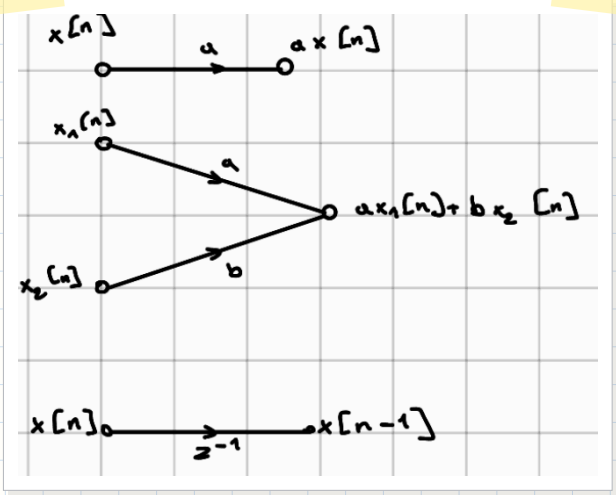
$$y[n] - \sum_{k=1}^{p} a_k y[n-k] = \sum_{k=0}^{q} b_k x[n-k]$$

a_k, b_k real

$$H(z) = \frac{\sum_{k=0}^{q} b_k z^{-k}}{1 - \sum_{k=1}^{p} a_k z^{-k}} \quad \text{rational}$$

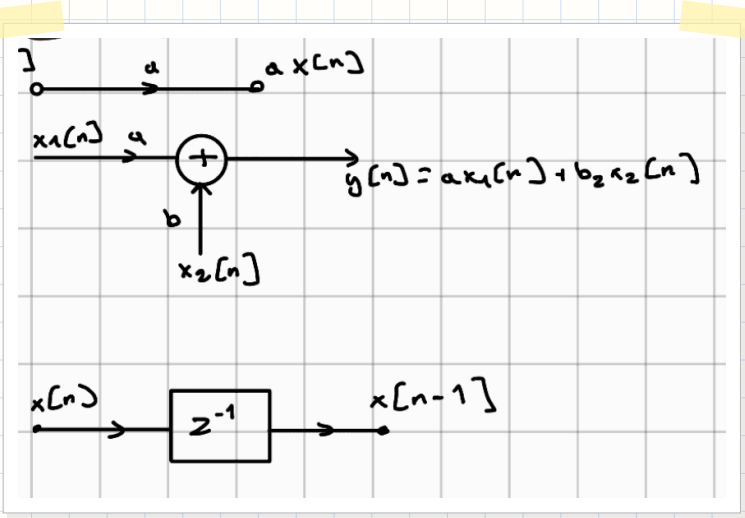
Özel sadeleşme durumu ? olmazsak bu IIR bir filtrenin sistem fonksiyonu

Flow notation



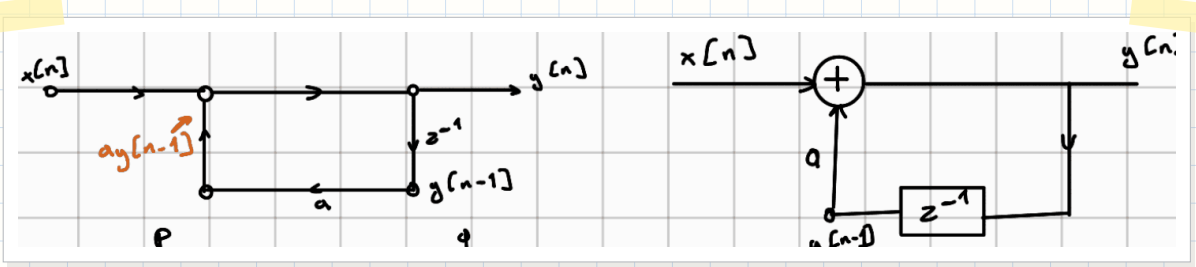
orijinal çizim, s. 2

Block diagram notation



orijinal çizim, s. 2

$$y[n] = a\,y[n-1] + x[n]$$



orijinal çizim, s. 2

$$y[n] = \sum_{k=1}^{p} a_k y[n-k] + \underbrace{\sum_{k=0}^{q} b_k x[n-k]}_{v[n]}$$

Direct Form-I

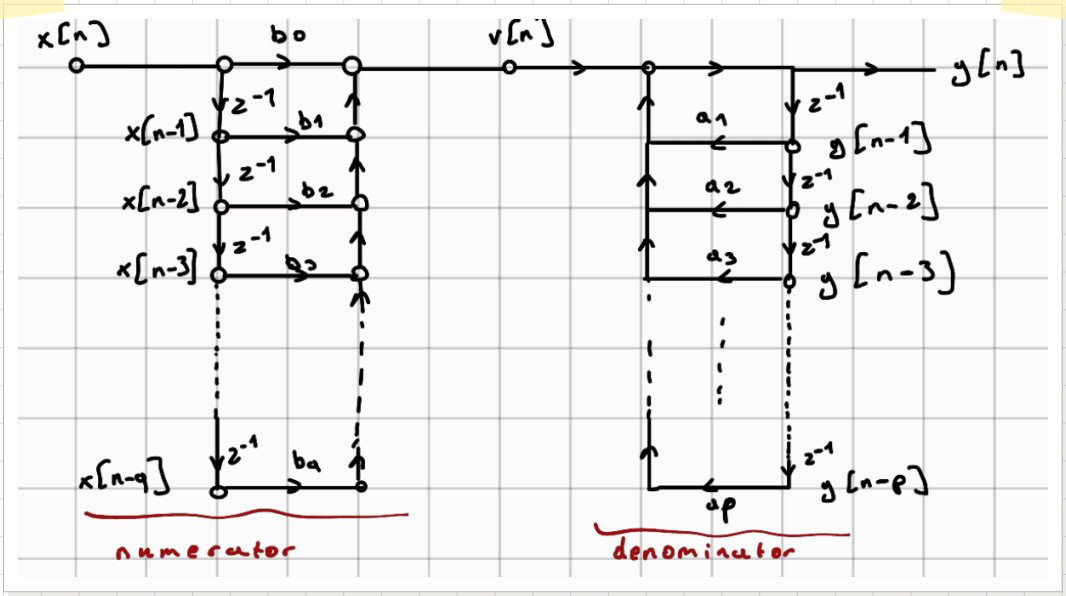
$$y[n] = \sum_{k=1}^{p} a_k y[n-k] + v[n]$$
$$= a_1 y[n-1] + \cdots + a_p y[n-p] + v[n]$$

$$H(z) = \frac{\sum_{k=0}^{q} b_k z^{-k}}{1 - \sum_{k=1}^{p} a_k z^{-k}} \qquad \text{rational}$$

orj. s. 3

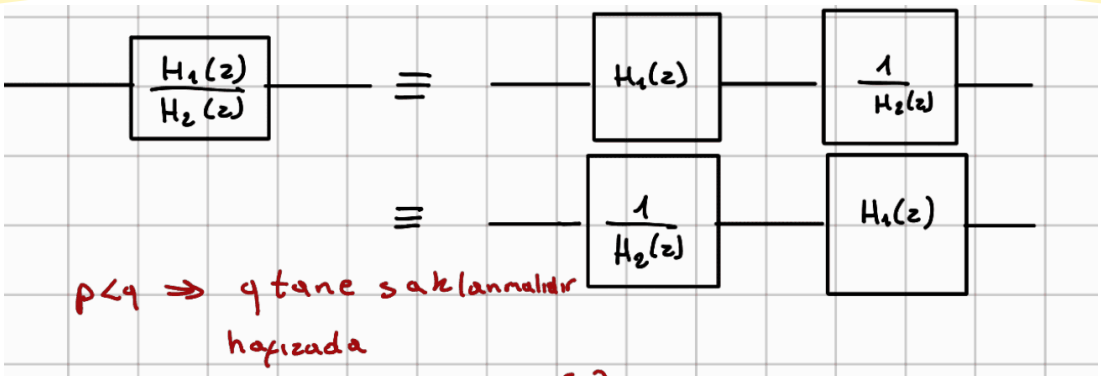
Önceki girişlerden q tane değer saklanmalı

Önceki çıkışlardan p tane değer saklanmalı

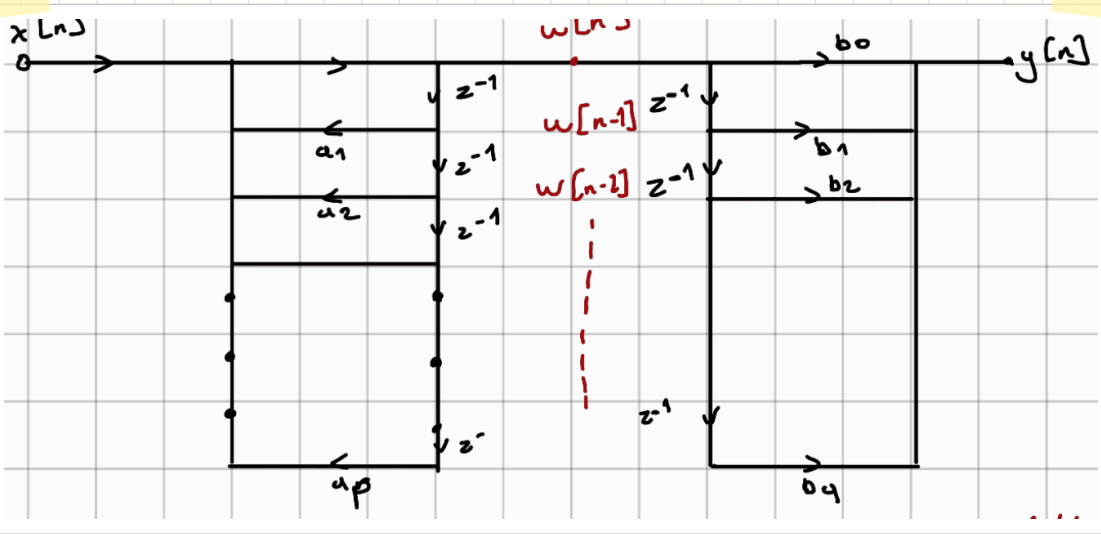


orijinal çizim, s. 3

$$H(z) = \frac{H_1(z)}{H_2(z)} = (H_1(z)) \cdot \left(\frac{1}{H_2(z)}\right)$$



orijinal çizim, s. 3



orijinal çizim, s. 3

artı artı diye yaptıklarımız artık eksi olacak a'larda sola geçirirken - oluyor

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 - a_1 z^{-1} - a_2 z^{-2}}$$

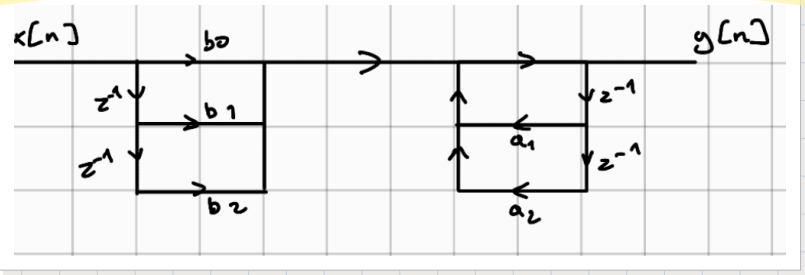
$$y[n] = a_1 y[n-1] + a_2 y[n-2] + b_0 x[n] + b_1 x[n-1] + b_2 x[n-2]$$

$$y[n] - a_1 y[n-1] - a_2 y[n-2] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2]$$

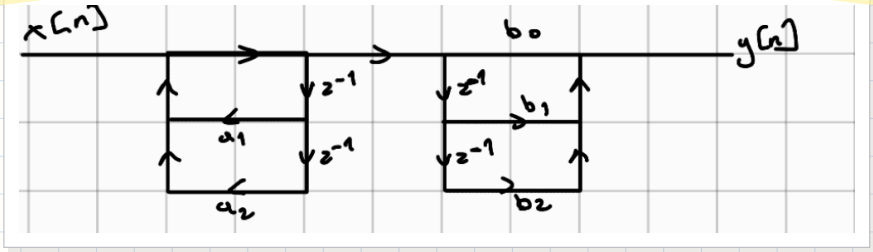
önceden bunlar artı idi genel gösterimizde

orj. s. 4

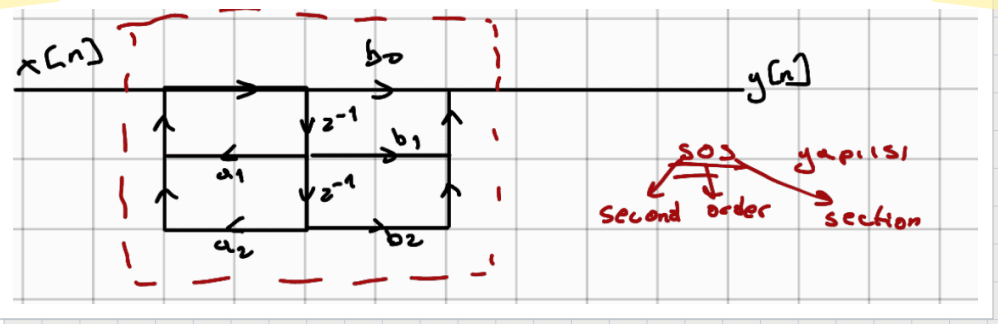
$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 - a_1 z^{-1} - a_2 z^{-2}}$$



orijinal çizim, s. 4



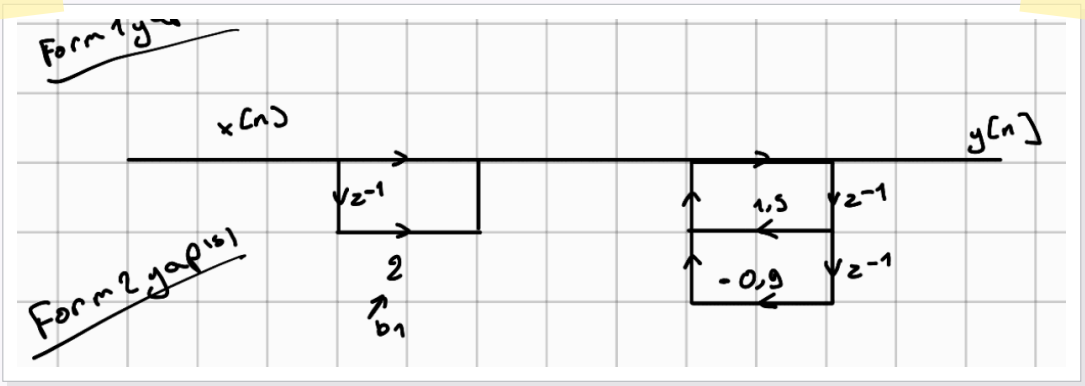
orijinal çizim, s. 4



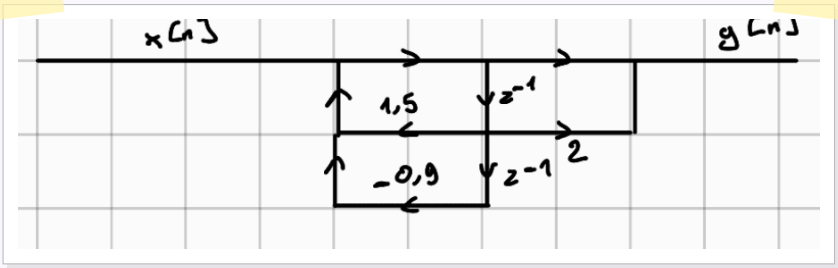
orijinal çizim, s. 4

Örnek

$$H(z) = \frac{1 + 2z^{-1}}{1 - 1{,}5z^{-1} + 0{,}9z^{-2}}$$



orijinal çizim, s. 4



orijinal çizim, s. 4

orj. s. 5

Cascade Structures

$$H(z) = A \frac{\overbrace{\prod_k (1 - e_k z^{-1})}^{\text{Reel sıfırlar}} \prod_k (1 - f_k z^{-1})(1 - f_k^* z^{-1})}{\underbrace{\prod_k (1 - c_k z^{-1})}_{\text{Reel kutuplar}} \prod_k (1 - d_k z^{-1})(1 - d_k^* z^{-1})}$$

$$\begin{aligned}(1 - d_k z^{-1})(1 - d_k^* z^{-1}) &= 1 - 2\,\mathrm{Re}[d_k]\, z^{-1} + |d_k|^2 z^{-2} \\ &= 1 + \beta_{1k} z^{-1} + \beta_{2k} z^{-2}\end{aligned}$$

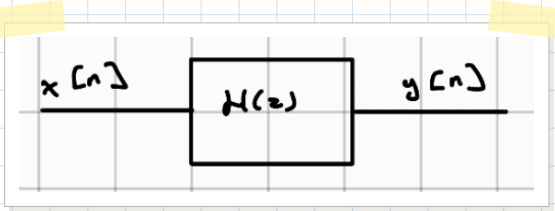
O zaman $H(z)$ tekrar bu şekilde yazabiliriz

$$H(z) = A \prod_k H_k(z)$$

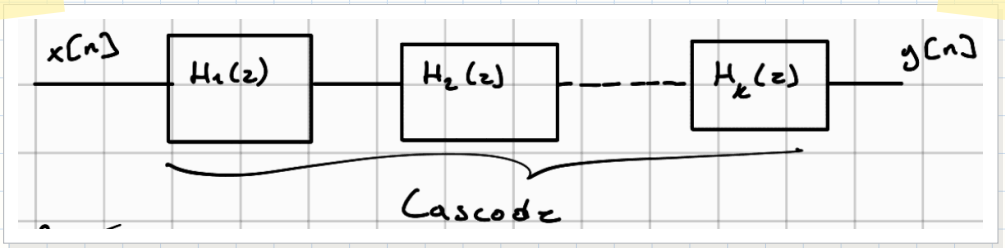
$$H_k(z) = \frac{1 + \beta_{1k} z^{-1} + \beta_{2k} z^{-2}}{1 - \alpha_{1k} z^{-1} - \alpha_{2k} z^{-2}}$$

s.o.s → section

↙ second ↘ order

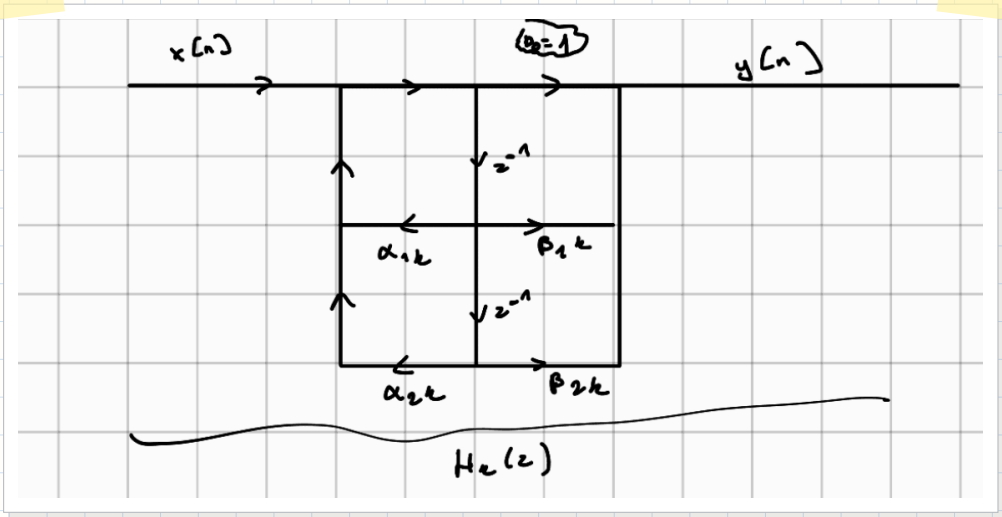


orijinal çizim, s. 5



orijinal çizim, s. 5

SOS Form



orijinal çizim, s. 5

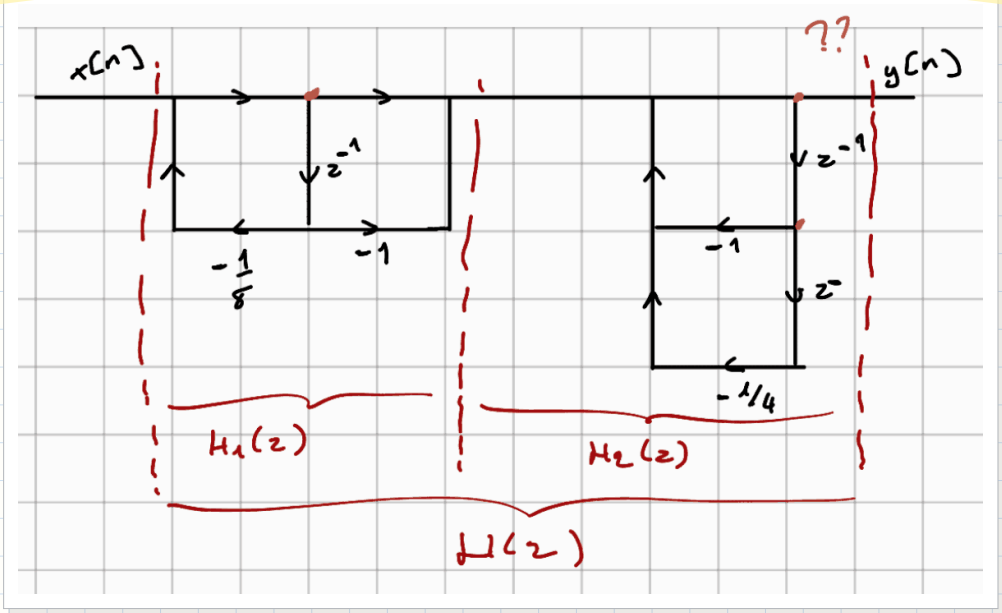
Örnek

$$H(z) = \frac{1 - z^{-1}}{\left(1 + \frac{1}{8}z^{-1}\right)\left(1 + z^{-1} + \frac{1}{4}z^{-2}\right)}$$

$$= \left(\frac{1}{1 + \frac{1}{8}z^{-1}}\right)\left(\frac{1 - z^{-1}}{1 + z^{-1} + \frac{1}{4}z^{-2}}\right)$$

$$= \left(\frac{1 - z^{-1}}{1 + \frac{1}{8}z^{-1}}\right)\left(\frac{1}{1 + z^{-1} + \frac{1}{4}z^{-2}}\right)$$

orj. s. 6



orijinal çizim, s. 6

Burada 3 tane değeri hafızada saklamak gerekir. (gecikmelerden dolayı)

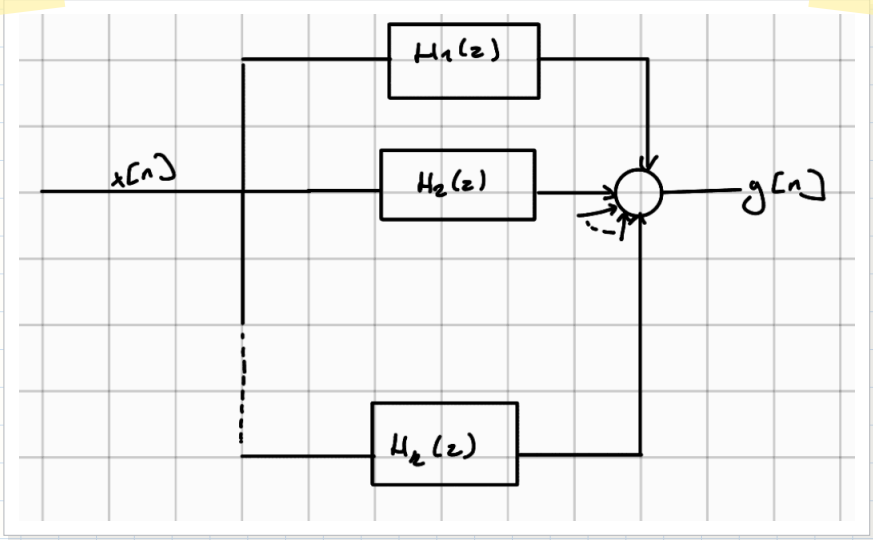
Parallel implementation

S.O.S'ların toplamı şeklinde ifade edilmelidir.

$$H(z) = \frac{\sum_{k=0}^{q} b_k z^{-k}}{1 - \sum_{k=1}^{p} a_k z^{-k}}$$

$$= \sum_k A_k z^{-k} + \sum_k \frac{B_k}{1 - g_k z^{-1}} + \underbrace{\sum_k \frac{C_k + D_k z^{-1}}{1 - h_{1k} z^{-1} - h_{2k} z^{-2}}}_{\text{en genel hal}}$$

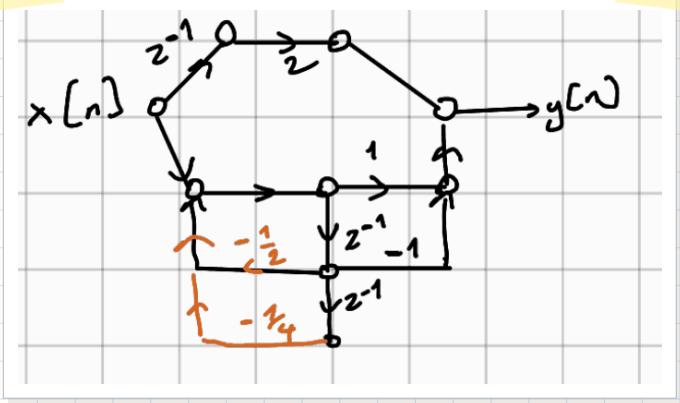
$$= \sum_k H_k(z)$$



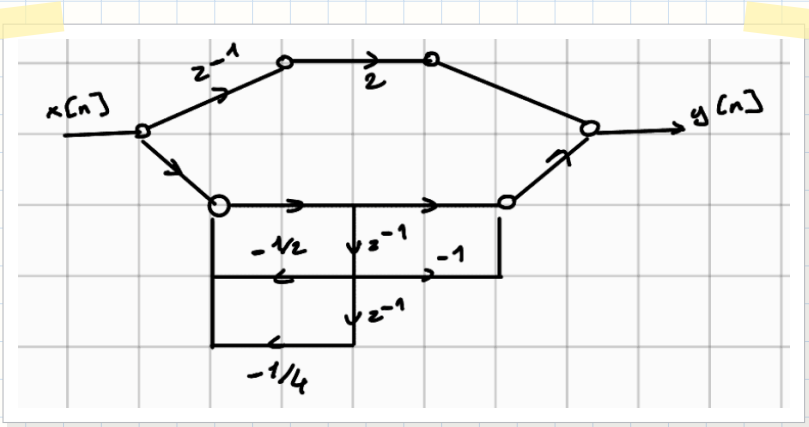
orijinal çizim, s. 6

orj. s. 7

$$H(z) = \underbrace{2z^{-1}}_{H_1(z)} + \underbrace{\frac{1 - z^{-1}}{1 + \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2}}}_{H_2(z)}$$



orijinal çizim, s. 7

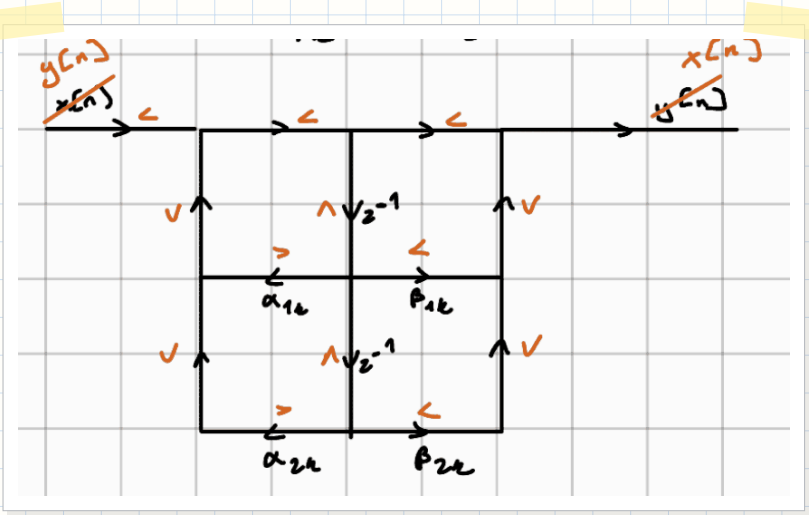


orijinal çizim, s. 7

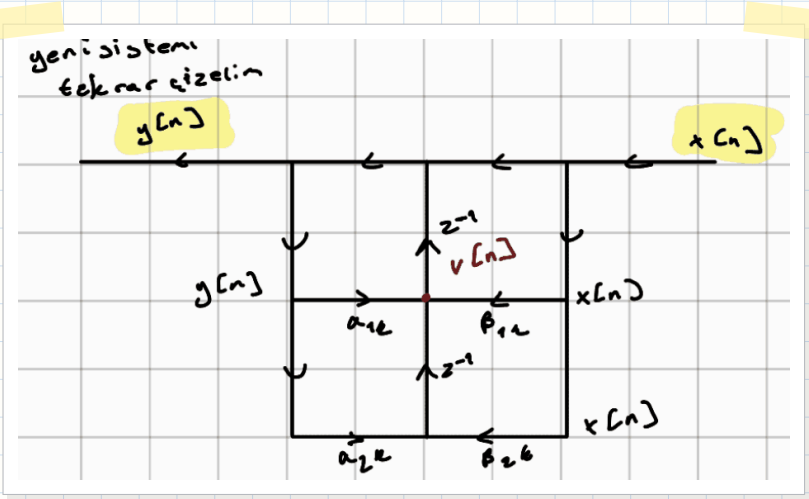
Transpozition theorem (Flow-graph reversal theorem)

Teoreme göre $x[n]$ ve $y[n]$'in yeri değiştirilip tüm okların yönü terslenirse yeni sistem eskinin eşidir.

$$H_k(z) = \frac{1 + \beta_{1k} z^{-1} + \beta_{2k} z^{-2}}{1 - \alpha_{1k} z^{-1} - \alpha_{2k} z^{-2}}$$



orijinal çizim, s. 7

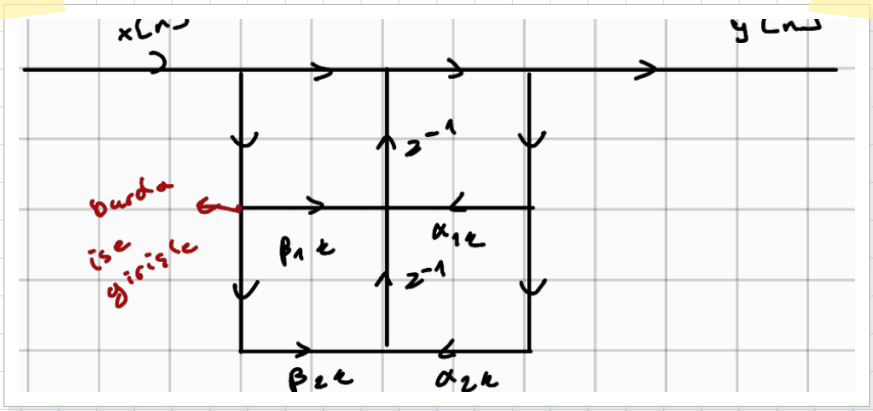


orijinal çizim, s. 7

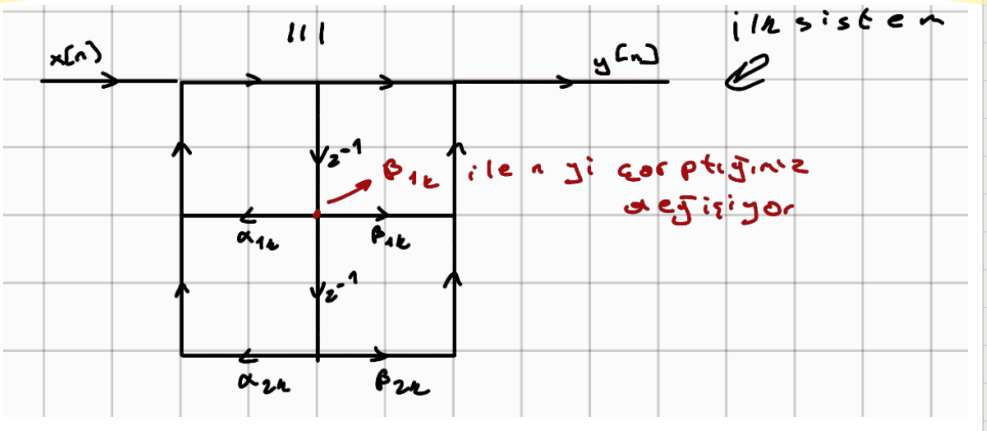
$$
\begin{aligned}
v[n] &= x[n]\,\beta_{1k} \\
&\quad + y[n]\,\alpha_{1k} \\
&\quad + y[n-1]\,\alpha_{2k} \\
&\quad + x[n-1]\,\beta_{2k} \\
y[n] &= x[n] + v[n-1] \\
&= x[n] + x[n-1]\,\beta_{1k} + x[n-2]\,\beta_{2k} + y[n-1]\,\alpha_{1k} \\
&\quad + y[n-2]\,\alpha_{2k}
\end{aligned}
$$

$$
\frac{Y(z)}{X(z)} = \frac{1 + \beta_{1k} z^{-1} + \beta_{2k} z^{-2}}{1 - \alpha_{1k} z^{-1} - \alpha_{2k} z^{-2}}
$$

orj. s. 8 Simetriğini alırsak bizim alıştığımız forma geçer



orijinal çizim, s. 8



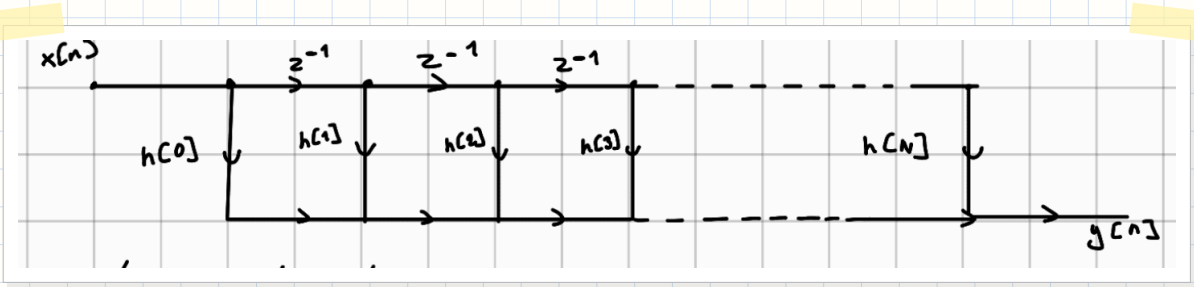
orijinal çizim, s. 8

FIR realization

$h[n]$: finite

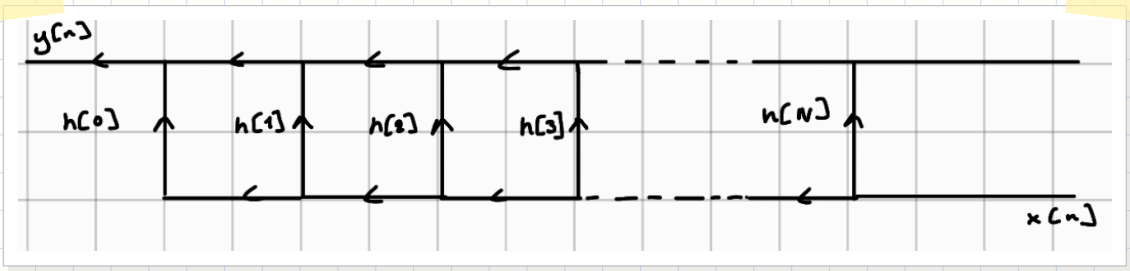
$$H(z) = \sum_{k=0}^{N} h[k] z^{-k}$$

$$\Rightarrow y[n] = \sum_{k=0}^{N} h[k] x[n-k] = h[0]x[n] + h[1]x[n-1] + h[2]x[n-2] + \cdots + h[N]x[n-N]$$



orijinal çizim, s. 8

use transposition theorem:

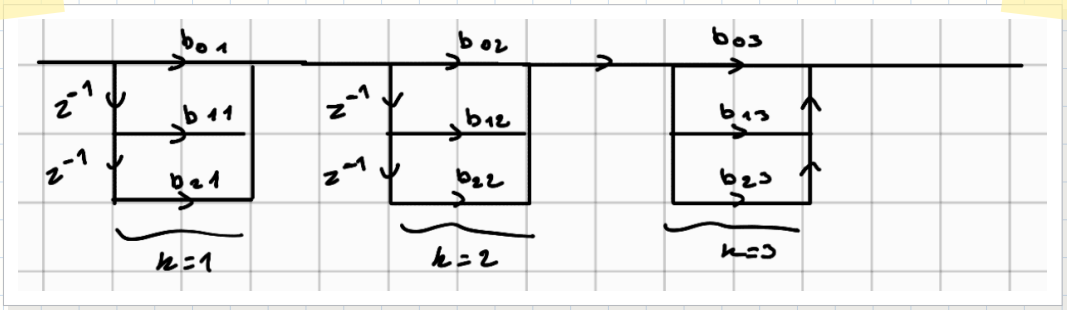


orijinal çizim, s. 8

orj. s. 9

In cascade Structure:

$$H(z) = \sum_{k=0}^{N} h[k]\, z^{-k} = \prod_{k=1}^{N} \left(b_{0k} + b_{1k} z^{-1} + b_{2k} z^{-2} \right)$$



orijinal çizim, s. 9

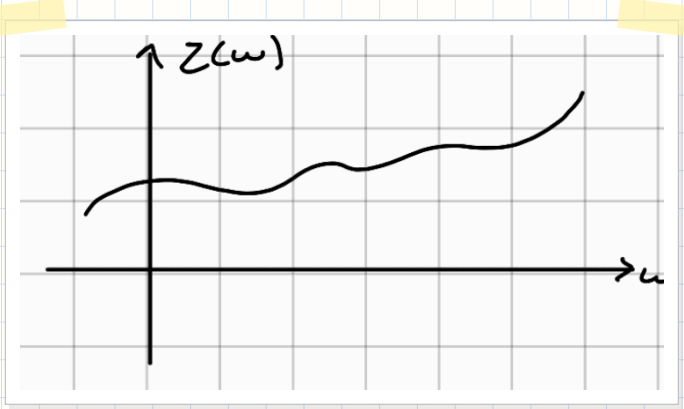
$H(e^{j\omega})$ $|H(e^{j\omega})|$ $\angle H(e^{j\omega})$

$$\tau(\omega) = -\frac{d}{d\omega} \angle H(e^{j\omega})$$ group delay

$\tau(\omega)$ = constant



orijinal çizim, s. 9



orijinal çizim, s. 9

Önceden böyle bir örnek göstermiştik

$0{,}2\pi$

$0{,}4\pi$

$0{,}8\pi$

$\angle H(e^{j\omega}) = \alpha\omega + \beta$ yapısında ise $\tau(\omega) = \text{constant}$

Linear Phase Filter

IIR × (hiç bir zaman sağlayamaz) FIR sağlayabilir

orj. s. 10

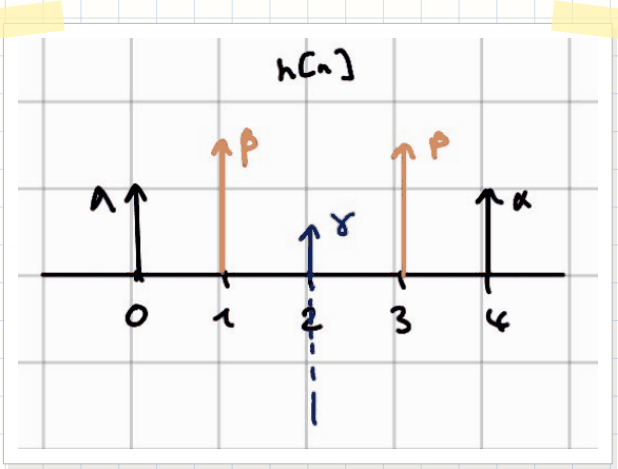
Type-I

$$h[n] = h[M-n] \qquad 0 \le n \le M$$

M: even integer (çift)

$$M = 4 \Rightarrow 0 \le n \le 4$$

$$h[0] = h[4]$$
$$h[1] = h[3]$$
$$h[2]$$



orijinal çizim, s. 10

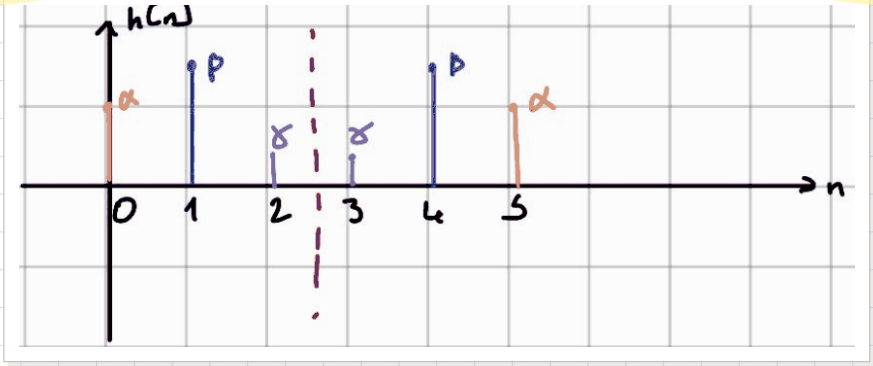
Type II

$$h[n] = h[M-n] \qquad 0 \le n \le M$$

M: odd integer (tek sayı)

$$M = 5 \Rightarrow 0 \le n \le 5$$

$$h[0] = h[5]$$
$$h[1] = h[4]$$
$$h[2] = h[3]$$



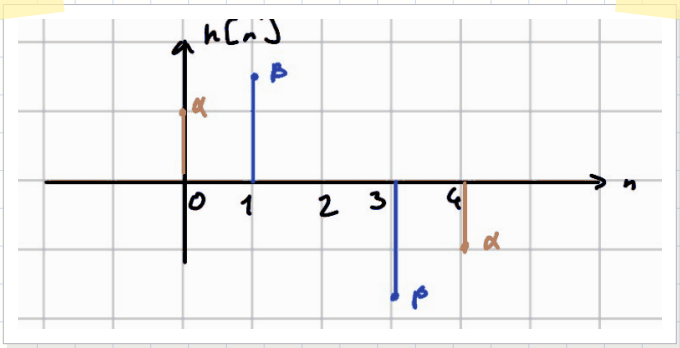
orijinal çizim, s. 10

Type III

$$h[n] = -h[M-n] \qquad 0 \le n \le M$$

M: even integer (çift sayı)

$M = 4$

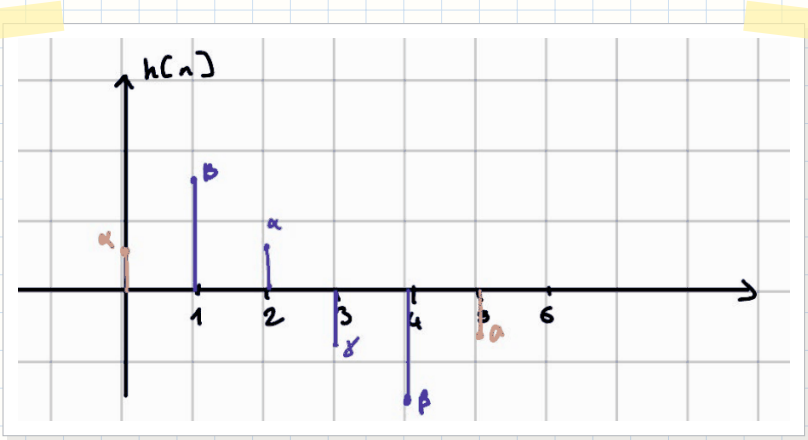


orijinal çizim, s. 10

Type IV

$$h[n] = -h[M-n] \qquad 0 \le n \le M \qquad M: \text{odd integer}$$

$M = 5$



orijinal çizim, s. 10